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EMIL ARTIN, HIS LIFE AND HIS WORK

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Emil Artin died from a heartfailure on December 20, 1962. The mathematical community has lost one of its most distinguished members.

Artin was born on March 3, 1898 as son of an art dealer in Vienna. After his father had died and his mother had remarried she went with her family to Reichenberg in Bohemia where Artin, except for a year's stay in France, attended high school and passed the high school examination in Summer 1916. Shortly after his matriculation at Vienna University he was draughted by the Army with which he served until the end of the first world war. After the war he continued his studies at the University of Leipzig where he studied mathematics, mainly under G. Herglotz, and chemistry. In June 1921, at the age of 23, he was promoted Ph.D. Then he went for a year to Göttingen and afterwards to Hamburg University which had been founded after the war. In July 1923 Artin obtained the Venia legendi for mathematics and was appointed Extraordinarius Eastern 1925, Ordinarius in Fall 1926, at the age of 28. For eleven years Artin together with Hecke and Blaschke directed the activities of the Mathematical Seminar of Hamburg University.

In Fall 1937 Artin emigrated with his wife and family to the United States of America where he was teaching for a year at Notre Dame University, thereafter from 1938 until 1946 at Indiana University, Bloomington, and finally from 1946 until 1958 at Princeton University. Since Fall 1958,he was teaching again at Hamburg University where his life suddenly came to an end while he was still active.

He was honored by many scientific societies. A 1962 honorary doctor's degree of the University of Clermont-Ferrant at the occasion of the tercentenary of Blaise Pascal's death was the last honor bestowed on Artin during his lifetime.

In my memory Artin stands out as a great teacher. Among his pupils were mathematicians from many countries who later became leaders in research and teaching as Max Zorn, Chevalley, Yanaga, Whaples, Thrall, Serge Lang, John Tate and Tim O'Meara.

A teacher in our field can work through many channels of communication: by formal lectures, by research papers, by textbooks, by private conversa-

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tion, and by generating an infectious spirit of doing research in a large group of students, working through a few. Artin was greatest in teaching a graduate class, it was then when he was a professor of mathematics in the most real sense of the word. The influence of Artin's person on the hearer was so powerful and in my memory still is so powerful that it takes a conscious effort to analyze the components of his success both as to the objective as well as to the means.

The saying goes that if Artin said that something was trivial it really was trivial. He coined the expression "enormously simple" which coming from Artin in reality meant the outcome of a life of commitment to working out the truth by a highly organized and restless intellect. Indeed, the effect of listening to a lecture of Artin was to believe the subject on which he lectured was enormously simple, but upon subsequent reflection it became clear that the simplicity was the result of hard work and a life of dedication to pure mathematics.

The intensity of Artin's desire for simplification and uniting various currents of research in algebra and number theory into one mighty stream was so enormous that it ultimately found its expression also in printed form. There are many lecture notes and manuscripts extant through which Artin's influence on his pupils was extended to a wider audience and which, because of their immediacy, freshness,and clarity of presentation deserve separate publication for the benefit of posterity. Besides this extension of Artin's graduate lectures he influenced decisively the basic organization of the books of van der Waerden on Algebra, Zassenhaus on group theory and Tim O'Meara on quadratic forms, moreover Artin's spirit of abstraction had an admittedly strong influence on the Bourbakists.

I was a witness how Artin gradually developed his best known simplification, his proof of the main theorem of Galois theory.

The situation in the thirties was determined by the existence of an already well developed algebraic theory that was initiated by one of the most fiery spirits that ever invented mathematics, the spirit of E. Galois.

But this state of affairs did not satisfy Artin. He took offense of the central role played by the theorem of the existence of a primitive element for finite separable extensions. This statement has no direct relation to the object of the theory which is to investigate the group of an equation, but it was needed at the time as a prerequisite for the proof of the main theorem. I remember many searching conversations of the years 1936 and 1937 prior to his departure to the United States when he discussed with me various possibilities to overcome the obstacle and rejected all sorts of compromises that I brought under consideration.

After having ploughed the field for at least a dozen years in as many courses Artin did two things. Firstly he restated the main theorem in the following form:

Given a field E and a finite group G of automorphisms of E, then the elements of E that are fixed by every element of G form a subfield F of E with the property that every automorphism of E over F belongs to G.

Secondly he proved the theorem by an ingenious application of methods of representation theory (see Artin's *Notre Dame Book on Galois theory*) thus initiating the fusion of structural methods and methods of representation theory in modern algebra. In this way Artin transformed the esoteric remarks of a few experts in the then remote field of representation theory into a cornerstone of the whole theory. Artin at several occasions himself analyzed the known methods of communication of scientific ideas and results. Following now already classical concepts of experimental psychology he distinguished between visual, acoustic and kinesthetic methods of communication corresponding to whether they are directed to perception by sight, hearing or sense of motion. Artin realized himself that he predominantly relied on kinesthetic methods of communication meaning that the emphasis of Artin's lectures was on motivation followed by logical deduction and terminated by a searching examination of the steps used in the logical deduction. For the hearer this meant two things: he was not permitted to rest at any stage to perceive the picture obtained, excepting possibly in the beginning of a lecture, and he gained the impression that right during the lecture he discovered and established deep theorems by a sequence of simple operations.

The appeal to the sense of motion and progressing in time was powerfully supported by the external signs of motion. Artin allowed himself to use the whole platform as a substitute for peripathetic motion, he supported the sequence of sentences, carefully sculptured in time, by enormously expressive gestures of his hands, which in his later years occasionally seemed to invite a particular person in the audience to supply the answer and ultimately succeeded in generating true intellectual perception in almost every one of his hearers.

One of his favorite expressions was: this is enormously simple, meaning that he had succeeded in breaking up a complex structure into a sequence of very simple steps. Let us remember that Artin and his pupils Tate and Chevalley developed the methods of cohomology theory in algebra to their highest known pitch, methods that in Hasse's words consist in a systematic sequence of trivial steps.

I have spoken of Artin's method of communication which largely emphasized kinesthetic perception. However the aim of his demonstration was clarity corresponding to a sharpened vision of the mathematical structures which either he himself had carefully created or which he had recreated from the work of other mathematicians. I remember him often saying: I want you to see this concept in your mind in full clarity (as group, ring, field or ideal). In order to reach his aim, in all his lectures even the ones of merciless abstraction he included numerous carefully worked out examples and ultimately based his speculations on the intimate study of mathematical experience, supported if necessary by electronic computation.

Turning to the influence which Artin had on his colleagues and pupils by his publications it is well known among Artin's pupils that this influence went far beyond what is evident from his papers. For example Artin had produced in 1933 a set of seminar notes on the structure of semi-simple

Lie-algebras over the complex field in which he anticipated Dynkin's monograph by 15 years and in some respect gave a superior presentation of the results of E. Cartan and H. Weyl. These never published notes strongly influenced the work of W. Landherr and of myself. His lecture notes on complex multiplication in reality were a modern text book on the subject which influenced greatly Soehngen's work. In geometry of numbers Artin has no research paper to his credit and yet he influenced pupils like Ankeny and MacBeath very much also in that field of research. His highly original lectures on group theory started my own research in group theory as I have explained in the preface of my book on the subject. Together with P. Scherk Artin simplified the proof of Mann's theorem in additive number theory.

The best known example for the preference that Artin gave to the spoken word over the printed word is the influence which he had in the development of classfield theory which can be fully understood only by viewing many of the main workers in the field taking instruction from Artin at the formative stage of their career. This is documented by example by the thesis of C. Chevalley which faithfully reflects the state of the theory as expounded by Artin in the early thirties filtered through the original mind of the author.

Among the published non algebraic number theory contributions to the field of algebra by Artin I would like to mention the now famous series of three 1927 papers on: *Algebraische Konstruktion reeller Körper, Über die Zerlegung definiter Funktionen in Quadrate* and *Eine Kennseichnung der reell abgeschlossenen Körper* in the Hamburger Abhandlungen in which the Hilbert problem concerning the decomposition of positive definite functions into the sum of squares of rational functions was solved affirmatively.

O. Schreier's and Artin's ingenious characterization of formally real fields as fields in which -1 is not the sum of squares and the ensueing deduction of the existence of an algebraic ordering of such fields started the discipline of real algebra. Really, Artin and his congenial friend and colleague Schreier set out on the daring and successful construction of a bridge between algebra and analysis. In the light of Artin-Schreier's theory the fundamental theorem of algebra truly is an algebraic theorem inasmuch as it states that irreducible polynomials over really closed fields only can be linear or quadratic.

It is interesting to follow the change of attitude that Artin experienced in regard to the question to which extent the problems of construction solved by Artin and Schreier can be done in a finite number of steps. In the thirties Artin was very much aware of the threat of intuitionism to classical mathematics, several pupils of Artin were working on these questions, one of which (A. Hollkott) anticipated in his thesis part of the later work of Tarski and Henkin. However, when I spoke again with Artin in the fifties on the discoveries of G. Kreisel and A. Robinson that indeed Artin's solution of Hilbert's problem could be turned into a finitistic construction he reacted with philosophical calmness and even went so far as preferring a mere existence proof to a construction that required $2^{2^{100}}$ steps.

The two papers on arithmetics of hypercomplex numbers together with Käthe Hey's dissertation under Artin's direction inaugurated the extension of algebraic number theory to non-commutative rings which still offers many fascinating unsolved problems.

In group theory Artin studied the order of the known finite simple groups in two 1955 papers. Based on this study he conjectured that the structure of a simple group apart from one notable exceptional series may be decoded from the order, a conjecture that begins to be verified, at least for the minimal simple groups, by Thompson.

Artin, since the beginning of his research activity, took a very active interest in topology. He invented the notion of braids in mathematics and established the theory of braids in his now classical papers of 1926 and 1947.

Artin was fond of scientific discussion, many times in this country and in Germany he has worked together with other mathematicians. He has stimulated their work by the clarity of his judgement in scientific questions which he exercised with discretion, great depth of insight and always with benevolence. I know of many pupils and colleagues who fondly remember the hours of their life spent in scientific contact with Artin, who thanks to the universality of his studies and the originality of his responses to the work of other mathematicians always seemed to give more than he received.

Artin was truly a scientist-philosopher in the undiminished sense of the 17-th century, the century of Blaise Pascal, René Descartes, Newton and Leibniz.

In Hamburg he gave lectures about general mechanics and about relativity theory in which he carefully discussed both the mathematical theory as well as the outcome of the decisive experiments. He owned a microscope with which he made his own observations in biology, and a telescope the mirror of which he himself had polished to perfection. Through his enduring contact with outstanding astronomers he was well familiar with modern astronomical research and cosmological theories.

From the best recipe for cooking rice to the job of tuning a cembalo to the task of building an organ and the significance of medical research for psychoanalysis Artin always attacked the problem freshly and responded with original solutions.

Since the beginning of his academic office Artin took a lively interest in the problems of teaching mathematics on all levels. This interest showed its first fruits in his outstanding courses on analysis in Hamburg and Bloomington. He showed no mercy for donkey bridges, but with wonderful clarity and patience he understood to guide his students to the full appreciation and mastery of the subtle concepts of limit and continuity, differentiability and integral from the beginning of their studies. His introductory course in analysis which I attended at the age of 17 converted me from a theoretical physicist to a mathematician. I know of several law students who attended regularly the first half of the course in appreciation of the clarity of his presentation.

The now classical book on the introduction to the theory of the gamma function is another outcome of these courses, its exposition being an ideal synthesis of research and teaching.

In Princeton Artin gave outstanding honors courses to mathematics freshmen beyond the call of duty and upon his return to Germany he took active interest in world wide modernization of the teaching of mathematics on the high school level by participation in several projects sponsored by European and Indian organizations.

The 1957 book on geometric algebra is the most mature fruit of Artin's ideas how to modernize the teaching of mathematics from the high school level to the senior undergraduate level, inasmuch as it carries out in original fashion his own recommendations: to place the definition and properties of linear spaces into the center of the theory and to expound geometry and algebra as two sides of a unified structure.

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