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LOGICAL DETERMINISM

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The question as to whether we are really free, have a genuine or only an apparent freedom of choice, has haunted and bewildered the philosophers of ancient and modern times. According to Friedrich Waismann, this question becomes a disturbing philosophical puzzle because it is emotionally charged with an obsessional anxiety whose source is the doctrine of logical determinism or logical predestination. Logical determinism is the view which states that the "entire future is somehow fixed, logically preordained."[1] To put it differently, logical determinism asserts that strict determinism, known and discussed in ancient Greece and reformulated in modern times by Laplace in his *Essai philosophique sur les probabilités*, inevitably follows from the principle of logical bivalence. Although Waismann referred to the law of the excluded middle, he actually had in mind either the metalogical (semantic) formulation of this law:

$$\mathbf{V}`p' \,.\, \mathsf{v} \,.\, \mathbf{F}`p' \,,$$

or the principle of bivalence:

$$[p] : p \,.\, \mathsf{v} \,.\, \sim(p) \,,$$

every proposition is either true or false: it has one and only one of two possible truth-values—truth and falsity.[2] Logical determinism claims that if the principle of logical bivalence is accepted, as it must be, the Laplacean conclusion can be inferred from it: the dynamic state of the world at the instant t_n, which is the effect of or is strictly determined by its state at every instant earlier than t_n, is the cause of, or strictly determines, the state of the world at every instant later than t_n.[3]

There are two additional assumptions which seem to underlie the argument for logical determinism. The first of them is the Aristotelian or so-called classical conception of truth. According to this conception, a proposition is true if it is in agreement with reality or if there is a fact to which it corresponds, or if it designates an existing state of affairs. The second assumption has to do with the timelessness or absolute character of truth. As the schoolmen put it, *veritas significata per aliquam complexionem est*

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aeterna. A true proposition, if it is true at all, is true once and for all; that is, it is true irrespective of whether it is ever expressed or not, and independent of the time at which it is expressed. Similarly, a false proposition, if it is false at all, is false once and for all. These two assumptions should be included in the premisses of the argument in favor of logical determinism.

The argument itself might be briefly presented in the following way. If for every p it is always true, i.e., at any time t, that either p or Np, then either p or Np at any time earlier and later than t. But if 'p' is true at any time t, what p states must have been determined in advance and have causes existing from eternity. For otherwise 'p' could not be true at some arbitrary time. A proposition is true in the Aristotelian sense of this term, if there is some fact to which it corresponds and which makes it true. A state of affairs existent at a specified time, e.g., Socrates' death in Athens in 399 B.C., cannot necessitate the truth of the respective proposition at a time earlier than a definite date on a certain day in 399 B.C. Therefore, if the statement 'Socrates died in Athens in 399 B.C.' is always true, Socrates' death in 399 B.C. must have been strictly determined prior to his death that year. In other words, if the proposition 'Socrates died in Athens in 399 B.C.' is to be true before Socrates died at time t, Socrates' death in 399 B.C. must have been laid down far in advance and eternally. Similarly, if it is true to say now that this pod of peas, which I hold in my hand, contains five peas, and this has been and remains true for ever, then the fact expressed by the sentence 'this pod of peas contains five peas' must be uniquely determined *ab aeterno.* It is a cause or a sequence of causes existing from all eternity which is a real correlate of a true proposition and makes it timelessly true. If it is true at all, it was true when the present cosmic cycle began and will be true when the cosmic cycle completes its course.

The same applies to any statement about future events. Such a statement p does not refer to any fact when made, and, consequently, can be neither true nor false unless there are some other facts which at any earlier time t determine the occurrence of the state of affairs designated by p. If 'p' is true today, this state of affairs must be determined in advance by some causes existing today. It is a contradiction to assume that every proposition is either true or false, true once and for all or false once and for all, and at the same time to reject the conclusion that every event is causally determined in a unique way by a sequence of antecedent events. Therefore, not only *quod fuit non potest non fuisse*, but also what is to be cannot not occur. Whatever happens always was to happen, and whatever is to be, will have to be. Both the past and the future are strictly determined by unlimited causal chains extending in both directions of time.

If every proposition about future contingent events is either true or false *ab aeterno*, the statement 'I will be in Warsaw in December next year' is either true or false at any time t. Let us assume that this statement is true. According to the doctrine of logical determinism nothing can prevent me from being in Warsaw in December next year. For if 'I will be in Warsaw in December next year' is a true statement at an arbitrary time t, there is a natural cause that determines my being in Warsaw next December. No act of mine can have any more effect upon the future than it has upon the past and it is not possible for me not to be in Warsaw at the appointed time. On the other hand, if the statement under discussion is false, whatever efforts are made to secure my presence in Warsaw in December next year, they are bound to fail. For if an event is not to occur, there is nothing that can cause it to happen. We can only fulfill what was to be, we cannot avert what is predetermined. The course of human life remains unaffected by any action or failure to act. An outside observer with a complete knowledge about the totality of conditions which determine the behavior of an individual can predict any of his actions in advance.

Fatalism is the view which states that the whole course of human life is unambiguously pre-determined. Waismann was right to regard 'logical determinism' and 'logical predestination' as closely related expressions. If the principle of bivalence entails strict determinism and strict determinism entails fatalism, the principle of bivalence entails fatalism.

II

It was Aristotle who (in chapter 9 of *De Interpretatione*) was the first to notice that fatalism seems to be deducible from the principle of bivalence. In our time Aristotle's arguments were recalled by Jan Łukasiewicz in an article published in 1930[4] and since then often discussed by many writers, particularly in recent years.[5] According to Łukasiewicz, Aristotle believed that strict determinism inevitably follows from the principle of bivalence, or, more precisely, from the metalogical formulation of the law of the excluded middle. But Aristotle did not consider the thesis of strict determinism to be true, and, consequently, restricted the universal validity of the principle of bivalence. (*De Int.* 19a 36) However, since he expressed his views in an extremely condensed form, he failed to make this restriction clear, and was either misunderstood (with the notable exception of the Epicureans and the Stoics) or misinterpreted up until the discovery of the trivalent and later the polyvalent systems of logic in this century.[6]

Aristotle argued in the following way. If all statements, whether positive or negative, are either true or false:

$$\mathbf{V}`\varphi x' \,.\, \mathrm{v} \,.\, \mathbf{V}`\sim\varphi x' \,.\, \mathbf{F}`\varphi x' \,.\, \mathrm{v} \,.\, \mathbf{F}`\sim\varphi x' \,,$$

it is necessary that a given event to which a statement refers either does or does not occur:

$$(x, \varphi)\,(\varphi x \,.\, \mathrm{v} \,.\sim \varphi x)\,.$$

For if every affirmation and negation is either true or false, then it is necessary for what is affirmed to be the case and for what is negated not to be the case. But if it is necessary for something either to occur or not to occur, for instance, to-morrow's sea battle, then it is impossible that it could not occur, or, respectively, that it could occur. Therefore, if all positive or negative statements are either true or false, nothing is or takes place fortuitously. This conclusion is, however, false; for it cannot be said without qualification that all that is or takes place is the outcome of necessity.

If some things do not happen of necessity, the disjunction of two statements, of which one affirms what the other denies, is a necessary truth; that is, a law of logic. But neither of its component propositions is thereby necessarily true or necessarily false. For if the propositions refer to a contingent future event, the affirmation may be neither more true nor more false than the negation. The disjunction: 'either there will be a sea battle to-morrow or there will not be a sea-battle to-morrow' is a necessary truth, a logically true proposition; symbolically, L'*ApNp*'. But it is not necessary that there will be a sea battle to-morrow (*NLp*) and it is not necessary that there will not be a sea battle to-morrow (*NLNp*).[7] To put it differently, while the equivalence:

$$ELApqALpLq$$

is valid, neither *Lp* nor *Lq* can be derived from it, and, within Łukasiewicz's axiomatized basic modal logic, *Lp* and *Lq* are not even assertable. There is, therefore, no contradiction in accepting *LApNp* and asserting the conjunction *KNLpNLNp* or each of its components. Thus, the conclusion that either of the statements 'there will be a sea battle to-morrow' or 'there will not be a sea battle to-morrow' is a true statement today is also invalid. The alternative must be left undecided (*De Int.* 19a 30). One of its components may indeed be more likely to be true than the other (or, as A. N. Prior suggested, it might differ from the other by a finite or infinite sequence of probability grades), but it cannot "be either actually true or actually false." The conclusion: either it is true that *p* or it is true that *Np*, cannot be inferred from the premiss: it is true that *p* or *Np*. The inference would be valid if the principle: every proposition has one and only one of two possible truth-values—truth and falsity, were assumed. But Aristotle did not accept this assumption without restriction.[8]

On the other hand, we cannot prove the theorem: every proposition is either true or false (a), from the definition of the alternative: '*Apq*' is false if and only if both its components are false (b), and from that of the negation: the negation of a true proposition is false and the negation of a false proposition is true (c). To establish (a) by means of (b) and (c) we need another premiss, namely that every proposition has at least and at most one truth-value. Should we, however, accept (a), (b) and (c), we can easily derive from them both the metalogical and the logical principle of the excluded middle. "For either *p* is true and then the principle '*ApNp*' is true according to (b), or *p* is false and then this principle is true according to (b), because *Np* is true according to (c)".[9] Nor is the principle of bivalence inferentially equivalent to the metalogical principle of non-contradiction: two contradictory propositions are never true together (d), and the metalogical principle of the excluded middle: two contradictory propositions are never false together (e). For the conjunction of (d) and (e) does not exclude the possibility that both *p* and *Np* are neither true nor false. We need again the additional premiss: every proposition has one and only one of two possible truth-values, truth and falsity (f). Aristotle's modern critics having tacitly assumed either (a) or (f) arrived at the conclusion that either (b) and (c) or (d) and (e) logically imply the principle of bivalence.[10]

To return to Aristotle, he seems to have believed that the principle of bivalence entails strict determinism and fatalism, but he considered strict determinism and fatalism to be false views. If whatever happens comes about by necessity, contingent or ambivalently possible events, such as to-morrow's sea battle, which can but also can not take place, are not among the articles of the world's furniture. Aristotle rejected this conclusion, for not everything is decided in advance. Things in which there is a "potentiality in either direction" may either be or not be and an event which "inclines this way no more than that" may either take place or not take place. Therefore, there are contingent propositions, which are neither as yet true nor as yet false; the division of propositions into true and false is not exhaustive; and the metalogical law of the excluded middle is not universally valid, for it does not apply to some propositions about the future. But the law of the excluded middle does apply to all propositions. If we call the logic of propositions which includes the *tertium non datur* theorem a bivalent logic of propositions, a view widely held, although not correct, we should include Aristotle among the adherents of the two-valued logic. On the other hand, Aristotle rejected the metalogical law of the excluded middle, and, thus, went beyond the bivalent logic of propositions in the proper meaning of this term. Łukasiewicz's view, which is now contested, that Aristotle actually restricted the validity of the principle of bivalence, appears to be justified. For one conclusion seems to follow clearly from Aristotle's examinations in chapter 9 of *De Interpretatione*, namely, that a complete system of the logic of propositions cannot be constructed on the assumption that all propositions are either true or false.

Aristotle was a logical determinist in the sense that he did not question the validity of the inference which derives from the principle of logical bivalence the conclusion: the world is subject to strict determinism and human destiny to fatalism. But he did question the material truth of this conclusion, its agreement with reality, and thus rejected logical determinism as a false belief. The Stoics accepted both the truth of the premiss and of the conclusion, thus becoming supporters of logical determinism.

According to Cicero, the Stoics argued that it is self-contradictory to assume that every proposition is either true or false and to deny that all events are strictly determined, "spring from eternal causes governing future events." First, the assumption that every statement is either true or false implies that there are natural causes which exist from all eternity and prevent anything from taking place otherwise than it will take place. If a statement corresponds to a state of affairs, this state of affairs must have its cause or causes which are its sufficient and necessary conditions. For to a thing or event not possessing efficient causes there would correspond a statement neither true nor false and this contradicts the assumption.

Therefore whatever is, had to be and whatever will be, must come about of necessity. It is also false to assume that anything takes place without a cause. For this would entail the conclusion that something comes out of nothing, and this is absurd. Since truth consists in the conformity of thought to reality, and whatever is or will be results from an eternal chain of causes, every proposition is either true or false. Thus, the principle of bivalence and the principle of strict determinism are inferentially equivalent. It is impossible to accept the one and to reject the other without contradiction.[11]

Recently the view has been voiced that the libertarians and some logicians, among whom Łukasiewicz alone is mentioned by name, still adhere to the Stoic argument. Since they accept the validity of the inference which derives strict determinism from the principle of bivalence and fatalism from strict determinism, they reject the principle of bivalence in order to escape the conclusion of fatalism. "This sort of argument", wrote Waismann, "was actually propounded by Łukasiewicz in favour of a three-valued logic with 'possible' as a third truth-value alongside 'true' and 'false'".[12]

Waismann's statement might be justly applied to some supporters of a trivalent logic[13], but it is unjustifiable with respect to Łukasiewicz. Łukasiewicz did believe that strict determinism implies fatalism. If all that takes place is the outcome of necessity, there are no chance events, no Aristotelian "real alternatives" (contingency), and, as the poet said, "the book of Nature is the book of Fate." To accept unbroken causal continuity entails the admission that no man could ever have acted otherwise than he actually did. It would be inconsistent to assert the former and to reject the latter. Łukasiewicz, following in Aristotle's footsteps, regarded as false the view that the behavior of the individual is uniquely determined, and, consequently, doubted the truth of strict determinism. His arguments did not differ from those which Cicero had used against the Stoics[14], that is, fatalism is incompatible with human experience and common knowledge. If strict determinism is a valid hypothesis, Łukasiewicz wrote, there is no room in the world for "creative action, which does not result from a law but from a spontaneous impulse. Impulses too, would be subject to laws, they would occur of necessity, and could be predicted by an omniscient being. All my actions would have been preordained before I was born into the world."[15] Now, Łukasiewicz believed that neither he nor anybody else did follow a path in life fixed in advance, that is, that no omniscient observer could predict every decision made by him. Whatever else acting of one's own free-will may mean, it involves multiple choice and unpredictability. Both these concepts could not be consistently used, if every event were uniquely causally determined by a chain of antecedent events. Such a chain would offer to the performer no alternative or multiple, but always a single choice, fully predictable to an omniscient being in complete knowledge of all the laws, the state of the environment and the particular man concerned. Waismann was right in saying that the falsity of fatalism led Łukasiewicz to the rejection or restriction of the idea of a strict causal determination of all natural events.

Strict determinism is a special interpretation of the principle of causality. To reject strict causal determinism does not entail that this principle and every other kind of determination have also to be rejected. For we can accept the existence of causal bonds without accepting them as the only possible kind of determination or without endorsing the view that everything has causes from all eternity. This was the standpoint to which Łukasiewicz adhered and which he called *indeterminism.* This term is perhaps misleading in view of various uses of 'indeterminism' in the history of thought, including the denial of every kind of natural orderliness of events or the insistence on the absence of determinate conditions for their occurrence, but is used in this sense also by others.[16] It is true that to account for creative action it is not enough to deny the universal validity of strict causal determinism. While according to some philosophers compulsion can be derived from necessity and chance from contingency, it is surely false to derive indeterminism from chance. But strict causal determinism implies that there is no creative action. This was the point which Łukasiewicz, and, before him Tadeusz Kotarbiński–another philosopher greatly concerned with the relations between the principle of bivalence, strict determinism and freedom of the will–tried to establish and to explore in its logical implications.

On the other hand, Łukasiewicz did not argue, as Waismann suggested, that either the principle of bivalence or the principle of trivalence is universally valid; that the principle of bivalence is false, since it entails false conclusions, namely strict determinism; and that consequently the principle of trivalence should replace that of bivalence. For obvious reasons such an apagogic proof would be unsound. It is true that the incompatible implications of strict determinism and of indeterminism, in which Łukasiewicz was deeply interested throughout his life, provided one of the psychological motives prompting him to go beyond the two-valued logic. This is, however, only a biographical detail. The avowed personal interest of the inquirer which inspires his investigations in a certain direction is irrelevant for the evaluation of the logical validity of the results obtained.

It is also true to say that in Łukasiewicz's mind the problem of strict determinism and indeterminism was somehow associated with that of the number of truth-values admitted in the various systems of the propositional calculus and that he believed that he would be able to elucidate the relation between these two sets of problems by his investigations of polyvalent logics. For he believed that the principle of bivalence and strict determinism on the one hand, the principle of trivalence and indeterminism on the other, are related in some way. The relation in question was not, however, that accepted either by Aristotle or by the Stoics. Łukasiewicz did not share either Aristotle's view that the truth of the principle of bivalence involves the truth of strict determinism or the belief of the Stoics who had considered the principle of bivalence and strict determinism to be inferentially equivalent.[17]

This is also clear from the history of the systems of polyvalent logics. The first of these systems was constructed in connection with the investigations concerned with the concepts of logical possibility and necessity and with the unsuccessful attempts to establish a consistent formalized system of modal logic based on the principle of bivalence. In his investigations Łukasiewicz used exclusively the methods of formal logic and the polyvalent systems were defined by him by means of the matrix method. Słupecki, Sobociński and Wajsberg then proceeded to apply the axiomatic method to the system $\mathbf{L}_3$ of the propositional calculus, and they succeeded in finding an independent axiomatic basis of the complete and consistent system $\mathbf{L}_3$. In a summary of a lecture given to the Polish Philosophical Society in Lwów in 1920, in which Łukasiewicz announced the discovery of the trivalent propositional calculus, he emphasized that this system "has, above all, a theoretical significance as a first attempt to construct a system of non-Aristotelian logic." Whether it may be shown to have also a "practical significance" cannot be decided until the consequences of the principle of trivalence are investigated in their relation to empirical knowledge.[18] The polyvalent systems of the propositional calculus, wrote Łukasiewicz a few years later, "do not form part of any philosophical doctrine with rejection of which they would have to be abandoned; they are as much an outcome of objective investigations as any established mathematical theory".[19] The question of the application of the trivalent system of logic, of finding a set of objects in which the axioms of this system are satisfied, is a distinct problem and independent of the theoretical discovery which should be judged by itself, irrespective of its application.

The emphasis laid upon the statement that the polyvalent systems of the propositional calculus do not contain, explicitly or implicitly, any definite philosophical outlook was due to the fact that Łukasiewicz was accused at that time, though not in Poland, of advocating non-Aristotelian logics merely because of their alleged usefulness in justifying the belief in the freedom of the will.[20] To answer this objection Łukasiewicz stated over and over again that the polyvalent systems remain abstract structures as long as a valid interpretation of their axioms is not found. In particular, since it turned out that modal sentences do not provide a model of his trivalent system of the propositional calculus, the demonstration of the existence of propositions which are in fact neither true nor false must be given.

This problem of interpretation and verification is not, however, a problem of logic alone; it can only be resolved by the investigations of the ontological structure of reality. Every philosophical theory of logic makes some ontological assumptions. Łukasiewicz believed that the ontological assumptions underlying various systems of logic may one day become testable, the test being conducted in the way familiar to us from natural science.[21] He rejected the view which regards logic and mathematics as systems of materially empty tautologies, which disclose nothing about the world, and, in particular, Carnap's principle of tolerance. "I believe", Łukasiewicz wrote, "that one and only one system of logic is really valid, that is, is satisfied in the real world, as much as only one system of geometry is really valid." It is experience that must resolve the issue as to whether the relations between facts, states of affairs or events satisfy the axioms of the bivalent or of one of the polyvalent systems of the propositional calculus. Łukasiewicz was inclined to think that this issue might be resolved, if natural science succeeds in providing decisive evidence either for strict determinism or indeterminism.[22]

It is clear, therefore, that Łukasiewicz's original position does not contain the erroneous views which his critics ascribed to him. He was not prompted by the fear that the principle of bivalence makes him a hostage of fate.[23] He did not subscribe to the Aristotelian or Stoic views that if every proposition is either true or false, all events are strictly determined by an eternal chain of antecedent causes. For the consequent of this conditional cannot be established by logic alone. An event cannot be made to occur by the true antecedent statement that it was going to take place. By making use of the metalogical principle of identity, non-contradiction, and of the excluded middle, in the manner of Aristotle and the Stoics we can reach the conclusion that the future is logically determinate. But this conclusion does not imply that the future is thereby uniquely causally determined or fated. Such an inference would be based on the confusion of causal determination with logical determinateness; of the logical necessity involved in descriptions and inferences in terms of an accepted logical theory with causal necessity which refers to an exceptionless sequential or concomitant order of events and thus explains why concrete events succeed each other as they do. Logical necessity consists in the absence of choice in the use of the formation and transformation rules applied to symbolic expressions. When we wish to describe the world according to a particular system of logic, we have to adhere to the rules of this system. The question then arises as to whether the descriptions of events in terms of the accepted logical theory represent only a possible way of their coincidence and succession or that in which they do follow each other. For the methods of description vary in accordance with the accepted logical theory and we are faced by a real choice in deciding which of the logical theories available conforms and is appropriate to the structure of reality. To find this out we have to go beyond logic and be guided by experience, which alone can confirm the validity of the choice. Since the choice of the system of logic determines the limits of the possibilities and restricts the domain of possible experience, we wish to adopt a logical system such as would allow the possibilities in agreement with facts and exclude those which do not correspond to facts.

III

Following Waismann's definition, we shall call *logical determinism* the view according to which the principle of bivalence entails strict determinism. It should be clear in the light of the foregoing remarks that Łukasiewicz was not a logical determinist. His investigations of the problems involved deserve, however, a closer examination because of their inherent interest and wider implications.

The dispute among the ancient philosophers over the principle of bivalence indicates that the problems concerning its universal validity involve both logical and ontological issues. While those supporting the universal validity of the principle of bivalence were adherents of strict determinism, the philosophers who advocated its restriction were inclined to reject strict determinism. The close connection between the two problems reflected the ontological conception of logic prevailing at that time, in sharp contrast to the formalistic conception of logic dominant in the last forty or fifty years.

According to the ontological interpretation, while logic deals with the forms of valid inference it also reveals the ontological structure of reality. This assumption underlies the ancient logic and has also inspired some modern logicians; for instance, Bertrand Russell (at the early stage of his career), Brouwer, Leśniewski and Łukasiewicz. Moreover, the ancient view that the restriction or rejection of strict determinism requires the adoption of a polyvalent logic in our methods of describing the world has been revived and has found recently an increasing number of eminent supporters among philosophers and scientists.

This trend has been opposed by arguments not easy to refute. Thus it is claimed that no formalized logical system can ultimately do without some non-formalized reasoning and this reasoning involves an intuitive residual logic that is bivalent. Consequently, if we accept the view that different systems of logic determine a varying range of permissible descriptions and that ultimately we cannot escape the bivalent logic, we are driven to the conclusion that it is reality which imposes upon us the acceptance of the principle of bivalence.[24] This, however, confronts us with other difficulties originating from the conception of the timelessness or the absolute character of truth which involves us in certain ontological implications as difficult to accept as the eternal causes of strict determinism. The thesis of the eternity of truth presupposes the ontological belief that the totality of facts or events is "spread out eternally", as it was put, "in the dimension of time as well as the dimensions of space."[25] Upon a closer examination the recent discussions of logical determinism raise a whole complex of problems difficult to disentangle and to elucidate. Like in ancient Greece, this complex of problems combines logical and ontological questions.

To clear the ground for further discussion let us return to logical determinism. In one of his early contributions (1907) Łukasiewicz examined and rejected the view that from an implication of determinate sentences, reduced to the form: Cpq, the conclusion can be inferred that the state of affairs designated by p is the cause of the state of affairs designated by q.[26] We cannot infer causal relationships between events from inference relations between respective statements, nor physical necessity from logical necessity. If this were the case, the best method of discovering causal relationships of events would consist in discovering that some propositions entail some other propositions, and this consequence is false. Causal relationships hold solely between real objects, for only a real object can be a cause or an effect. On the other hand, only certain abstract objects can satisfy the relation of inference. Causal relationships and logical relations apply to objects of different sorts. If an object can be either cause or effect, it cannot be either premiss or conclusion. While one truth can entail or be entailed by another truth, truths cannot either make or fail to make one event follow another event.

The logical connection between causal relationships and inference relations is asymmetrical. It allows us to establish the relation of consequence between the statements which designate certain states of affairs, provided that a causal interdependence obtaining between these states of affairs is known, but its converse fails to hold. The conditional: 'if a stone hits against a window-pane, the glass is broken', states what happens if and when certain causally determined events occur. We do not accept this occurrence because of having discovered the inference relation between the antecedent and the consequent of the conditional, but because we have discovered the described succession of events, we accept the logical relation between the respective statements.

Necessary relations which hold between real objects cannot be deduced from those logically established. The schema:

$$(x)\,(\varphi\, x \supset \psi\, x)\ ,$$

may signify some causal interdependence and be expressed by the statement: the state of affairs designated by the antecedent is the sufficient condition or cause of the state of affairs designated by the consequent, and the state of affairs designated by the consequent is a necessary condition or effect of the state of affairs designated by the antecedent. The necessary relations between cause and effect on the one hand, between premiss and conclusion on the other, share certain formal characteristics. These common characteristics account for the fact that a causal relationship may be expressed in terms of a causal implication, and a logical relation in those of a formal or material implication. While having some characteristics in common, they differ in others. It is true for both causal and material implications that

$$CCpqNKpNq\ ,$$

but the thesis:

$$CNKpNqCpq$$

is clearly false, if '*C*' stands for causal implication.[27] If p causally implies q, p implies q materially. But from the material implication: Cpq, we cannot deduce that p causally implies q. For in this case the thesis:

$$CNKpNqCpq$$

would apply to both material and causal implications, and this conclusion has been shown to be false.

We could, therefore, possibly deduce from the premiss of strict determinism the conclusion concerning the universal validity of the principle of bivalence. We could, for instance, argue that if the future is as much determined as the past, every proposition is either true or false. But the assumption that every proposition is either true or false does not involve the truth of the proposition that there exist eternally immutable causes which strictly determine every event in advance. The principle of bivalence and of strict determinism are not, as the Stoics asserted, inferentially equivalent.

On the other hand, if we assume that not every event is uniquely determined by a causal chain existing from all eternity, statements about some future events are neither true nor false at the time when they are made, though they will become true or false at the instant when the respective events actually occur or fail to occur. It does not follow from the admission that some statements become true or false that a true statement might change into a false one or a false statement into a true one; more generally, that what is true at one time may not be true at another. For it has been assumed that the statements about the future are neither true nor false at the moment of their utterance. When they become true or false, they are true or false once and for all (this would not apply to the indeterminate statements of Quantum Mechanics). They are not, however, "eternal truths" or "eternal falsehoods", since there is such a definite time *t* that at any instant earlier than *t* the statements considered are neither true nor false and at any instant later than *t* they are either true or false once and for all. Consequently, the principle: every proposition is either true or false at any time *t*, is not universally valid.

We can, of course, adopt the view that if a statement is neither true nor false at time *t*, it is no statement at all, or, as C. D. Broad put it, at most enjoys this name as a "courtesy title by anticipation, like the oldest sons of the higher nobility during the lifetime of their fathers".[28] But this approach seems to be unrewarding or merely therapeutic; it makes use of a nominal definition to declare that the problem involved is no problem at all; it administers a pain relieving drug without removing the cause of pain. For statements neither true nor false are not a fictitious invention of the logician or an unintelligible phantasy of the metaphysician.

If we reject the suggestion that statements in the future tense expressing no propositions are not genuine statements and for this reason alone they are neither true nor false (like the imperative: 'close the door, please' is neither true nor false being no declarative sentence), viz. that it is a syntactic error to attribute truth and falsehood to them, or that these statements, contrary to all appearances, do not convey singular but only general propositions,[29] we have to follow the course adopted by Kotarbiński and later by Łukasiewicz. They both believed that besides statements which are either true or false there are also statements which are neither true nor false, the "bilaterally possible", the "undecided", or the "statements of the third kind." The latter have a truth-value different from truth and falsity, and refer to what can best be described as ambivalent possibility. "If a 'statement of the third kind' is true", wrote Kotarbiński, "both the positive and the negative statements are neither true nor false; if the 'statement of the third kind' is not true, either of the two contradictory statements, the positive or the negative, is true."[30] The principle of the excluded middle only states that contradictory statements cannot be false together. This does not imply, however, that one of them must be true (a view often voiced even today). For if a proposition is not false, this does not necessarily mean that it is true; nor is it necessarily false if it is not true. This again is often asserted today[31] without making clear that these equivalences are valid if and only if the principle of logical bivalence is assumed, and not otherwise. Kotarbiński saw through this and concluded: "Every proposition is either true or false or neither true nor false and *Quartum non datur.*"[32]

Łukasiewicz, like Kotarbiński before him, starts from an assumption entirely different from that of logical determinism. His assumption is an ontological hypothesis, namely that both necessary events and ambivalent possibilities are articles of the world's furniture. From this premiss he derives the conclusion that besides true and false statements there must also be those of the "third kind", neither true nor false at the time when they are made.

The ontological assumption can also be interpreted as the rejection of causalism—causation is the only mode of determination—and as a restriction of strict determinism. The view that there are some observable physical entities for which no uniquely fixed predictions are possible in principle (that is, they are contingent), is known in present-day terminology as indeterminism. In the case of Łukasiewicz's discovery of polyvalent logics, it was the admission of the indeterministic structure of the world which constituted the ontological background of the new logic. We are bound to draw the conclusion that the dichotomous division of propositions into true and false is exhaustive if and only if the hypothesis of strict determinism is valid.[33] On the other hand, if this hypothesis is rejected or revised and restricted, there is no *a priori* reason why the dichotomy of truth and falsity should be accepted. An indeterministic world picture admits the possibility of some propositions being neither true nor false.

Łukasiewicz made the decisive step forward when he went beyond Aristotle's statement concerning "real alternatives, in which case the affirmation is no more true and no more false than the denial" and constructed by means of the matrix method a system of trivalent logic.[34] This system, being consistent, showed that we can operate with the "statements of the third kind" and thus became a logical apparatus of the indeterministic world picture. If this world picture presupposes a trichotomous division of propositions (into true, false and neither true nor false), an indeterministic world picture remains an unsatisfactory intellectual structure as long as there is no proof that we can make use of the "statements of the third kind" without contradicting ourselves. The construction of a trivalent calculus of propositions resolved this issue and demonstrated the formal consistency of an indeterministic world picture. But its formal consistency is insufficient for establishing its material truth, which can only be shown if experience confirms the consequences deduced from the hypothesis of indeterminism. Łukasiewicz believed that in principle this can be done, although he doubted as to whether any decisive test is technically feasible in the present state of knowledge. In this respect there is, however, no substantial difference between strict determinism and indeterminism. We adopt the one or the other according to our belief as to which of them is better confirmed and the relative strength of evidence in favor of either of these decisions continues to be a highly disputable question.

Two points seem to emerge from the preceding discussion. First, the universal character of the principle of bivalence is a necessary condition for the validity of strict determinism; contrary to what logical determinism asserts, it is not its sufficient condition. The truth of the principle of bivalence does not involve the truth of strict determinism. Second, the principle of trivalence is a necessary condition for the validity of indeterminism; it is not, however, its sufficient condition. The dispute between strict determinism and indeterminism is not thus resolved, for they are both hypotheses with an empirical meaning. But the demonstration that the principle of bivalence is not an absolute principle of logic is of considerable significance. While it does not resolve the underlying ontological dispute, it allows us to formulate precisely the differences of a logical nature involved in the dispute and to show that logically both strict determinism and indeterminism are consistent and legitimate. As Łukasiewicz put it, the construction of a consistent trivalent system of propositions makes it clear that strict determinism is not logically a better established world picture than is that of indeterminism.[35]

IV

It is now a matter of crucial importance to demonstrate the existence of propositions which are in fact neither true nor false, and once having found such, to provide the proof that they cannot be accommodated within the bivalent system of propositions.

Let us assume that statements about future contingent events are instances of the propositions of the required sort. In the definition of contingent statements the modal concepts of necessity and impossibility are involved. We now know that a consistent formalized system of modal logic has to be polyvalent. The first proof to this effect was given by Łukasiewicz in *Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls*, where he demonstrated by means of the matrix method that the functors *M* and *L* have no interpretation in two-valued logic (this proof has been greatly simplified in Łukasiewicz 1957, §49). He also tried to show that the traditional modal logic can be successfully formalized on the basis of the principle of trivalence. Later this last attempt proved to be abortive, for Łukasiewicz himself and others showed that the traditional modal logic is not a possible realization in which the trivalent system of propositions is satisfied (if the intuitively grasped meanings associated with modal concepts are not to be abandoned). Serious doubts were also expressed as to whether a more satisfactory result had been obtained in another attempt of Łukasiewicz, published in 1953, in which he tried to show that the traditional modal logic is a model of a certain four-valued system of propositions. These developments are not, however, of direct relevance to the problem under discussion. The essential point is that contingent propositions can only be defined in terms of modal concepts and that a consistent system of modal logic cannot be based on the principle of bivalence.

Let us now return to the assumption that statements about future contingent events are in fact neither true nor false. We have to show that these statements themselves are such that if we applied to them the accepted procedure by means of which the truth and falsity of a statement is verified, they would reveal themselves as neither true nor false. To put it differently, what is required is the demonstration that contingent statements are in fact neither true nor false, and thus not owing to our lack of knowledge whether they are true or false. Logic does not deal with statements of the latter category.[36]

The demonstration given by Łukasiewicz proceeds as follows:

> I can assume without inconsistency that my presence in Warsaw at a definite date of next year, for instance, at noon on December 21, is not decided, either in a positive or in a negative sense, today. It is, therefore, *possible* but not *necessary* that I shall be in Warsaw at the appointed time. On this assumption, the statement 'I shall be in Warsaw at noon on December 21 next year', is neither true nor false. If it were true today, then my future presence in Warsaw would have to be necessary, and this contradicts the assumption; and if it were false today, then my future presence in Warsaw would have to be impossible, and this also contradicts the assumption. The statement under discussion is, therefore, neither *true* nor *false* today, and must have a third value different from '0' or falsity and from '1' or truth. We can denote this value by '1/2'; it is exactly the 'possible', which turns up as a third value beside the 'false' and the 'true'".[37]

This argument offers one considerable difficulty in connection with the meaning to be attached to the terms 'necessary' and 'impossible'. How should we understand them in the sentence 'If it were true today, then my future presence in Warsaw would have to be necessary, and this contradicts the assumption; and if it were false today, then my future presence in Warsaw would have to be impossible, and this also contradicts the assumption'?

It is a view of Aristotle that some true statements are necessary and some false statements are impossible. This view results from Aristotle's ontology and is connected with his theory of modal statements. According to Aristotle, the modal functors are not sentence connectives (proposition-forming functors of one propositional variable); they are operators qualifying the inherence relation between the characteristics of things or facts denoted by the subject and predicate of a modal sentence. A statement which attributes an essential characteristic to the object designated by the subject is not only true, but also necessary or necessarily true (see, e.g. *An. Post.* 74b 5-10). Let 'L' stand for 'it is necessary that. . .' and 'α' for a propositional variable, the values of which are sentences attributing essential characteristics to objects. Aristotle's rule can then be written down symbolically:

$$\alpha \longrightarrow L\alpha,$$

α, therefore it is necessary that α.

Aristotle's view that some true (analytic) propositions are necessary and some false propositions are impossible has frequently and in varying form been voiced by ancient, modern and contemporary philosophers. Łukasiewicz too, seemed to have embraced it at some time of his life only to reject it later. "True propositions are simply true without being necessary and false propositions are simply false without being impossible."[38] But also as far as his earlier views are concerned, it cannot be assumed that in the argument under discussion he applied as universally valid the rules: 'If it is true that p, it is necessary that p' and 'If it is false that p, it is impossible that p'. For he was aware at that time that the formula: $CpLp$, must be rejected. It must be rejected for otherwise modal logic would collapse. If we assume the expression: $CpLp$, we can easily derive from it the conclusion that p is equivalent to Mp.[39]

We cannot assume that the crucial point in the argument under consideration involves an inference from the logical necessity to the physical necessity. This follows directly from the preceding examinations of the Aristotelian and Stoics beliefs that the truth of the principle of bivalence entails the truth of strict determinism. As Donald C. Williams put it, to be determined is to be necessitated in some way, and to be determinate is merely to be definite or completely characterized. "Events cannot be determined without being determinate. . . But events can be determinate without being determined."[40] Łukasiewicz's argument does not apply the clearly erroneous rule according to which from the assumption that it is true today that 'I shall be in Warsaw next year' the conclusion is deduced that my future presence in Warsaw is factually necessary.[41] Truths do not make events to take place, events are not caused by the truth of antecedent statements or of predictive prognoses that they would occur. Only a logical determinist could possibly argue this way, and it has been shown earlier that Łukasiewicz was not a logical determinist.

The conditional: if the sentence 'I shall be in Warsaw at noon on December 21 next year' is true today, then my future presence in Warsaw is necessary, does not state a logically true proposition. For the conditional in question is neither a law of logic nor a correct substitution of a law of logic. In particular, it is not a substitution of the formula: Cpp. We can reject the conditional without violating a law of logic. If the conditional is valid, the ground of its validity should be sought elsewhere.

This ground may be called *the principle of semantic necessity*, which can be stated as a rule of inference:

$$\mathbf{V}\text{'}p\text{'} \longrightarrow p\,.$$

This rule, as well as its converse:

$$p \longrightarrow \mathbf{V}\text{'}p\text{'}\,,$$

where p is an asserted proposition were known to Aristotle who made use of them in chapter 9 of *De Interpretatione*.

Since '*p*' is true if and only if *p*, it would be self-contradictory to recognize the truth of *p* and to assert *Np*, and to accept the falsehood of *p* and to assert *p*. If I assert 'it is true today that I shall be in Warsaw at noon on December 21 next year', I cannot deny that I shall be in Warsaw at noon on December 21 next year. If I were not to be there at the given date, I could not say that the sentence stating this fact is true. The truth of my statement made today about my future presence in Warsaw and this fact itself are semantically related. They have to be jointly accepted or jointly rejected. The assertion 'it is true today that I shall be in Warsaw at noon on December 21 next year' is, therefore, also incompatible with the possibility of my not being in Warsaw at the appointed time as much as the denial of the above assertion is incompatible with the possibility of my being there. The semantic relation between a true proposition and what this proposition states does not allow us to accommodate a contingent statement within the bivalent logic of propositions. This is the gist of the above quoted argument produced by Łukasiewicz in *Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls* in favor of adopting for this purpose the principle of logical trivalence. This does not affect a different issue, namely whether or not my future presence in Warsaw is determined today, and, if so, in what way it is determined in advance. This issue has nothing to do with the question concerning the validity of the inference from the truth of *p* to the occurrence of what *p* states.

The connection between the truth of *p* and *p* is similar to that between a prediction and its confirmed prognosis. The schema of prediction is a conditional whose antecedent is a conjunction of two or more statements and the consequent a statement called *the prognosis.* If the predicted event actually occurs, the prognosis is confirmed and the conditional is true. The correctness of the prediction depends on the occurrence of the predicted event. But this event does not depend on and still less is determined by the prediction. If it does occur, it takes place irrespective of whether or not the prognosis was based on a valid inference or the prediction made, at all. In the course of his destructive criticism of the fatalist argument, which is, as it were, an abbreviated version of logical determinism, Gilbert Ryle made a point fully applicable to the issue at hand. "The questions, what makes things happen, what prevents them from happening and whether we can help them or not, are entirely unaffected by the logical truism that a statement to the effect that something happens, is correct if and only if it happens."[42]

However, Łukasiewicz might have wished to make a stronger statement than that based on the necessary semantic connection between the truth of *p* and *p*. For his argument might and actually was understood to mean that he considered *p* itself as necessary. If this were the case, 'necessary' could not have meant 'logically necessary'. Such an inference would presuppose the identification of truth with necessary truth and falsity with impossibility. This presupposition has to be rejected in view of its consequences for modal logic, which were mentioned earlier. Consequently, the necessity of *p* could be only causal necessity of the event stated by *p*. This interpretation would be closer to Łukasiewicz's views. For he believed that a statement about the future cannot be true today unless what it states is predetermined by causes existing today. This follows from the correspondence theory and the absolute character of truth.

But in the light of what was said before this additional assumption is not needed for the understanding of the argument under discussion. It is enough to recall that the argument assumes the existence of true contingent, "bilaterally possible" propositions. The definition of these propositions (Tp) can be written down symbolically as an equivalence:

$$ETpKMpMNp \ .$$

If the principle of logical bivalence is universally valid, Tp is never true. For the right-hand side of the equivalence is a conjunction which for both truth-values of p becomes falsehood. Therefore, we would have to conclude that contrary to the assumption there are no propositions which are both contingent and true. On the other hand, if a third value, different from truth and falsity is assumed, the existence of true contingent propositions can be formally established. Accepting the definition of the modal function Mp due to Tarski:

$$EMpCNpp \ ,$$

we obtain the equalities:

$$M0 = 0, \ M1/2 = 1, \ M1 = 1 \ ,$$

and hence for $p = 1/2$ Tp becomes 1.[43]

V

The argument considered above establishes only the formal conditions for the demonstration of the existence of the propositions which are in fact neither true nor false. To put it differently, the argument indicates certain syntactic characteristics of the language in terms of which propositions neither true nor false can be expressed. The question whether there are in fact such propositions or not does not belong to logic. To answer it, we have to turn to natural science and to the evidence brought in support of of the two rival hypotheses, strict determinism and indeterminism.

The logician can do more, however, than construct a consistent syntactic structure that would allow us to deal with statements neither true nor false. He can also examine the logical credentials supporting strict determinism, which, as a rule, are accepted as unquestionable.

This analysis was the subject of an address delivered by Łukasiewicz in Warsaw in 1922. Its original was never published and was subsequently lost in the destruction of war. Łukasiewicz reconstructed it in Dublin in 1946. Although he outlined his argument in the closing paragraph (§62) of the second edition of *Aristotle's Syllogistic*, the full text was only published posthumously in 1961.[44]

The thesis which Łukasiewicz wished to analyze shall be called *the semantic formulation of strict determinism.* This thesis states that if A is *b* at time *t*, it is true at any instant earlier than *t* that *A* is *b* at *t*. It corresponds very closely to the model of classical mechanics as a deterministic theory. For a distinctive characteristic of the equations of motion of classical mechanics allows us to establish the following thesis, "Given the force-function for a physical system, the mechanical state of the system at any time is completely and uniquely determined by the mechanical state at some arbitrary initial time".[45]

The semantic formulation of strict determinism refers to a certain important characteristic of true statements, namely that the predicate 'true' is not a relational, time-dependent and incomplete predicate but an absolute, independent of time and complete one. On this account the semantic formulation of strict determinism is also known (in Poland) as the thesis of the absolute character of truth. This thesis underlies the classical theory of logic which assumes the immutability of the truth-values. In logic we postulate for the purpose of simplification or recognize the existence of two abstract objects—truth and falsehood, and we assume that all true sentences denote truth and all false sentences denote falsehood. The relation between logical sentences and truth-values is defined syntactically. It is sometimes emphasized that this procedure is adopted in order that the relation 'having values' may be used independently of any interpretation which is given to the systems of logic. But if the concept of truth is syntactically defined, the correspondence relation 'having values' is conceived as timeless and altogether outside of time. We should, although we do not, abandon it when we deal with an interpreted logical system and assert that this system is satisfied in a physical model.

We can deduce the semantic formulation of strict determinism from the principle of logical bivalence or of from the metalogical law of the excluded middle. Let us make two assumptions:

$$\mathbf{V}'p' \,.\, \mathrm{v} \,.\, \mathbf{V}'Np' \qquad \text{(A)}$$

$$\mathbf{V}'p' \longrightarrow p \,. \qquad \text{(B)}$$

The first of them is a particular case of the metalogical law of the excluded middle, and the second is a semantic rule of inference, to which reference has been made earlier. If every proposition is either true or false, the expressions 'It is true that *p*' and 'It is false that *p*' are equivalent to '*p*' and '*Np*' respectively. Tarski's partial definition of truth makes use of this semantic rule, which allows us to assert *p* if '*p*' is true. From the assumptions (A) and (B) the conclusion can be derived that if '*p*' is once true it is always true, that is, that a true statement *p* is true at any time, regardless of date of its utterance, and, thus, irrespective of whether it has ever been formulated or not. Should the truth be mind-dependent, a true statement could not be true at any time.[46]

If the principle of logical bivalence is accepted as universally valid, every proposition is either true or false, and thus, also, every proposition about the future. We can then prove that if it is true at time *t* that John will be home tomorrow noon, then it is true at any instant earlier than *t* that John will be home to-morrow noon. If we accept that 'John will be home to-morrow noon' is true now, we have to accept, as Aristotle already said, that it is true "at any time you like", viz. from all eternity (*De Int.* 18b, 38).

What sense, however, should be attached to the expression 'it is true that *p*' made earlier than the occurrence of what *p* states to be the case? If truth consists in the conformity of thought to reality, a statement cannot be true unless the real correlate to which thought conforms exists or is given in some way. If this correlate did not exist, the statement could not be true, nor could it be false. It is not sufficient to say that a statement *p* is true provided that at a specified time what *p* states is a fact.[47] For according to the classical conception of truth a statement is true if there is a fact to which it corresponds and a statement cannot either correspond or not correspond to a non-existent fact, to a mere non-entity, to use C. D. Broad's expression. It is essential to the truth of statements about the future that they should have some correlate. What is required is some connection between present reality and future reality, which would account for the truth of a statement made today about an event or state of affairs to occur only to-morrow. This connection is provided by strict determinism. The only other ontological justification for the thesis of the eternity of truth is that mentioned earlier, namely that the totality of facts, correlates of true propositions, is spread out eternally in time and space. This alternative is as hard or even harder to accept than that of strict determinism. If we reject both ontological presuppositions, the absolute conception of truth becomes an apodictic pronouncement, perhaps didactically useful but otherwise unaccountable. We are then faced by the problem of how to make use of the Aristotelian definition of truth, unless we modify it in some important respect. This problem, closely connected with the absolute conception of truth, will be considered in the closing part of this article. For the moment the justification of the semantic formulation of strict determinism must be explored.

A certain interpretation of the principle of causality states that for every event *B* there is such an event *A* that the statement '*A* is the cause of *B* and *B* is the effect of *A*' is true. *A* is the cause of *B* if *A* is a sufficient condition for *B*, that is, if *A* occurs, then *B* occurs. Therefore, if *A* occurs, *B* inevitably occurs. Since according to the principle of causality every event has its cause in an antecedent event, every event is a link in an unlimited causal chain:

$$\ldots . E_n, E_{n-1}, \ldots . E_3, E_2, E_1, E_0 ,$$

each of which occurs at an instant earlier than the subsequent one. Causal relation is a transitive relation, and, thus, every earlier event is the cause of every later event. At t_n, however large n might be, earlier than t_0 there occurred an E_n which is the cause of E_0. But if the cause occurs, then the effect inevitably follows the cause. Therefore, if *A* is *b* at t_0, *A* is *b* at t_n. For at every instant earlier than t_0 there occurs some event which is the cause of E_0, that is, an indirect correlate of the statement: *A* is *b* at t_0. If John is going to be home to-morrow noon, then there occurs now and at any instant earlier some event which is the cause of John's presence at home to-morrow noon, and it cannot happen that the cause occurs and the effect does not follow. Thus, provided that events strictly determine one another, statements about the future can be true. For while their direct correlates have not yet taken place, the facts which determine their occurrence exist, irrespective of the time of the utterance of the statement about the future. Consequently, although a true statement is independent of time, there is always a fact to which it corresponds, and, in a certain sense, a true statement always expresses what is the case.

This argument is, of course, independent of that based on the principle of logical bivalence. But they support each other and jointly exclude the possibility of chance and contingency. Without the support of the argument based on a certain interpretation of the principle of causality, the thesis of the absolute character of truth hangs, as it were, in the air. If there were no unlimited sequences of causally connected events stretching in both directions of time, statements about the future could not be either true or false and the division of propositions into true and false would not be exhaustive. The principle of logical bivalence gives universally valid rules of inference provided that every event has its causes existing *ab aeterno*.

While strict determinism cannot be deduced from the principle of bivalence, it could be argued that strict determinism justifies the acceptance of the universal validity of this principle in the language of science and in the construction of scientific theories. Since we are very familiar with the principle of bivalence, we are inclined to regard it as a *principium per se notum*, "final and self-evident", as D. C. Williams put it, and accepted unconditionally. A logical principle as long as it is applied in the constructions of abstract structures does not require any justification, excepting that of consistency. But the claim of its universal validity, to the exclusion of any other, in the language of science, would be dogmatic unless it is supported by some evidence. The only evidence available in its favour seems to be that provided by strict determinism. The value of this justification is not, however, beyond suspicion, and is indeed doubtful.

There is no necessity to accept strict determinism. If this hypothesis is confirmed, the bivalent logic is the most appropriate for the construction of scientific theories. On the other hand, if strict determinism has to be restricted or rejected, there is no impelling reason for accepting the logical principle according to which every statement is either true or false. The question as to whether scientific theories have to make use of the bivalent logic remains open and cannot be decided in advance.

The great achievements of modern science are closely associated with strict determinism. On this account the principle of the strict causal determination of all events has been regarded as the permanent foundation of scientific knowledge. The elevated position of strict determinism should perhaps also be accounted for by the fact that strict determinism closely approximates the idea of universal orderliness and of the conformity of all events to law. However, from a logical point of view the idea of a strict causal determination is not without blemish.

The world picture of Laplace inclines us to conceive of every event as a link in a chain of events connected by a necessary causal bond. Every event is regarded as the effect uniquely determined by an unending sequence of antecedent events—as a matter of fact, according to Laplace's conception, nothing short of the state of the whole universe immediately preceding a given event could be regarded as its cause—and is itself a sufficient condition of an unending sequence of future uniquely determined events. This conception of the universal causal interconnection cannot be considered as either clear or consistent. For one thing, while it makes use of the principle of causality to construct its picture of the universe, the admission of universal causal interconnection renders the application of the concept of causation practically useless.

The description of the world in terms of an unending sequence of events tidily tied up to each other by a causal bond is inadequate and confusing by virtue of its simplification. This model of becoming has its origin in the supposition that each concrete event has its proper place in a sequence of events and that its occurrence and characteristics are determined exclusively by the antecedent part of this sequence. This supposition, advanced e.g. by Spinoza (*Ethics*, Part I, Prop. XXVIII), does not render justice to the number and diversity of ways in which events determine each other, nor to the indefinite variety of their characteristics for which no single causal chain could account. On the other hand, if each event is considered as a point of intersection of many causal chains, that is, as an effect of multiple independent causation, as the Stoics seemed to have suggested, the advantage of the chain analogy in explaining change vanishes.

While sometimes we have to assume the existence of causal chains in a finite time interval, the extension of this methodological procedure, necessitated by the regulative principles of scientific investigations, to the totality of events in the space-time continuum reifies this methodological device. The described procedure is useful to discover with a certain approximation some causal or other connections of events, but having served this purpose it becomes, as a rule, redundant in further investigations. Łukasiewicz suggested that to ignore these circumstances is to replace scientifically testable assertions by "whimsical speculations" and "imaginary lines of development".

The principle that every event is determined in accordance with a law or a set of laws is again a methodological regulative principle to guide us in research which cannot be used as a premiss for a valid inference of a very general nature. Since this principle can be shown to be satisfied in some cases, we are justified in claiming that some events are determined in accordance with laws and if these laws happen to be causal, that they are related by the connection of cause and effect. If we were ignorant of these laws we could not say whether B is the effect and A its cause, or whether B follows A by chance. No event is a cause or an effect or a link in a causal chain unless we can establish a causal relationship between them. There is a considerable gap between our knowledge of causal determination and the thesis that the totality of events is connected by unlimited causal chains, that these chains can bridge any events, however distant from each other, and that without the support of causal chains the world would disintegrate into a chaos of distinct, disjointed and disorderly parts linked by mere coincidence.[48]

There is as yet another flaw in the principle of a strict causal determination. Strict determinism implicitly assumes that an unending sequence of events has no limit. For instance, John's presence at home to-morrow is produced by an unlimited sequence of events that reaches back through the present instant to the most distant and unrecorded past. It is neither self-evident nor justified to assume that the chain of causation can be traced backward to the creation of the world. It might be the case that John's presence at home to-morrow is determined in advance by an unlimited causal chain and could be predicted at any earlier instant. But the case might also be quite different. The sequence of events which determines John's presence at home at t_0 might have a lower limit and this lower limit is an event which occurs at such an instant t_n that t_n is earlier than t_0 and later than a certain instant t_{n+m}. If this is the case, it could not be said that John's presence at home at t_0 is decided either way at t_{n+m}. The causal chain which determines this fact would not reach the events occurring at t_{n+m} and would, with respect to the instant t_{n+m}, belong entirely to the future.

Now, we know that there are infinite sequences with a lower limit. An instance of such a sequence in question is the ordered set of rational numbers $\{a_n\}$ such that $1/2 < a_n \leq 1$. This sequence in infinite, it has a first but no last term and each of its terms is greater than any rational number b_n defined by the inequalities $0 \leq b_n \leq 1/2$.

Let us assume that the terms of the sequence $\{a_n\}$ are time indexes of events which can be ordered into a sequence of causes univocally determining event E_1. E_1 is then determined by a sequence of causes which is infinite and limited. Every event of this sequence occurs at an instant later than the time index of its lower limit which is also later than the present instant t_0. All events of the sequence in question are with respect to t_0 future events. At t_0 the infinite sequence of causes which strictly determines E_1 has not yet begun.

If this assumption is accepted, the world or some of its parts would never have been completely and strictly determined. Since for two arbitrary events, E_1 at t_1 and E_2 at t_2, there is always another event later than E_1 and earlier than E_2, and, generally, an event has no immediate predecessor and successor, the system comprising E_1 and E_2 is not fully determined at any instant of the interval $t_1 - t_2$. It is not by any means certain that any sequence of events is strictly determined by past events. There might be some causes in the past the action of which is entirely extinguished and thus cannot be causally connected with any subsequent event. There might also be other causes whose action influences present events as well as those to occur in the future. This does not provide sufficient reason to claim that any future event is strictly determined at any earlier instant. We can speculate on the chain analogy for there are certain suggestive similarities between an ordinary and a causal chain. But there are also important differences between chains and accounts of natural events. These differences show that to derive inferences from the assumption of causal chains about what is actually the case is an unjustifiable procedure.

While strict determinism sets no restrictions on causal connections and assumes that causal chains are unlimited, the denial of strict determinism implies that there are such limits and admits the possibility of broken causal chains and of causal chains which can be established only within finite time intervals. The view that every future event is determined by a causal chain from all eternity should not be dogmatically rejected but neither can it be regarded as logically and satisfactorily justified. It may be true with respect to some events, for which the motions of celestial bodies provide an instance. But the assertion that my presence at home to-morrow has been determined from all eternity seems to be highly improbable.

If at any *t* there are some future events determined by causal chains which have not yet commenced at *t*, it is logically permissible to assume the hypothesis that some future events are ambivalently possible, neither necessary nor impossible. For to follow Leibniz's argument in the *Discourse on Metaphysics*, "nothing is necessitated whose opposite is possible". This indeterministic hypothesis does not exclude a strict causal determination of some future events. It is compatible with some interpretations of the principle of causality. To say that a future event is not strictly determined does not mean that when it occurs it will be taking place without cause. It is also compatible with the principle of determinacy: everything is determined in accordance with laws by something else. The restriction of strict determinism does not commit us to the view that things just happen and are subject to no order and law. Indeterminism does lead, however, to the rejection of causalism, that is, the doctrine that everything is strictly determined according to a set of causal laws.[49]

The hypothesis of indeterminism admits that we find among the articles of the world's furniture both strictly determined and ambivalently possible events. Contingent statements refer to what Aristotle called *potentiality in either direction.* It is this strange kind of physical reality which stands in between possibility and reality, that Heisenberg ascribes to energy, ontologically conceived of as *materia prima* and defined in terms of the Aristotle's concept of *potentia.* In his view, a probability wave function, by means of which the elementary particle of modern physics is described, physically corresponds to Aristotle's potentiality and designates "a possibility for being or a tendency for being."[50] Heisenberg did not know Łukasiewicz's contributions but referring to those of Birkhoff, von Neumann and Weizsäcker, who were familiar with Łukasiewicz's writings, he suggested that the syntactic structure of language, in terms of which the *potentia* discovered by quantum physics could be adequately described, should not include the rules of inference based on the bivalent logic. "Quantum logic" requires that "undecided statements" be accepted as theses of the language of physics and that this language includes the rules prescribing the ways of dealing with such statements. The syntactic structure of language which provides for these two requirements "corresponds precisely to the mathematical formalism of quantum theory. It forms the basis of a precise language that can be used to describe the structure of the atom".[51]

VI

From a logical point of view, strict determinism and indeterminism may raise an equal claim to logical validity. But in the light of present-day knowledge strict determinism is a less probable hypothesis than that of indeterminism.

As Heisenberg's view, to which reference has just been made, has already indicated, Łukasiewicz's criticism of strict determinism is now strongly supported by the theories of contemporary physics. The developments in this field have undermined or invalidated the expectations concerning the capacities with which Laplace endowed his omniscient being. Laplace assumed that his calculator would have a complete knowledge both of all the laws of the universe and of the initial and boundary conditions which at a given instant determine the state of a part of the universe. It is not the assumption of a complete knowledge of all the laws of the universe but that of a complete knowledge of the state of an isolated system that is now seriously doubted. The doubt is not prompted by the fact that only a finite number of observations can ever be made and that the state of a physical system cannot be determined by a finite number of observations. What is claimed by a great number of physicists is that even granted the possibility of an infinite accumulation of observations, this would not suffice to give us a complete knowledge of the initial state of the system on which an accurate and unequivocal prediction of its future behavior depends. For there is a rigid limit to the accuracy of observation, and this limit, being a constant of nature, can never be surpassed. Therefore, however accurate our knowledge of initial conditions may be, it does not lead to an unequivocal prediction of their effects, but at most to a probability prediction expressed in terms of an identical relative frequency of their possible effects.[52]

Thus, if strict determinism implies that a complete knowledge of the state of a system at one instant provides the sufficient conditions for making a single-valued prediction of this state at some other instant, physical theories of to-day are not in their entirety strictly deterministic. Since we can no longer accept the assumption that the independent parameters of any physical occurrence or state can be measured simultaneously and as precisely as we wish, the cause does not determine the effect unequivocally. Consequently, complete knowledge cannot, in principle, be achieved and there is a strong objection either against regarding the respective laws as being causal or, more generally, against the universal validity of the principle of causality in its strictly deterministic interpretation. Moreover, if the values of independent parameters cannot be known as exactly as we wish, we are unable to make predictions of future events and thus we are also unable to describe physical occurrences meaningfully in terms of causal chains. Unlimited causal chains are a construct by means of which strict determinism can be applied in the descriptions of the world, causally relate any events, generate by an infinite regress unlimited causal sequences and establish universal conformity to laws. They constitute a bond which ties things together, makes the world intelligible, and reveals the order of nature. The supposition that there are such chains cannot be either confirmed or disconfirmed. It becomes a physically meaningless, unverifiable statement, from which no deductive inferences can be derived to describe relations between observable physical occurrences.

If the thesis of strict determinism is regarded as equivalent to that of complete predictability, as is sometimes maintained, the failure of predictability invalidates strict determinism. But this equivalence should never have been accepted. On the one hand, the strict determinism of Laplace is a stronger assertion than the predictability of all "the movements of all bodies from the largest to the smallest." Strict determinism is an ontological thesis; it refers to facts and their connections which are not dependent on minds and which hold irrespective of whether we know them or not. Predictability is an epistemological thesis, concerned with the knowledge of facts and their connections, and knowledge is relative and mind-dependent. On the other hand, what is predictable does not need to be strictly determined according to a set of causal laws. We have to know the laws to make predictions, but these laws do not need to be causal. In particular, predictions do not make any use of causal chains, unless these are limited and closed within finite time intervals.

The concept of cause, as used in science, is far removed from the commonly accepted idea of causal bond, from which originates the model of an unlimited causal chain connecting events, as N. R. Hanson put it, by a kind of cosmic glue. This idea rests on the assumption that events can be completely isolated from each other and regarded as a closed system; that is, that they occur irrespective of all the other events which take place simultaneously. The concept of causal relationship in contemporary physics does not apply to isolated events, whose succession is uniquely fixed and proceeds with necessity to comply with a pre-determined order. Causal chain is no longer a model of change. Causal statements expressed in terms of plural and independent causes, of functional relations and differential equations describe 'horizontally'–and not 'vertically'–related events. Functionally related events are regarded as interdependent states of a system, in which changes of one variable take place together with changes of another variable. Time sequences of states are not necessarily ordered by causal relationship, that is, they need not represent uniform successions of events governed by a causal law. Functional relations are symmetrical. We need, therefore, some semantic rules to be able at all to interpret statements about functional relations as statements about causal relationships.

Richard von Mises showed how Laplace's picture of the world, in which everything is tightly connected to everything else and yet, paradoxically enough, is strictly determined and uniquely predictable, established itself owing to the tremendous success of the Newtonian mechanics of rigid bodies. He also showed the enormous difficulties which were encountered when the attempt was made to apply Newtonian mechanics to all phenomena of motion and to carry out the deterministic program to other parts of physics outside of mechanics. Mises concluded his examination with the statement that despite great achievements the claim cannot be made that "in our day the high aim of deterministic physics has been reached even approximately."[53]

When faced by the huge gaps in the world picture of Laplace we cannot disregard the possibility of more adequate deterministic theories being developed which would take care of these gaps and supersede the non-deterministic theories. It is unlikely, and, indeed, impossible that we shall ever be able to provide a decisive proof demonstrating that no strict deterministic model of nature capable of doing justice to the facts can be found. It is, therefore, possible that the difficulties encountered are, as it is sometimes claimed, of a technical nature and that they would be removed one by one by the advancement of knowledge. But while the advancement of knowledge does show that some of the problems in question are of a technical nature, it also brings along new discoveries which increase the difficulties of finding an adequate strictly deterministic model of nature. Quantum Mechanics is one of these discoveries about which it is said that when it makes use of the traditional observables of physics, of position, momentum, energy, and so forth, it is incompatible with deterministic theories. We do not need, however, to enter into the complexities of Quantum Mechanics to justify the opinion that Laplace's world picture has to be abandoned.

It appears now to be certain that Laplace's picture of the world involves one basic assumption which can no longer be maintained. In Newtonian mechanics the forces which act according to some definite laws are propagated instantaneously, that is, with infinite velocity, and in relativistic physics with finite speed. For this reason infinite causal chains reaching back to the beginnings of the cosmic cycle were physically possible in classical physics but have to be excluded in relativistic physics. The finite velocity with which the action of forces is propagated through space restricts the temporal length of causal chains. Since causal relationships can be established only within final intervals of time, there must be events and systems which are not causally connected to one another.

According to the relativity theory light propagates with maximum speed, which no physical action can surpass. Light is the fastest signal, but it takes time for light to be emitted and reflected, and it travels with a finite speed c. An example given by Heisenberg explains very clearly the consequences of the structure of space and time revealed by the theory of special relativity. Two remote events—A and B—cannot act on each other before a definite time interval I_t has elapsed. They have to be unrelated if I_t is shorter than the time a light ray would take to link them. Consequently, any change in the state of A occurring during I_t remains unrelated to the changes in the state of B and no causal connections can be established between A and B during I_t. If during the interval in which a signal is transmitted from A to B, B originates a causal chain, this causal chain is not connected by any causal action to A. Simultaneous events cannot be a cause of one another or be related in any other way. For this would require a physical action to be transmitted with an infinite speed through space, an assumption no longer valid in the relativity theory. Not all events can, therefore, be causally connected to one another. Fields of forces spread with the speed c, physical action is not transmitted from one point of space-time to another instantaneously and, consequently, causal chains cannot be arbitrarily extended to establish causal bonds between any two events.[54]

We speak of causality when we wish to refer to a physical action, which is now conceived of as a transmission of energy from one point of space-time to another with a finite velocity. As a consequence, there is no empirical meaning to be attached to a world picture in which the events of today are the effects of some remote causes to be reached by an infinite regress of causes and effects. Since an infinite causal regress assumes continuity and excludes any gaps in causal lines of change, unlimited causal chains are an imaginary or insular construct, as Henry Margenau calls it, whose "insertion into a theoretical system makes no difference whatever". They are not operationally definable and have no epistemic correlations with the data of physics. Moreover, it can actually be shown that causal chains are bound to have a limited finite length and that they break down completely at some points.

> There are always events between which no causal relation can be established since action cannot reach from one event to every other event. This reduces the scope of causality: though all events may be members of some causal sequence, these sequences need not, and often cannot, be related to one another. The telephone call of a friend to tell me that he cannot keep the appointment with me does not affect my actions, if he rings after I have already left my house. Whatever he does while I am on my way to the appointment does not cause me to do anything: the causal chain is broken and my actions are truly independent of his.[55]

Even an omniscient intelligence cannot view the universe as if it were unfolding itself from the first causes and kept together by continuous, uninterrupted and unlimited chains of causes and effects. This world picture is incompatible with our knowledge about the structure of space and time and the restrictions which this structure imposes upon the possibilities of causal connections.

VII

It is time to sum up the conclusions to be drawn from the foregoing discussions. Łukasiewicz believed that there are certain relations between various systems of propositional calculus based on the principle of bivalence and of polyvalence on the one hand, and different conceptions of causality, that is, strict determinism and indeterminism, on the other. Unlike some logical determinists, he did not believe, however, that this relation is that of deductive inference. Strict determinism cannot be derived from the principle of bivalence and indeterminism from that of trivalence.

As formal structures, logical systems refer to possible worlds. Only upon being interpreted or built into a language of science do they limit the possibilities of description. Logical theorems do not say anything about reality, they do not state any natural connections among real objects, and remain valid irrespective of what we learn from experience about these objects. The logical structure of a proposition is exactly the same both when it is materially true and when it is materially false; logical necessity has nothing to do with what is the case. On this account connections holding between real objects cannot be derived from logical relations. "Logic is not a science of the laws of thought or of any other real objects", wrote Łukasiewicz towards the end of his life. "It is, in my opinion, only an instrument which enables us to draw asserted conclusions from asserted premisses".[56]

The principle of bivalence and of polyvalence, assumed in different systems of propositional calculus, determines by means of adequate matrices which expressions of a given system are tautologies. The change in the number of truth-values entails a redefinition of the range of valid tautologies, for tautologies are not absolutely valid but only within a specific logical structure, determined by its matrices or axiomatic basis. But the principle of causality and its various interpretations are not a tautology. When we pass from a bivalent to a polyvalent system of logic we are not obliged to change the way in which the principle of causality is conceived and formulated. A purely formal decision does not have and could not have such consequences.

On the other hand, the various ways of interpreting the principle of causality had nothing to do with the investigations on formal structures. The relinquishing of Laplace's world picture resulted from the theory of the electro-magnetic field, and, finally, from the relativity theory. Since in relativistic physics there are no instantaneous actions and actions at a distance, the conception of causality, applied in classical physics, had to be restricted and given a new definition. The construction of polyvalent systems of logic did not influence the evolution of physical concepts and theories. The converse relation, however, applies. For the evolution of concepts and theories of physics prompted the construction of various polyvalent systems which had been discovered before with the purpose of their applications in the formulation of a new syntactic structure of the language of physics.[57] This was a new departure, for it originated from an implicit or explicit recognition that there are more ways than one of describing the world of physics, these various ways being determined by the syntactic structure of the language used to describe reality. Those advocating the adoption of the polyvalence in the syntactic structure of the language of physics are prompted by the belief that such a language fits best the data of physics. A polyvalent logic provides a logical apparatus which allows us to incorporate "indeterminate statements" of Quantum Mechanics into the language of physics (instead of ruling them out as meaningless) and to combine them consistently with other statements by means of logical rules which preserve the tautological character of theorems.

This, and not logical determinism, was indeed the objective pursued by Łukasiewicz, although his approach, unlike that just described, was inspired by logical and ontological considerations. Logical determinism is a speculative doctrine. It tries to demonstrate that certain characteristics of formal structures determine the relations which hold among things and events and hardly distinguishes deduction from causation, a causal chain from a deductive chain.[58] Logical determinism should be distinguished from quite a different view with which Łukasiewicz was concerned. Different formal structures determine different possibilities for the description of reality. Consequently, when in our method of describing nature we adopt in our language instead of the bivalent a tri- or another polyvalent system of logic we are bound to use a language differing essentially in its syntactic structure. The differences in the syntactic structure of language are then reflected in the respective descriptions of the world and their ontological implications. The classical theory of deduction, based on the principle of bivalence, is the oldest and the most widely used. But neither its antiquity nor its familiarity guarantee its exclusive suitability and adequacy. Thus, for instance, we need a 'higher' logic than a bivalent one to define and use consistently modal functors. A bivalent logic excludes the possibility to which Aristotle referred when he spoke of contingency. His "potentiality in either direction" cannot be admitted and described without contradiction unless the principle of bivalence is abandoned and an alternative system of logic is accepted. The choice of formal structure for the language in terms of which we wish to describe the world is not, according to Łukasiewicz, a matter of expediency, for one and only one of the methods of representing nature is true. The question as to how the choice should be determined in order that it leads to an adequate description of nature cannot be solved by purely logical means.

There are two world views which involve one common conception but are opposed to each other in some other respects. Their common factor is the correspondence theory of truth, their differences involve their respective logical principles and conceptions of causality. The language of each reveals a different syntactic structure and different semantic rules. They will be called *the world view of strict determinism* and *the indeterministic world view*.

The world view of strict determinism is based on three basic assumptions:

I. The principle of logical bivalence: every proposition is either true or false and this dichotomic division is exhaustive.

II. The absolute conception of truth or the semantic formulation of strict determinism: if A is b at time t, it is true at any instant earlier than t that A is b at time t.

III. The hypothesis of strict determinism: everything takes place of necessity, has a cause in some earlier event and is determined in advance by causes existing from all eternity.

In the indeterministic world view we can also distinguish three basic assumptions:

IV. The principle of logical trivalence: the division of propositions into true and false is not exhaustive, there are propositions which are neither true nor false.
V. The principle of temporal relativity of truth: the assumption 'if *A* is *b* at time *t*, it is true at any instant earlier than *t* that *A* is *b* at time *t*' is true only for some substitutions of the variables '*A*', '*b*' and '*t*'.
VI. The principle of causality: every event has a cause in some earlier event (what is denied here is the continuity of causal connections and not the genetic relation of one event to another).

Łukasiewicz showed that (II) can be deduced from (I). If every proposition is either true or false, every proposition is either absolutely true or absolutely false; what is once true or once false is true once and for all or false once and for all. But (III) cannot be derived from (II). The absolute conception of truth is a kind of bridge which connects the principle of bivalence and the hypothesis of strict determinism. Similarly, (VI) is not inferable from (IV) and (V) provides their connecting link.

What kind of relation holds between assumption (II) and assumption (III)? Although it is clear that the relation between them is not that of deductive inference, they are related in some way. Assuming the classical definition of truth, the expression 'it is true at *t* that *p*' would have no meaning if it had no referent at *t* or if at *t* there existed no cause to be reached by a causal regress of the state of affairs designated by *p*. This sequence of causes is correlated with the expression 'it is true at *t* that *p*'. Generally, the assumption (II) and (III) are correlated by the semantic relation which holds between the expressions of a language and the objects or states of affairs to which these expressions refer, between a linguistic fact and extra-linguistic reality.

A set of objects *S* is a model of language *L* if language *L* is satisfied in *S*, or, more generally and freely, if language *L* can be used to speak of *S*. If this purpose is to be realized, language *L* must have a syntactic and a semantic structure. If language *L* had only a syntactic structure, we could not communicate meaningfully with each other by means of this language, to convey and to receive information about *S*.

Language *L* which serves the purpose of meaningful communication always includes semantic rules, although they do not need to be explicitly formulated. In particular, this applies to languages in terms of which scientific theories are expressed. Apart from the rules of syntax, which follow some system of logic, a scientific language includes semantic rules correlating theoretical constructs with experience (also sometimes called *operational definitions, epistemic correlation rules* or *rules of correspondence*). When we make use of language *L* to convey a certain theory or knowledge about the world, what we speak about in terms of language *L* shall be called *the proper model of language L*.[59]

A theory or a world view like that of strict determinism or indeterminism is a linguistic system consisting of sentences which express asserted propositions. Both the deterministic and indeterministic world views are a frame of reference or a schema for an adequate and detailed description of the world and not this description itself. It would not be possible, therefore, to determine with precision their respective proper models. These models can be, however, indicated to fix the distinctive connections between the world pictures and their referents.

Let us call the language of the deterministic world view L_1 and that of the indeterministic world view L_2. A proper model of L_1 is the set of events in which both strict determinism and the absolute conception of truth are satisfied. On the other hand, a proper model of L_2 is the set of events in which both the principle of causality (assumption (V)) and the principle of temporal relativity of truth are satisfied. The principle of bivalence provides the logical apparatus on which the syntax of language L_1 and the principle of trivalence that on which the syntax of L_2 is based. The logical apparatus of L_1 and L_2 also serves the purpose of investigating the ontological structure of the proper models of L_1 and L_2 respectively.

Łukasiewicz's views concerning the relations between various systems of propositional calculus and various conceptions of causality can now be formulated in the following way: The bivalent system of logic provides a logical apparatus adequate for the investigations of the ontological structure underlying the proper model of language L_1. This logical apparatus is, however, no longer suitable if we wish to investigate the ontological structure of the proper model of language L_2; it has to be replaced by a logical apparatus based on the principle of polyvalence. If the consequences derived from a set of hypotheses and logical principles are not in agreement with experience, two courses are open to us. We can either retain the logical principles and revise the hypotheses, or revise the logical principles and retain the hypotheses. But to revise the logical principles requires the revisions concerning the number of truth-values which the propositions may have.

The question arises whether only a polyvalent system of logic can provide a logical apparatus adequate for the investigations of the ontological structure of events in which strict determinism is not satisfied. In this connection the role played by assumption (II) and (V) in the deterministic and indeterministic world views should be emphasized. It is the absolute conception of truth, that is, a semantic rule which makes the bivalent logic an inappropriate tool for the investigations of the proper model of language L_2. If this time-honoured conception of truth is revised and restricted (for no reason is apparent why we should accept its absolute validity), we may reach the conclusion that the bivalent system of logic can be applied to ontological investigations irrespective of whether or not the more or less strict interpretation of the principle of causality is accepted. Varying semantic rules of correspondence can be attached to invariant syntactic structures. It is a well-known fact that in physics equations remain but their interpretations come and go.

NOTES

1. Waismann 1956, p. 455-457. The term 'logical determinism' was introduced by Moritz Schlick; see Schlick 1931, p. 158b.

2. Łukasiewicz indicated in one of his early contributions (Łukasiewicz 1920, p. 197, 199) that the principle of bivalence can also be expressed in the following formula:

$$[f] : [p] \,.\, f(p) \,.\equiv.\, f(1) \,.\, f(0) \,.$$

Now, it has been shown by Tarski in 1923 that this formula is inferentially equivalent to the principle of extensionality:

$$[p, q, f] : p \equiv q \,.\, f(p) \supset f(q)$$

(Tarski 1956, pp. 13-16). This gives to the bivalent calculus of propositions a distinctive superiority over any polyvalent propositional calculus. For in the latter systems the principle of extensionality must be either admitted axiomatically or be derivable from the axiom system which requires the addition of a new axiom. See, e.g. Łukasiewicz 1953, pp. 122-124.

3. The term 'cause' means here 'total cause', that is, 'a sufficient condition for the occurrence of the event of which it is the cause'.

The doctrine of strict determinism is a conjunction of several theses of which sometimes one and sometimes another is singled out to stand for the whole doctrine. The following component theses should be distinguished: (a) Causalism: causation is the only mode of determination, all scientific laws are causal laws. (b) Necessity and uniqueness: causal relationship is a necessary and unique relation. (c) Continuity: causally connected events constitute a sequence with no gaps or a causal chain which is broken nowhere. (d) Universal determinacy: every event is determined in accordance with a causal law or a set of causal laws. (e) The principle of predictability: every event is in principle predictable.

4. Łukasiewicz 1930, pp. 75-76. Cf. Łukasiewicz 1961, p. 125; Łukasiewicz 1952, p. 207; and Łukasiewicz 1957, p. 205, where the earlier interpretation of Aristotle's views is somewhat more cautiously stated.

5. Following Prior 1953 the question of what Aristotle had really meant in chapter 9 of *De Interpretatione* was taken up in Butler 1955, Anscombe 1956, Taylor 1957, Albritton 1957, Strang 1960.

6. This view is not entirely true, see Michalski 1937, Baudry 1950, Prior 1955, pp. 241-242.

7. Functor '*L*' has a different meaning in the expression "**L**'*ApNp*'" and in the expressions '*NLp*' or '*NLNp*'. While in the first it is a metalogical functor, in the second and third it is a functor of the object language.

8. W. V. Quine rejected as phantastic Aristotle's view to the effect that 'It is true that *p* or *q*' is an insufficient condition for 'It is true that *p* or it is true that *q*' (to put it differently, that **V**'*ApNp*' does not always entail **V**'*p*' . v . **V**'*Np*'). But Quine assumes the universal validity of the principle of bivalence which Aristotle did not do. See Quine 1953, p. 65; Linsky 1954, p. 251. According to Cicero, *De Fato* 16, 37-38, Aristotle's view was shared by the Epicureans.

9. Łukasiewicz 1952, p. 207.

10. Łukasiewicz himself thought at one time that the principle of bivalence corresponds, as he put it, to the conjunction of (d) and (e). See Łukasiewicz 1958, pp. 67-68. He abandoned this view soon after.
11. Cicero, *De Fato* 7, 13-14; 10, 20-21; Łukasiewicz 1930, pp. 75-77; Łukasiewicz 1961, p. 218.
12. Waismann 1956, p. 456; Cf. Bradley 1959, p. 194, 197; Williams 1951, p. 285; Kłósak 1948, p. 218. Waismann's objection against Łukasiewicz's views and his criticism of these views are essentially those produced by Schlick thirty years ago. See Schlick 1931, p. 158b.
13. See, e.g. Butler 1955, p. 272.
14. *De Fato* 19, 45.
15. Łukasiewicz 1955 (originally published in 1918).
16. See, e.g. Reichenbach 1959, pp. 154-157.
17. Łukasiewicz 1955; Łukasiewicz 1961, p. 126, 203.
18. Łukasiewicz 1919-1920, pp. 170b-171a.
19. Łukasiewicz 1961, p. 212, 217; Cf. Łukasiewicz 1958, pp. 69-70.
20. Łukasiewicz 1961, p. 218. Łukasiewicz had probably Schlick in mind; see footnote 12.
21. Łukasiewicz 1961, p. 218.
22. Łukasiewicz 1961, p. 206, 207.
23. Łukasiewicz expressed such an opinion in his address delivered at Warsaw University in 1918, but he never returned to it later in his life. In this address Łukasiewicz mentioned that his discovery of a trivalent calculus of propositions, which he published in Łukasiewicz 1919-1920, was actually made "last summer", that is, in summer 1917. "This system", Łukasiewicz stated, "is as coherent and consistent as that of Aristotle and greatly surpasses the latter by the richness of its theorems and formulae."
24. Sobociński 1956, pp. 29-31.
25. Williams 1951, p. 282.
26. Łukasiewicz 1961, pp. 33-37 (originally published 1907).
27. When the Stoic Chrysippus suggested that the equivalence

$$ECpqNKpNq$$

should be accepted, his purpose might have been to eliminate the differences between causal and material implications. The ridicule and the way in which Cicero poured it upon Chrysippus for advancing the above equivalence are very suggestive in this respect. See Cicero *De Fato* 8, 15-16.
28. Broad 1927, p. 73. Similar opinions were voiced in Ryle 1954, p. 20, and Ducasse 1941, p. 334.
29. Ryle 1954, p. 27. I do not understand why a statement like 'I will get up to-morrow at 8 a.m.' should be a general statement.
30. Kotarbiński 1957, p. 137 (originally published 1913); Łukasiewicz 1958, p. 68 (originally published 1929).
31. It was already asserted by Cicero who asked the Epicureans: quod

autem verum non est, qui potest non falsum esse? aut quod falsum non est qui potest non verum esse? (*De Fato* 16, 38). See also Baylis 1936, p. 166.

32. Kotarbiński 1957, pp. 156-157.
33. Cf. the quotation from an abstract of a lecture *On the Principle of the Excluded Middle* (1910) cited in Borkowski-Słupecki 1958, p. 15.
34. Under Leśniewski's influence (see Leśniewski 1913, pp. 350-352), Kotarbiński came to the conclusion that the principle of trivalence must lead to contradictions and abandoned the views, which, according to Kazimierz Ajdukiewicz, greatly impressed Łukasiewicz and set him upon the course crowned with the discovery of a trivalent propositional calculus. See Kotarbiński 1957, p. 13, footnote.
35. Łukasiewicz 1961, p. 126.
36. Łukasiewicz 1920, p. 190, 192. It was Schlick who contended that while there are statements which are not known to be either true or false, there are no statements neither true nor false. See Schlick 1931, p. 159a.
37. Łukasiewicz 1930, p. 64.
38. Łukasiewicz 1953, p. 135.
39. Łukasiewicz 1930, p. 57; Łukasiewicz 1957, p. 153.
40. Williams 1951, p. 293. Williams' argument can be found already in Cicero's *De Fato* 11, 28.
41. This objection can be found in Bradley 1959, p. 205.
42. Ryle 1954, p. 22.
43. Łukasiewicz 1930, pp. 65-72.
44. Łukasiewicz 1961, pp. 114-126.
45. Nagel 1961, p. 279.
46. Łukasiewicz 1961, pp. 117-119.
47. This argument was put forward in Scholz 1959, p. 77; Baylis 1936, p. 162.
48. Łukasiewicz 1961, pp. 16-17, 42-43, 48; Łukasiewicz 1957, p. 207.
49. The principle of determinacy states that events determine each other and that this determination, whether of causal, statistical or other nature is expressed in laws concerned with the co-existence and succession of events. See Łukasiewicz 1961, p. 122.
50. Heisenberg 1959, p. 42, 67, 139.
51. Heisenberg 1959, p. 158.
52. Schrödinger 1957, pp. 79-80.
53. Mises 1956, p. 180.
54. Heisenberg 1959, pp. 140-141; Cf. Bunge 1959, pp. 62-68; Margenau 1950, pp. 39-44.
55. Hutten 1956, p. 206.
56. Łukasiewicz 1952, p. 208.
57. Reichenbach 1948, §32.
58. Hanson 1955, p. 290, 307.
59. Suszko 1957, p. 47.

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