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A THEORY OF TRANSLATION AND TRANSFORMATION OF LANGUAGES

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The aim of this paper is to introduce several points of approach for the study of colloquial and artificial languages in the hope that they open the road for the construction of a *general theory* for their translation and transformation.

A careful observer of the widespread activities aimed at the construction of a fully automatic high quality translation (FAHQT) may ask the following questions: Where is the general theory of the foundation of languages (and similar interpretation bound formal structures) justifying the efforts put into the construction of complicated machinery? How dare we hope to succede and arrive at a highly efficient automation as long as the machines are based on sporadic partial theories *unconnected by the framework of a general theory?* How dare we start with only the partial understanding of the logically uncomplete systems acting within any single language and with practically no knowledge at all of the mechanism of the evolution, the natural development of the colloquially used linguistic systems?

Let us start with the following three properties of all colloquial languages, fundamentally important when we regard a language as a totality from an external, *metalogical* point of view:

1) The point of view given by the evolutionary approach:

Any colloquial language represents the momentary state of a long process of development, a process in which one may detect some influence of causality, but quite often taking a very haphazard way with no causality detectable.

2) The two-domain point of view:

We regard "language" on any of its possible levels of development as a construction erected over the coordination of two domains, i.e. *being constructed over a coordinative-domain*, whereby

2a) the first domain consists of sign vehicles, the *rules* governing the combinatorics of sign-vehicles and affronting us, according to the level of

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evolution as grammatics, syntax, algebrosyntactical algorithm or constructed algebraic formalism.

2b) The range of the second domain extends over structures of greatly varying composition used for linguistic interpretation. They are represented, for comparatively simple cases, by the "designatum".

2c) The term "semantic" stands for a coordinated occurrence of a permissible pair taken from these two domains.

3) The composit structure point of view states:

We cannot arrive at the structures occurring in the colloquial languages by starting with the set of constituents of any well formed calculus. If we succeed in analysing a language in a set of its constituents, then these are not taken potentially from any one or several well formed calculi and they do not belong to any single level of linguistic evolution, if we regard them from a genetic point of view. The detected set of constituents of any colloquial language turns out to be a conglomerate of constituents corresponding to diverse possible calculi and systems of interpretation and to greatly different genetic levels, techniques and methods.

Very few of the detected constituents and structures are acceptable as approximations to those occurring in formal logic. The greater part of them are quite unsophisticated contraptions and need much good will to be regarded as approximations. But they do exist, have a restricted efficiency and cannot be disregarded. Such structures are not approximations to any *one* single calculus, but possibly to a mixture of the constituents of different schemata of calculi. Mixtures, as they are, by usage they are superimposed into an apparent unity. We characterize these apparent units, but actual mixtures of different possible target-calculi and different levels of evolution by the metalinguistic predicate "composite".

Within these composite units we find structures of different levels of a hierarchy of types—or approximation of a hierarchy. The totality or unit may be regarded, for the sake of logical systematization, as an approximation of a calculus with a system of interpretations.

A composite character may occur in any of the domains and structures erected over them, in single and coordinated domains as well.

We suppose that the target structures of an evolutionary process are well formed and not "composite".

After having stated these three points of view, two further methodological steps should be explained: A) The preference for concepts of a high level of types, e.g. "interpreted calculus", "calculus coordinated to a system of interpretation with the range of the calculus restricted to interpreted values of its formulas". Any single concept should occur as one within this general framework, and technically, as a symbol derived from the symbol of this high level source concept.[5] But one should not forget that expressions like "calculus" are abbreviations of a formally strictly defined nature.

B) By putting the source concept task on calculus-type concepts, we need some system of orientation and reference for such concepts. We are, therefore, endeavouring to construct metalinguistic predicates having ordered sets of values, and to apply them as the ordinates of a system of coordinates. One of them may be a constructivistic hierarchy of types, represented by an operator with indices for the levels of types. It solves the technical problem, how to represent concepts with reference to their source calculus if this calculus (or approximation of a calculus) is supposed to exist.

A second and not less important ordered set of predicate values should express in a formalized way the intuitistic content of the ordered succession of levels of evolution. This intuitive content may be replaced by a set of basic constituents and the existence of a partial set as a characteristic of a certain group of such levels. We are not interested in the further iteration of the same principle for expression of the secondary, ... , subdivisions of the "ladder" of the tree of evolution. The subset of the existing constituents, taken from the total and ordered set of constituents, is the definition of a certain level and there is a possibility to arrange the levels amongst themselves with reference to the arranged source-set.

The existence of the elementary sentence, or of a modal connective are two obvious examples of constituents the appearance of which heralds a new level of development. A considerable part of the existing languages does not possess the structure of an elementary sentence (e.g. the Basque language in northern Spain, having instead the "sentenceword"; many primitive people have languages without modal connectives, or with the only one "and" without the plain understanding of its modal possibilities.) A few modal connectives do not constitute yet a linguistic or a syntactical calculus and a syntactical calculus does not represent the highest possible level of logical and linguistical evolution. To reinforce this last assertion we point to the two-domain approach with its possible generalisations to n and to ∞ many domain approaches.

Thus, the evolutionary approach, if reconstructed into a metalogical theory, yields a tool of orientation of the greatest importance. It is actually one of the ordinates within a system of coordinates for source structures of the calcul-level of types.

Colloquial languages are approximations to calcul-level compound units consisting of an (approximation of a) calculus coordinated to an (approximation of a well formed) system of interpretations.

This is a generalisation by inclusion of the evolutionary aspect of the fundamental approach of Carnap in his "Introduction to Semantics" and is restated by emphasizing the concepts on a high level of types.

Even a very primitive colloquial language is a tool of intercommunication with some limited degree of efficiency. A few basic constituents are already present. The study of the development of colloquial languages suggests the following hypothesis: The natural development of the colloquial languages tends towards an upper limit consisting of the coordination of a well formed calculus to a well constructed system of interpretations by a set of well coordinating rules.

The colloquial languages actually in use are still very far from this (external) limit, but all of them are exhibiting several of the basic constituents of the limital case.

Just as in the case of well constructed concepts we advised to use as source concept a well formed one of the calcul level of types, so we are suggesting to arrange the intermediate states of linguistic evolution with reference to such an external upper limit: with reference to the set of basic constituents of the well formed upper limit. The upper limit is the focal point of evolution—natural and artificial as well—and growing distance from this focal point (or points) means a reduction of the efficiency, the disappearance of some basic constituents or components of constituents.

No actually existing language possesses a partial set of the total set for a well constructed calculus. As far as it is possible to detect some correspondence or approximation, it is not to a set making up a well constructed target unit. This has been mentioned already when describing the composite character of living languages. But inspite of it, the evolutionary approach with its target and the constructivistic hierarchy of types with its high level source concept are two coordinates of a system of ordering of calcul-level concepts and concepts created by a reduction of the source or limital concepts along their respective "scales".

The constructivistic order leads to easy methods of symbolic expressibility.

Now we concentrate our attention on the technical possibilities of the two-domain approach. Its first assumption is the introduction of two mutually exclusive domains, $\mathbf{D}_1$ and $\mathbf{D}_2$ and their coordination to a derived "coordinated domain" $\mathbf{D}_3$. Thus, the precise denotation should be "two basic-domain approach". Over $\mathbf{D}_1$ one or more calculus, or approximations of the limital calculus cases are constructed; over $\mathbf{D}_2$ "systems of interpretations" are constructed. As the terms "calculus" and "system of interpretations" are abbreviations, in the following the using of these terms implies the structures signified by their respective abbreviations. Thus we are able to speak of one-one coordinations of a calculus to a system of interpretation, whilest the coordinations are valid actually between their constituents and components of constituents.

We assume that coordinations are existing at any level of the types of the structures occurring in the two domains.

We shall name $\mathbf{D}_1$: The domain of forms and structure for sign-vehicles" in short 'the domain of sign vehicles'.

We name the second domain: The domain for structures and forms starting on the lowest level with the elementary designata, on a higher level the propositions, propositional calculi and approximations of a "calculus of designata" and in general, systems of *non-material* interpretations: in short, the domain of designata. Its application is the interpretation of the structures, etc. of the first domain. This is done by a coordination, or set of such rules, Z.

The accepted expression "system of interpretations" means the totality of structures and arguments erected over the domain $\mathbf{D}_2$ and having an existence independent of its usage for interpretation. (As an opposite example to the non-material interpretation erected over $\mathbf{D}_2$ we point to a wire circuit with diodes; suitable to materialize propositional functions as "and", "or": such 'material interpretations' are limited by conditions inherent to the physical qualities of their components. The occurring propositional functions are materialized by electronic device components and not by sign-vehicles only.) We do not look for the merits of $\mathbf{D}_2$ and its structures: we have to accept their existence and grasp the way of their functioning.

Both in $\mathbf{D}_1$ and in $\mathbf{D}_2$—and later in their coordination to $\mathbf{D}_3$—we erect hierarchies of types, starting with the arguments of the lowest level and reaching a level suitable for the abbreviating term "calculus". The hierarchy in $\mathbf{D}_3$ presupposes conventions regarding to the mapping of $\mathbf{D}_2$ on $\mathbf{D}_1$. The simplest structures of $\mathbf{D}_3$ are obviously the ones corresponding to type levels with the same serial index.

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We intend to demonstrate in the following that some well known structures and operations of linguistics are applications of a combinatorics over two domains, using the components of the two hierarchies as arguments for coordinations in the form of one-one or one-many relations.

No generally accepted system of notations and no terminology appears to be in usage. As the fate of science is greatly affected by the technics of notations and terminology, we turn our attention to the introduction of a simplified metalogical notation, restricted to our present problem.

Herewith the *notations* for the fundamental problems of languages for a two basic domains approach with a variability of the level of types and degrees of development:

A. Sign vehicles for the subdivision of a linear arrangement of other sign-vehicles:

, ; ( ) · . :

B. Blanks:

. . . --- for basic domains, —— for derived domain.

C. Operators,

C1) within any of the domains:

$=$, S, and its possible modification shown on Z in C2).

C2) connecting symbolically different domains or elements of different and heterogenous domains:

Z	an instruction for simultaneous consideration or "coordination".
Z_{1-1}	'Z' used in the form of an one-one relation
$\bar{Z}$	The conversely directed 'Z'.
Z_{1-n}	'Z' used in the form of an one-many valued relation.
$Z_{m\,(1-n)}$	m different one-many relations having a common first argument.

Schemata for formulas:

D1) within a given domain the usage of the same blank is mandatory.

1) $\cdots . S . \cdots$; 2) $\cdots a . S_{1-1} . \cdots b$ 3) $\cdots . = . \cdots$

D2) between heterogenous domains:

1) $\cdots . Z .$ --- 2) $\cdots x1 . Z .$ ---$y1$ 3) --- $. \overline{Z}_{1-n} . \cdots$;

('–' does not occur in D2)

D3) Symbolic operators. Their usage does not state that the possibility of the requested operation has been proved already.

T . . . . Operator for variation of the level of *type* within a given constructivistic hierarchy:

T^{-n}. . . . The requested reduction of the level of types by n levels.

T^{+j}. . . . The requested elevation of the level of types by j levels.

$T^{\pm n}$. . . . The general T-operator

i-Operator used for the variation of the degree of development along a supposed ordered set of degree values, constructed with reference to the ordered set of basic constituents of an upper limit case.

i^{m} . . . . Operator for the elevation by m degrees.

i^{-j}. . . . Operator for the reduction by j degrees.

$i^{\pm}$ . . . . or $i^{\mp}$. . . . the general i-Operator.

Three letters with fixed meaning are sufficient addition to the notations A) - D):

C . . . . . Calculus; ${}_{u}C_{j}$ the j-th C of the u-degree.
I . . . . . System of interpretations; ${}_{u}I_{j}$.
L . . . . . Language ${}_{u}L_{j}$ the j-th language of the u-degree.

C, I, L are supposed to be of a high level in their respective hierarchies.

The definition of any language based on the presupposed two domain approach, and with a single system of interpretations will be:

$$L_{j} \; . = . \; C_{j} . Z_{1-1} . I_{j}$$

over the domains

$$\mathbf{D}_{1.2} \; . = . \; \mathbf{D}_{1} . Z_{1-1} . \mathbf{D}_{2}$$

This may be read as follows:

A "Language" L_{j} is a structure of the calculus-type erected over the coordinated domain $\mathbf{D}_{1.2}$ by means of a set of one-one valued coordinating definitions Z_{1-1}, if and only if the first domain $\mathbf{D}_{1}$ contains algebrosyntactic forms, rules, and genetic precedessors, and the vehicles for them, and the second domain $\mathbf{D}_{2}$ contains a system of interpretations with its own 'physical' forms, argumental elements or occurrences and other rules.

Let us denote by the prefix $_u$ the low degrees characteristic of the colloquial languages and by the prefix $_a$ the supposed upper limit of the genetic development:

$$_uL_j \;.=.\; _uC_j \cdot Z_{1-1} \cdot {_uI_j} \;; \quad _aL_j \;.=.\; _aC_j \cdot Z_{1-1} \cdot {_aI_j}$$

The indices "j" are, for the first approach, the same .- $_u\mathbf{L}_1$ should appear with an index, expressing the possibility of more $_a\mathbf{L}$ cases for limits.

The prefix for any single member of the formula defines the degree of development if an additional rule restricts all the members to the same level of development. But in general, this is an oversimplification. We cannot deal with the manyfold problems arrising in this connection here.[1]

In the case of a *multiple interpretatillity* of a text or a language, we are facing as many separate results of coordinations $Z_1, Z_2, \ldots, Z_n$ as the value of the index n is. C is common for all the cases, thus

$$\begin{aligned} _a\mathbf{L}_{1.\,\ldots,\,n} \;.=.\; & _uC_x \cdot Z_{n\,(1-1)} \cdot ({_uI_1}, {_uI_2}, \ldots, {_uI_n}) \\ .=.\; & _uC_1 \cdot Z_1 \cdot {_uI_1} \;; \; _uC_2 \cdot Z_2 \cdot {_uI_2} \;; \ldots, {_uC_n} \cdot Z_n \cdot I_n \\ .=.\; & _u\mathbf{L}_{1,1} \;; \; _u\mathbf{L}_{1,2}, \ldots, {_u\mathbf{L}_{1,n}} \end{aligned}$$

which means that a language system with a first domain capable of n different interpretations is not a *single* language, but actually n different languages, having the same first domain for sign vehicles. Usually the structures chosen for interpretation from $\mathbf{D}_1$ are different too.

This formula, if reduced to the level of types corresponding to simple expressions and single words, defines the concept of "synonymes". If "a" gives the necessary number of level reductions:

$$_u\mathbf{L}_1, \ldots, n \,.\, T^{-a} \;.=.\; _uC_x T^{-a} \cdot Z_{n\,(1-1)}\, T^{-a} \,.\, ({_uI_1}, \ldots, n) \,.\, T^{-b} \;.=.\; _u\mathbf{L}_1 T^{-a} \;; \; _u\mathbf{L}_{1,2} \,.\, T^{-a}, \ldots, {_u\mathbf{L}_{1,n}}\, T^{-b}$$

where $b \leq a$, (and the single occurrence of b should point to an open possibility, not detailed here).

In the case of multiple interpretations to a single structure of vehicles over $\mathbf{D}_1$ we face different languages with a common vehicle domain. An analogous schema, but with a distribution to the domains, which is just the opposite to the multiple interpretation, leads us to the concept of *translation.*

The fundamental condition of "translation" is the conservation of the "content" during a change of the sign-vehicle structures and arguments. The content of the target-language should have the same structures of interpretations and argumental cases used in course of interpretation as it has been in the source language. The optimal case is an invariance of the "I" (over the second domain) during the exchange of the sign vehicles and the formal structures connected with the sign-vehicles. If the source language is

$\mathbf{L}_1 .=. C_1 . Z . I_1$, and the target language $\mathbf{L}_2 .=. C_2 . Z . I_2$, then

the order of the operations to be carried out may be demonstrated, accepting $Z_1 .=. Z_2 .=. Z$

$$\mathbf{L}_1 .=. C_1 . Z . (I_1 = I_2) . \bar{Z} . C_2 = \mathbf{L}_2$$

$I_1 = I_2$ is the condition of invariance. As a consequence of carrying out Z first in the direct, then in the converse direction the remaining formula may be regarded as a homogenous one; we may use for it the homogenous "S". Thus, the schematic formula for translation will be, emphasizing the invariance of I:

$$\mathbf{TR} :=: \mathbf{L}_1 . S . \mathbf{L}_2 .=. C_1 . Z_{1-1} . I_{inv} . \bar{Z}_{1-1} . C_2$$

over

$$\mathbf{D}_{1,1} . Z . \mathbf{D}_{2,(1=2)} . \bar{Z} . \mathbf{D}_{1,2}$$

The result of the comparison is as follows:

"Translation" is the change of the elements and structures of the first whilst the second domain remains unchanged;

"Multiple interpretation" is the change of the elements and structures (models) of the second domain for a fixed first domain and its elements and structures. These two operations are *dual* in relation to an underlying schema.

The comparison shifted our attention to the occurring schemata. Let us symbolize schemata by means of blanks as follows:

The blank of 3 dashes: --- for designata and their formalism at any level of types.
The blank of 4 length: —— for the coordinated occurrence of . . . and ---.

The simplest extradomain coordination has the schema

$$\cdot\cdot\cdot\cdot . Z . ---$$

The simplest coordinations within a single domain are

$$\cdot\cdot\cdot\cdot . S . \cdot\cdot\cdot\cdot \quad \text{and} \quad --- . S . ---$$

Coordination of the already coordinated domains in a homogenous manner:

$$—— . S . ——$$

as e.g. the coordination of an object-language to a meta-language.

Now, let us have a look for accepted usages corresponding to these schemata.

$$\cdot\cdot\cdot\cdot . S . \cdot\cdot\cdot\cdot$$

corresponds to an operation called "to chiffre".

$$\cdot\cdot\cdot\cdot . S . \cdot\cdot\cdot\cdot . \bar{S} . \cdot\cdot\cdot\cdot$$

corresponds to the concatinated operations "to chiffre and dechiffre".

In the case of more chiffres for the same source language:

$$\cdots . S_{n(1-1)} . \quad \cdots 1, \quad \cdots 2, \quad \cdots\cdots, \quad \cdots n$$

with a separate S for each $\cdots j$, if they are not variants generated by means of cyclic alphabetic changes.

The schema for linguistic translation **TR** :=: $\mathbf{L}_1 . S . \mathbf{L}_2$ or —— $. S .$ —— completed with alphabetic notations is:

$$\mathbf{L}_1 .=. \frac{C_1}{\cdots} . Z . \frac{I}{---} . \bar{Z} . \frac{C_2}{\cdots} .=. \mathbf{L}_2$$

A comparison with the formula **TR** demonstrates, that "S" is only apparently a simple and homogenous operation, if connecting two languages. S is in $\mathbf{L}_1 . S . \mathbf{L}_2$ a homogenous result of a closed loop of heterogenous steps, and should be written as "S'".

$$—— .=. \cdots . Z_{1-1} --- . \bar{Z}_{1-1} \cdots .=. ——$$

is the schema for a single, one-one related translation.

In the case of a multiple translation, i.e. the translation of one source language into several target languages:

$$——_{o} . Z_{n(1-1)} . --- . \bar{Z}_n . \cdots 1, \cdots 2, \cdots\cdots, \cdots n .=.$$

$$——_{o}, 1; \quad ——_{o}, 2; \cdots\cdots, \quad ——_{o}, n$$

In comparison the schema for multiple interpretation:

$$——_{o} .=. \cdots_{o} . Z_{n(1-1)} . ---_{1, \ldots n}$$

$$.=. \cdots_{o} . Z_{n(1-1)} . ---_{1}, ---_{2}, \cdots\cdots, ---_{n}$$

$$.=. ——_{1, o}; ——_{2, o}; \cdots\cdots; ——_{n, o}$$

Both of the schemata lead to n different languages, but the first has a common domain of interpretations, the second a common domain of sign-vehicles. There exists a metatechnical duality with regard to this pair of schemata: disregarding the direction in the arrangement of the indices and Z to $\bar{Z}$, we used in both cases the same schema.

In the previous examples we took tacitely for granted that there exists a far reaching structural paralellity of the source and target cases. Further, we supposed that the argumental objects are of the type-level corresponding to calculi: If we are permitted to speak of a unit called "language" in the colloquial language, we may assume units of such a level of types for our formalism. Now let us analyze examples illuminating the tacit suppositions.

Example 1. The concept of the complex number may be explained on the two-basis principle as follows:

The second domain $\mathbf{D}_2$ appears as an identity case, as $\cdots_2$, instead of ---. The distinctness of $\mathbf{D}_1$ and $\mathbf{D}_2$ remains. For such elementary pair and for any structure erected over elementary pairs the schema will be:

$$\cdot\cdot\cdot_1\, T^a \,.\, S'.\cdot\cdot\cdot_2\, T^b$$

and if $a = b$,

$$\cdot\cdot\cdot_1 \,.\, S'\, T^a.\cdot\cdot\cdot_2\,.$$

Example 2. Approximation in programming. Reducible functions are based on $\cdot\cdot\cdot_1$, but functions presupposing infinitesimal analysis are based on $\cdot\cdot\cdot\,.\ \,.\,S'.\cdot\cdot\cdot_2$. If a computer is digital and as long as it has a single domain of digits as $\mathbf{D}_1$, programming means the single-basic approximation of two-basic structures. Thus, programming is not a translation and certainly not a chiffre, but in a part of the schema for "programming" we may recognize the schema of example 1). The schema for programming is:

$$\cdot\cdot\cdot_1 \,.\, S'.\cdot\cdot\cdot_2 : S_p : \cdot\cdot\cdot_o$$

S_p being an approximation, it is not possible to revert the result over $\cdot\cdot\cdot_o$ into the source quantity without any loss. In the case of a good chiffre, we arrive to our source structure without any loss as there are no transitions from two to one domains involved. The same applies if more than two domains are reduced to a single one, as long as the number of domains is finite.

Examples 1) and 2) involve one or more domains of integers and a high level of exactness. "Translation" is usually not an exact operation, but it may be constructed on any level of exactness and genetic development.

Two domain coordinations, transgressing levels of types and degrees of genetic developments are used for indexing of card registry, for machines retrieving books of a given subject in libraries and in many other practical applications.

Example 3. A group of important examples are the coordinations for which the predicate "meta-" applies. Its most general case is the coordination of a language to a second language, $\mathbf{L}_{\text{object}} \,.\, S.\ \mathbf{L}_{\text{meta}}$ over the coordinated domains $\mathbf{D}_{1.2}\ .\, S'.\ \mathbf{D}_{1.2}'$, whereby there exist rules for the reduction of the type level as far and to an extent which is necessary 'to speak in the language with the suffix "meta"' of the language with the suffix "object" and of any structure occurring in the object language, by means of expressions on a lowered type of the meta language. The schema is, if there are no genetic differences:

$$\text{——}_{\text{obj}} \,.\, S'\, T^{-r}.\ \text{——}_{\text{meta}} \quad \text{or} \quad \text{——}_{1.2} \,.\, S'\, T^{-r}.\ \text{——}_{1.2}'$$

The replacement of "——" by either "$\cdot\cdot\cdot$" or "---" results in special cases, as "meta-syntax", or meta language in relation to an object-syntax or of an algebra, etc.

Speaking in a non-exact metalanguage of an exact algebra involves an additional operator. one reducing in a symbolic way the degree of genetic development of "algebra" to the lower level of the colloquial languages. We represent inequalities in the degrees of development by the operator "i", symbolizing the degrees (not yet defined) by writing "$i^{\pm m}$", and use it analogously to the operator for symbolic equalization of level differences. Thus, if the speaking of an exact algebra in a non exact meta-language with the blank —— involves in a given theory of degrees of development the difference of "b" degrees and the reduction of "a" levels of types, we have to construct a formula over the following schema:

$$\text{——}\ 1,2 :=: \text{---}_{2} \,.\, \overline{\overline{Z}} \,.\, \cdots{}_{2} : S'\ i^{+b}\ T^{-a} : \cdots 1\ .$$

Example 4. Common meta coordinations. If there are more than one object cases to a single meta-structure, the many-one coordination including the necessary reductions of type levels T^{-r} and, if necessary, i-operators, the meta structure is a common-meta structure of all the occurring object cases. Indicating by the prefix "i" the general degree, and by "a" a high degree of genetic development, we have two examples:

4a) Common meta coordination on the same degree of development:

$$\text{——}_{\text{obj.}} \cdot S'_{(n-1)}\ T^{-r} \cdot \text{——}_{\text{meta}}$$

4b) Common meta coordination with equalizing operator for different degrees of development

$$i_x\ \text{——}\ \text{obj},\ 1, \ldots, n : S'_{(n-1)}\ T^{-r}\ i^{+x_1, x_2, \ldots, x_n} : a\ \text{——}\ \text{meta}$$

The n object languages with the indices 1, - - -, n and of the various degrees of development "i_x" are coordinated to the single meta language of the degree "a" by means of the many-one coordinative relation $S'_{(n-1)}$ and by means of the symbolic operators equalizing the differences in the degree of development "$i^{+x_1, x_2, \ldots, x_n}$" and creating the reductions of type levels for names in the meta language, expressing structures of the object language.

This schema has a great importance by virtue of its unifying quality. The concept of "unification within the range of semantics" is the application of this schema. The classical dream of Descartes and Leibniz, known as the "unification of sciences" receives its modern redefinition using similar schemata over n heterogenous domains.

With respect of this important aspect, we may attach a second reading of the above schema, concentrated to the unifying capacity of it:

Unifying transformative translation is a chain of sets of operations starting at greatly different source "object-languages" and resulting in a single target meta-language of a high level of technical development.

Now, let us collect systematically the schemata, for which we have shown different examples. They demonstrate, that "translation" is a special case within a long list of various schemata. For each of the special cases there are practical examples existing.[1] The gist of this paper is to demonstrate the comparatively simple operations, reduced to a one-many relation with two operators, making up the formal skeleton of these seemingly unconnected structures used in many fields of our daily life.

Relational schemata for the two domain approach

A. The general form of the constituent operations:
A1). heterogenous:

$$\cdots . Z_{n(1-1)}\ T^{\pm r}\ i^{\pm m} . \cdots_1, \cdots_2, . , --- n$$

A2). homogenous, vacuogenous:

$$\cdots . S_{n(1-1)}\ T^{\pm r}\ i^{\pm m}\ \cdots_1, \cdots_2, . , \cdots n$$

B. Constituents as specific cases of A1) and of A2).
2. Simple constituent coordinative operations.

2a) $\cdots . Z_{1-1} . ---$ with the variants $\cdots . \bar{Z} . ---$

$--- . Z . \cdots$

$--- . \bar{Z} . \cdots$

2b) $\cdots . Z_{n(1-1)} . ---_1, . \quad , ---_n$

2c) $\cdots . S_{1-1} . \cdots$

2d) $\cdots . S_{n(1-1)} . \cdots_1, . \quad , \cdots n$

3. Constituent coordinative operations with T operator.

3a) $\cdots . Z_{1-1} . T^{\pm r} . ---$

3b) $\cdots . Z_{n(1-1)}\ T^{\pm r} . ---_1, . \quad , ---_n$

3c) $\cdots . S_{1-1}\ T^{\pm r} . \cdots$

3d) $\cdots . S_{n(1-1)}\ T^{\pm r} . \cdots_1, . \quad , \cdots_n$

4. Constituent coordinative operations with i-operator.

4a) $\cdots . Z_{1-1}\ i^{\pm m} . ---$

4b) $\cdots . Z_{n(1-1)}\ i^{\pm m} . ---_1, . \quad , ---_n$

4c) $\cdots . S_{1-1}\ i^{\pm m} . \cdots$

4d) $\cdots . S_{n(1-1)}\ i^{\pm m} . \cdots_1, . \quad , \cdots n$

5. Constituent coordinative operations with T and i operators.

5a) $\cdots . Z_{1-1} . T^{\pm r}\ i^{\pm m} . ---$

5b) See A1)

5c) $\cdots . S_{1-1}\ T^{\pm r}\ i^{\pm m} . \cdots$

5d) See A2).

Variants may occur as shown at 2a).
Remark to 2.-4. : --- . S . --- is an abbreviation for --- . Z . $\cdots$. $\bar{Z}$. ---.

C. General form of the constituent operation with its result.
C1) heterogenous:

$$\cdots . Z . \text{---} :=: \text{———}$$

C2) homogenous, vacuogenous,

$$\cdots . S . \cdots :=: \cdots$$

C3) homogenous coordination of results of former coordinations:

$$\text{———} . S' . \text{———} :=: \text{———}$$

T and i operators may be inserted, if necessary in C1) - C3).
For each of these schematic cases examples are existing.

D. *Connected constituent operations.*

Example 1).: Translation and retranslation. Source and target structures are of the same T and i levels.

If $\text{---}_a \cdot = \text{def.}\ \text{---}_b \cdot =\text{def.}\ \text{---}$

then $\text{———}_a \cdot = \cdot \cdots_a . Z . \text{---}_a$

$$\text{---}_b . \bar{Z} . \cdots_b \cdot = \cdot \text{———}_b$$

$$\overline{\text{———}_a \cdot = \cdot \cdots_a . Z . \text{---} . \bar{Z} . \cdots_b \cdot = \cdot \text{———}_b}$$

is the schema of translation from the source language $\mathbf{L}_a$ into the target $\mathbf{L}_b$.

Retranslation means to give the task of source language to $\mathbf{L}_b$ after completion of the first translation. If the resulting C-structure in $\cdots a$ is identical with the original one, the concatinated set of operations closes without loss (or with a "zero balance").

High quality fully automatic translation is supposed to work with a zero balance if retranslated. In general, the upper limit of efficiency is given by a closed ring of concatinated operations of direct and converse direction without any loss. This condition remains valid with and without the occurrence of T and i operators.

Example 2). Transformation of an object-source language in a meta-target language, (both of the same degree of development).

Condition of transformation:

$$\mathbf{D}_{1.2,a} . S' . \mathbf{D}_{1.2,b}$$

or: the coordinated domains and the structures erected over them are the concatinating elements.

$$\text{——}_a \,.\, S'\, T^{-r} \,.\, \text{——}_b \quad \text{and} \quad (\exists r)\, \mathsf{L}_a \,.\, T^{-r} \,.=.\, \mathsf{L}_b$$

E. Single domain expressed by means of the second single domain and the given relation of the two domains.

To express one of the single domains by means of the coordination-result and the other domain the following method of notations is useful:

E1) $$\cdots \,.=.\, \frac{\text{——}}{\text{- - -}}$$

E2) $$\text{- - -} \,.=.\, \frac{\text{——}}{\cdots}$$

E3) $$\text{If } \cdots_1 \,.\, S \,.\, \cdots_2 \,.=.\, \cdots_{1.2}$$

$$\cdots_1 \,.=.\, \frac{\cdots_{1.2}}{\cdots_2} \quad \text{resp.} \quad \cdots_2 \,.=.\, \frac{\cdots_{1.2}}{\cdots_1}$$

Thus, if two of the three structures are known, we may express the third by these relational schemata.

Further, we may include the operators $T^{\pm r}$ and $i^{\pm m}$ into the schemata E) and create a notation of additional expressive possibilities, e.g.:

$$\cdots\, T^2\, i^2 \,.=.\, \frac{\text{——}}{\text{- - -}}$$

F. Schemata for valuation and similar meta-coordinations. An evaluation (over two domains) presupposes:

a) a meta-relation, i.e. a T^{-a} operator attached to Z or to S.
b) an ordered series of evaluator arguments or "values".
c) one-many valued case of the relation, with the evaluators reserved for the one-valued domain, the value being coordinated to *a class* with members considered as equals in respect of the "value".
d) An intermediate occurrence of $\mathbf{D}_2$ within the concatination. Schemata without $\mathbf{D}_2$, e.g. $\cdots .S. \cdots$ or $\cdots .S\ T. \cdots$ belong to mathematic or pure syntax and may be used for mapping or for Gödel-numbers, but not for evaluation in the above or in the intuitive sense.

F1) Value over $\mathbf{D}_1$ or logical evaluation.

F1a) $\cdots .\, Z_{1-n} \,.\, T^{-a} \,.\, \text{——} \,.\vee.\, \text{- - -}$

F1b) $\cdots_1 \;.\, S_{1-n} \,.\, T^{-a} \,.\, \cdots_{1.2}$ and $\cdots_2 \;.\, \bar{S}_{1-n} \,.\, T^{-a} \,.\, \cdots_{1.2}$

F2) Semantical evaluation.

F2a) $\text{——} \,.=.\, \text{- - -} \,.\, \bar{Z}\, T^{-a} \,.\, \cdots : Z_2 : \text{——} \,.\vee.\, \text{- - -}$

F2b) $\text{———} \;.\!=\!.\; \text{---}\;.\, S\, T^{-a} . \cdot \cdot \cdot : S_2 : \cdot \cdot \cdot$

"True" and "false" are the highest and lowest members of the ordered series for Fb).

Conclusions. "Translation" is a special case of a general operation presupposing two mutually exclusive domains, their coordination and a simultaneous occurrence of structures taken from these two domains.

Many well-known scientific and other structures are examples of a simple combinatorics and simple concatinations of constituent structures of different levels of types, taken from the two domains. A few of them were shown as examples:

Language; Code; multiple interpretation, translation into a common target language; transduction; 'common-meta' transformative translation or "unification"; valuation. But this list may be continued to include two-basic formulas, e.g. for phrase-structures, for translatory-automatons, for punch-card methods, etc., etc.

The two basic method and the two-basic explanations are special cases of the n-basic approach.

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