1. As a rule modern textbooks of traditional logic distinguish only two kinds of syllogism: the categorical syllogism, which has originated with Aristotle, and the hypothetical syllogism, which goes back to the early Peripatetics and to the Stoics. Rarely, if ever, is mention made of the third kind of syllogism namely the prosleptic syllogism. Yet, the prosleptic syllogism, for which we seem to be indebted to Theophrastus, appears to have been regarded at least by some logicians in later ages of antiquity as a legitimate part of logical theory.

Like the expressions 'categorical' and 'hypothetical' the expression 'prosleptic' is a technical term and its full significance can only emerge at a later stage of our enquiry. At this stage suffice it to say that 'prosleptic' is meant to render the Greek expression 'κατὰ πρόσληψιν' in its adjectival use.

Although the prosleptic syllogism has not played as important a rôle in the development of logic as the other two kinds of syllogism, it deserves our attention particularly for the following two reasons. First, the validity of prosleptic syllogisms is based, as we shall see, on certain logical notions which in modern logic find their expression in the use of the universal quantifier. Secondly, the theory of prosleptic syllogism bears witness to the resourcefulness of Theophrastus as a logician.

In what follows I propose to reconstruct the theory of prosleptic syllogisms to the extent to which the scarcity of textual evidence permits, and to examine it from the point of view of modern logic.

2. A very brief and fragmentary exposition of the theory of the prosleptic syllogisms can be found in the anonymous scholium preserved in the Codex Parisinus Graecus 2064, f. 261v-263v, and published by M. Wallies in the Preface to his edition of Ammonii in Aristotelis Analyticorum Priorum Librum I Commentarium, Commentaria in Aristotelem Graeca, Vol. 4, pt. 6, Berolini 1899, p. IX sq. The scholium is entitled 'On all the forms of syllogism' (Περὶ τῶν εἰδῶν πάντων τοῦ συλλογισμοῦ). It consists of three parts. Having stated that there are three forms of simple syllogism, the categorical, the hypothetical, and the prosleptic,\(^1\) the anonymous scholiast

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distinguishes, in the first part of his compendium, the simple categorical syllogism, which falls into the three Aristotelian figures, and the composite categorical syllogism, which according to Galen falls into four figures. Then he goes on to explain the four Galenian figures basing his account on Galen’s *De Demonstratione*, which unfortunately is not extant. The second part of the compendium deals with the hypothetical syllogism, both the simple and the mixed, while the third part concerns the prosleptic syllogism. The scholiast explicitly attributes the theory of the prosleptic syllogism to Theophrastus. He then shows that this type of syllogism also falls into three figures. The first figure is exemplified with the aid of the following inference:

(1) (i) *whatever* (is predicated) of *man* universally, 
    *substance* (is predicated) of *it* universally; 
    now, (ii) *animal* (is predicated) of *man* universally 
    therefore, (iii) *substance* (is predicated) of *animal* universally.

Examples of the remaining two figures are given in abbreviated form. We can, however, easily expand them. On doing so we get, for the second figure, an inference which can be formulated as follows:

(2) (i) *whatever* is predicated of *man* universally, 
    *it* is predicated of *horse* universally; 
    now, (ii) *animal* is predicated of *man* universally; 
    therefore, (iii) *animal* is predicated of *horse* universally.

Finally, the example which was meant to illustrate an inference of the third figure can be expanded in this way:

(3) (i) of *whatever entity animal* is predicated universally, 
    *rational* is predicated of *it* universally; 
    now, (ii) *animal* is predicated of *man* universally; 
    therefore, (iii) *rational* is predicated of *man* universally.  

The scholiast continues by discussing 'the middle term' in his inferences, and the passage ends with some rather irrelevant criticism of the prosleptic syllogism. 

3. In the same codex we find yet another passage which throws further light on our subject. The passage, f. 255v-256r, is entitled 'On prosleptic syllogisms' (*Προς τον καρα προσληψειν συλλογισμόν*). It is included in a sort of appendix to the commentary to Aristotle’s *Prior Analytics* by Ammonius but it cannot be attributed to Ammonius with certainty. The anonymous contributor to the commentary, or his authority, tells us that the prosleptic syllogism has this in common with the categorical syllogism that like the latter it can be validly construed in all the figures. This is illustrated with examples, which, however, are not complete inferences. So in fact we are shown that the premisses which are characteristic of prosleptic syllogisms can be regarded as falling into the three figures.

In the first figure we have

*whatever* (belongs) to *c* in every instance, *a* (belongs) to *it* in every instance
The second figure is represented by

whatever (is predicated) of $b$ universally, $it$ (is) also (predicated) of $c$ universally

And the third figure contains

of whatever (entity) $a$ (is predicated) universally, $b$ (is) also (predicated) of $it$ universally.

A similar classification of our premisses can be found in an anonymous scholium to Aristotle's Prior Analytics, Book II, Chs. 5-7, published by C. A. Brandis.7

4. The weight of anonymous evidence may always appear to be dubious. Fortunately enough inferences analogous to inference (2) and inference (3) are given as examples of the prosleptic syllogism by Galen in his Institutio Logica.8 Galen was a very keen student of logic and made some original contributions to the Aristotelian syllogistic by working out a theory of the composite categorical syllogism. He discussed the theory of the prosleptic syllogism in his treatise De Demonstratione (Περὶ ἑταὶ ἀποδείξεως), where he showed which inferences were to be regarded as prosleptic syllogisms and how many they were. He criticised the theory on the ground that prosleptic syllogisms were, in his view, mere abbreviations of categorical syllogisms, and that consequently they were redundant altogether. However, the De Demonstratione has not been preserved, and neither the details of Galen's exposition nor the argument in support of his criticism are known to us. The few remarks which we find in the Institutio Logica are very sketchy and offer less help than we might have wished for. Galen does not mention the name of Theophrastus in connection with the theory of the prosleptic syllogism. He says, however, quite generally that Peripatetics had written about them and considered them useful. To sum up the significance of Galen's testimony lies largely in that in no respect does it refute our anonymous evidence.

5. It is evident from the examples already given that a prosleptic syllogism is an inference which consists of three propositions. Two of them, viz., one of the premisses and the conclusion, are categorical propositions. The third proposition, in our examples it happens to be premiss (i), is a different one. Propositions of this type were called prosleptic premisses and, according to our sources, it was Theophrastus who first called them so.9 He was also the first to study the logical significance of such propositions in some detail.

Now in order to proceed with our analysis let us translate the three inferences into the idiom of modern logic. In this way we shall be able to bring to light the structure of the prosleptic syllogism in general and that of the prosleptic premiss in particular.

The translation of the categorical propositions which occur as parts of a prosleptic syllogism presents no difficulty. The translation of the prosleptic premisses, (1i), (2i), and (3i), is equally simple once we have realised that without altering their meaning in the least we can paraphrase them as follows:
(1) (i') for all $x$, - if $x$ is predicated of *man* universally then *substance* is predicated of $x$ universally.

(2) (i') for all $x$, - if $x$ is predicated of *man* universally then $x$ is predicated of *horse* universally.

(3) (i') for all $x$, - if *animal* is predicated of $x$ universally then *rational* is predicated of $x$ universally.

From the paraphrase we immediately see that a prosleptic premiss is an implication preceded by the universal quantifier. The antecedent and the consequent of the implication are categorical propositions, or rather propositional functions, and the variable bound by the universal quantifier occurs in them either as subject or as predicate. It also becomes evident that, in fact, a prosleptic premiss contains three terms, which does not seem to have escaped the notice of ancient logicians. In his commentary to Aristotle's *Prior Analytics* Alexander remarks that "in a way they (i.e., the premisses which Theophrastus calls prosleptic premisses) have three terms. For in the premiss 'of whatever (entity) $b$ is predicated universally, $a$ (is predicated) of it universally' the two terms, '$b$' and '$a$', which are definite, already contain the third term of which $b$ is predicated except that this term is not definite or explicit in the sense in which the other terms are." And in the scholium published by Brandis we read that a prosleptic premiss consists of an indefinite middle term and two definite extreme terms, and that it is like the hypothetical implicative syllogism. Now, the hypothetical implicative syllogism is the one which Chrysippus had called the first indemonstrable (πρώτος ἀναδεικτος), and which later on became known as *modus ponens*. So we would say perhaps that a prosleptic premiss is like a hypothetical premiss, namely like an implication, in a hypothetical syllogism. Similarly, Alexander would have been more precise had he said that in a prosleptic premiss the third term was contained in 'of whatever . . . of it . . .' $(καθ' υπ. . . καθ' εἰσίν. . . ορ. . . τοῦτω . . .)$ rather than in the remaining two terms.

Let us, however, revert to our three inferences, and let us rewrite them in a symbolic language. For this purpose I propose to use Łukasiewicz's symbolism. In this symbolism, as is well known, expressions '$A a b$', '$E a b$', '$I a b$', and '$O a b$' represent, respectively, the universal affirmative proposition 'every $a$ is $b$', the universal negative proposition 'no $a$ is $b$', the particular affirmative proposition 'some $a$ is $b$', and the particular negative proposition 'some $a$ is not $b'$. The expression '$C a b$' stands for the implication 'if $a$ then $b$', and finally the expression '$Π x$' stands for the universal quantifier and is to be read 'for all $x$'.

Now if we put variables instead of the extra-logical constants 'man', 'substance', 'animal', 'horse', and 'rational', and if we remember the way in which the first premiss in each of the three inferences could be paraphrased, then we obtain the following three inference-schemata:

(1')

\[
\begin{align*}
&\text{(i)} \quad Π x \; C \; A x \; A x b \\
&\text{now, (ii)} \quad A a c \\
&\text{therefore, (iii)} \quad A c b
\end{align*}
\]
The validity of inferences constructed in accordance with any of the three schemata is fairly obvious. Thus, for instance, an inference constructed in accordance with schema (1") is valid in virtue of the following logical law

\[ C \Pi x C A a x A b x C A a c A b \]

which can be deduced from the law of identity 'C p p' by successively applying the rule of substitution, the rule which allows us to add a universal quantifier to the antecedent of any implication which is a law, and the rule of substitution again. In the same way the other two schemata can be shown to be valid.

6. Our symbolic formulation of inferences exemplified by (1'"), (2'"), and (3'") enables us to see at a glance that it was the position of the indefinite term (\( \alpha \theta \rho ι \alpha \) of \( \theta \rho ο \)), which corresponds to the bound variable in our inference schemata, that served as the principle of classification of prosleptic premisses, and prosleptic syllogisms, into three figures. A prosleptic premiss in which the indefinite term occurred as the predicate in the antecedent and as the subject in the consequent was said to belong to the first figure. And so was the corresponding prosleptic syllogism. A prosleptic premiss in which the indefinite term occurred as the predicate in both the antecedent and the consequent was said to belong to the second figure. The corresponding prosleptic syllogism was also regarded as belonging to the second figure. Finally, a prosleptic premiss in which the indefinite term occurred as the subject in both the antecedent and the consequent was said to belong to the third figure, and the same applied to the corresponding prosleptic syllogism.

The simple propositions involved in those prosleptic syllogisms which we find, in truncated form, in the Codex Parisinus Graecus 2064 are all universal affirmative propositions. This has been reflected in our inference schemata. We have, however, to note that the Brandis scholium makes no indication of the quantity or the quality of the antecedent or the consequent in a prosleptic premiss, which suggests that in this respect they could vary. And, indeed, from the comments in the Codex Parisinus we learn that there are valid prosleptic syllogisms with two 'negative' or two 'particular' premisses. Evidently, the anonymous scholiast must have had in mind inferences like these:

\[(i) \Pi x C E a x A b \]
\[\text{now, (ii) } E a c \]
\[\text{therefore, (iii) } A c b \]
(i) \( \Pi x C \ I a x A x b \)

now, (ii) \( I a c \)

therefore, (iii) \( A c b \)

He would probably describe the prosleptic premiss in the first inference as negative whereas the prosleptic premiss in the second inference would for him be an instance of a particular proposition.

In any case our inference schemata (1'), (2'), and (3') cannot be regarded as adequately summarising the theory of the prosleptic syllogism, and have to be generalised. We modify them as follows:

First Figure

(1''')

(i) \( \Pi x C \Phi a x \Psi x b \)

now, (ii) \( \Phi a c \)

therefore, (iii) \( \Psi c b \)

Second Figure

(2''')

(i) \( \Pi x C \Phi a x \Psi b x \)

now, (ii) \( \Phi a c \)

therefore, (iii) \( \Psi b c \)

Third Figure

(3''')

(i) \( \Pi x C \Phi x a \Psi x b \)

now, (ii) \( \Phi c a \)

therefore, (iii) \( \Psi c b \)

In these inference schemata the Greek letters \( \Phi \) and \( \Psi \) stand for any of the four functors which form categorical propositions. In other words they stand for 'A', 'E', 'I', or 'O'.

At this stage it is appropriate that we should consider a question which cannot have failed to suggest itself to our minds already. Is there a fourth figure of the prosleptic syllogism with the following inference schema?

Fourth Figure

(4''')

(i) \( \Pi x C \Phi x a \Psi b x \)

now, (ii) \( \Phi c a \)

therefore, (iii) \( \Psi b c \)

I have not been able to find any evidence to the effect that inferences of this type were regarded by ancient logicians as constituting a fourth figure. A syllogism which is constructed in accordance with schema (4''') with the functor 'O' and 'I' in the place of '\( \Phi \)' and '\( \Psi \)' respectively, can be found in the Commentary to Aristotle's Prior Analytics by Philoponus, and there is a passage in the same commentary which presupposes another syllogism of this type with the functors 'O' and 'A'. It is quite obvious that in view of the laws exhibited in the square of opposition the law of transposition \( 'C C p q C N q N p' \) enables us to reduce any prosleptic premiss of the fourth figure to one in the first, but to my knowledge there is no evidence that the ancient logicians knew that. Nor is there any evidence
that they considered inferences in which the functor of the categorical premiss of a prosleptic syllogism was contradictory to the functor in the consequent of the prosleptic premiss. One is left with the impression that the possibility of a fourth figure was ignored in order to save the analogy to Aristotle's classification of the categorical syllogism. In this connection it is perhaps of interest to mention that the so-called wholly hypothetical syllogisms or simple hypothetical syllogisms were divided by Theophrastus into three figures too.\textsuperscript{18}

7. One of the problems discussed by Galen in his treatise \textit{De Demonstratione} concerned the number of possible prosleptic syllogisms. Unfortunately no details of his calculations are known to us.\textsuperscript{9} It is clear, however, that the number of different prosleptic syllogisms will be the same as the number of different prosleptic premisses. Now, every prosleptic premiss requires two categorical functors, which means that with four such functors we have 16 different premisses in each figure. This makes 48 different premisses in the three figures, and 64 if we take into consideration the figure that is not explicitly mentioned in our authorities. In the course of my research leading to the present paper I was able to identify only eleven different syllogisms.

8. Our reconstruction of the theory of the prosleptic syllogism was based on rather late and fragmentary sources but there can be no doubt that the theory was first developed by Theophrastus. In this respect the anonymous evidence is supported by Alexander, whose testimony hardly calls for additional confirmation.\textsuperscript{10} It is, however, more than probable that the whole conception of the prosleptic syllogism was derived by Theophrastus from the writings of his master. In particular Chapters 5-7 of the Second Book of Aristotle's \textit{Prior Analytics} must have played a decisive rôle in directing the attention of Theophrastus to the possibility of a new logical theory. The chapters that have just been referred to are devoted to the discussion and application of what Aristotle calls the circular and reciprocal proof or demonstration (τὸ κύκλῳ καὶ τὴν αλήθην διέκδικον σοφατί). The procedure involved by this 'circular proof' can be described as follows. As the point of departure we take a valid categorical syllogism with premisses \textit{a} and \textit{β}, and a conclusion \textit{γ}. Then we consider two inferences, the one with \textit{γ} and the converse of \textit{a} as the premisses and \textit{β} as the conclusion, and the other with \textit{γ} and the converse of \textit{β} as the premisses and \textit{a} as the conclusion. If any of these two inferences turns out to be a valid syllogism, we say that we have derived it by means of the circular and reciprocal proof. The method, however, is not universally applicable. In some cases on effecting the prescribed transformation of a valid syllogism we derive another valid syllogism but in some cases the result of the transformation is invalid. In the chapters under consideration Aristotle systematically examines the results of applying the method of the circular proof to valid syllogisms, and lists the successful cases and also the cases in which the method breaks down.

Let us now illustrate the circular proof with the aid of concrete examples. Consider the syllogism in \textit{Barbara}
(5) (i) every \( b \) is \( a \) now, (ii) every \( c \) is \( b \) therefore, (iii) every \( c \) is \( a \)

On transforming this syllogism in the way described above we get the following two inferences:

(6) (i) every \( c \) is \( a \) now, (ii) every \( a \) is \( b \) therefore, (iii) every \( c \) is \( b \)

and

(7) (i) every \( c \) is \( a \) now, (ii) every \( b \) is \( c \) therefore, (iii) every \( b \) is \( a \)

In the present case the syllogisms derived by means of the circular proof are valid. Consider, however, a syllogism in *Celarent*

(8) (i) no \( b \) is \( a \) now, (ii) every \( c \) is \( b \) therefore, (iii) no \( c \) is \( a \)

By applying the circular and reciprocal procedure we get

(9) (i) no \( c \) is \( a \) now, (ii) every \( b \) is \( c \) therefore, (iii) no \( b \) is \( a \)

and

(10) (i) no \( c \) is \( a \) now, (ii) no \( a \) is \( b \) therefore, (iii) every \( c \) is \( b \)

Now, inference (9) is a valid syllogism but inference (10) is not. This was known to Aristotle. He remarks that by converting the original premiss 'no \( b \) is \( a \)' into 'no \( a \) is \( b \)' we do not get the required result, which can, however, be secured if we convert 'no \( b \) is \( a \)' into the proposition which says that

\[
\text{to whatever (entity) } a \text{ belongs in no instance, } b \text{ belongs to it in every instance}
\]

For then we have the following valid inference:

(11) (i) to whatever (entity) \( a \) belongs in no instance, \( b \) belongs to \( it \) in every instance  
now, (ii) no \( c \) is \( a \), i.e., \( a \) belongs to \( c \) in no instance  
therefore, (iii) \( b \) belongs to \( c \) in every instance, i.e., every \( c \) is \( b \)

The validity of (11) becomes even more perspicuous if we translate the inference into our symbolic language. On doing this we get:
Similar difficulties occur in the case of syllogisms in *Ferio*. Consider for instance the following inference schema

(12)  
(i) no b is a  
now, (ii) some c is b  
therefore, (iii) some c is-not a

In accordance with the circular and reciprocal procedure (12) yields

(13)  
(i) some c is-not a  
now, (ii) some b is c  
therefore, (iii) no b is a

and

(14)  
(i) some c is-not a  
now, (ii) no a is b  
therefore, (iii) some c is b

Now, neither of these two inferences is valid. In the case of inference (13) Aristotle does not even consider how to transform the original premiss 'some c is b' so as to effect the proof of the universal negative premiss 'no b is a' on the assumption that the proposition 'some c is-not a' is to be used as the other premiss. He simply points out that the premiss 'some c is-not a' being particular no universal conclusion is possible. As regards inference (14) he remarks that on assuming that some c is-not a one can prove that some c is b provided we convert the premiss 'no b is a' in a somewhat similar way to the way in which the conversion was performed in the case of *Celarent*, namely if the major premiss takes the form of the following expression:

\[ \text{to whatever (entity) } a \text{ does not belong in some instance, } b \text{ belongs to it in some instance} \]

Thus instead of (14) we get the following valid inference:

(15)  
(i) to whatever (entity) a does not belong in some instance, b belongs to it in some instance  
now, (ii) some c is-not a, i.e., a does not belong to c in some instance  
therefore, (iii) b belongs to c in some instance, i.e., some c is b

And the symbolic translation of this inference is as follows:

(15')  
(i) \( \Pi x C O x a I x b \)  
now, (ii) \( O c a \)  
therefore, (iii) \( l c b \)

It is obvious that inferences (11) and (15) are prosleptic syllogisms in the Theophrastian sense. Aristotle introduces them somewhat casually. He has no special name for them to distinguish them from categorical
syllogisms. The expression διὰ προσλήψεως in Prior Analytics 58b9 is regarded by scholars as an interpolation of post-Aristotelian origin.23

As we have seen inference (11) is used by Aristotle in connection with his attempt to apply his circular and reciprocal procedure to Celarent. He could have made use of it when he discussed Cesare but he seems to have failed to realise this. Inference (15) is mentioned three times, namely in connection with Ferio, Festino, and Ferison.24 No other instances of the prosleptic syllogism are to be found in Aristotle's discussion of the circular and reciprocal procedure although it is not very difficult to see that if we take any categorical syllogism then it is possible to prove any of its premises by using the conclusion and an appropriate prosleptic premiss.

I have discussed Aristotle's circular procedure at some length because it seems to me that Chapters 5-7 of Book II of the Prior Analytics constituted the starting point for Theophrastus theory of the prosleptic syllogisms. Theophrastus must have noticed that in addition to inferences (11) and (15) given by Aristotle other similar inferences could be constructed and that the number of different prosleptic premises could be increased. He also noticed that prosleptic premises could be arranged into three figures in accordance with a principle analogous to the one adopted by Aristotle in his classification of categorical syllogisms. This, of course, has no logical significance but it seems to have impressed Theophrastus so much that he overlooked the possibility of more interesting ways in which his new theory could be developed.

9. I have already mentioned that Galen criticised prosleptic syllogisms on the ground that they were abbreviations of categorical syllogisms.9 Perhaps the term 'abbreviation' (επιτομή) is not quite appropriate in this connection since it is the categorical syllogism to which a prosleptic one is supposed to be reducible that is in fact shorter and simpler of the two inferences. In any case, Galen's point was that prosleptic syllogisms were, as it were, categorical syllogisms in disguise. This would be so if it could be shown that every prosleptic premiss was equivalent to one categorical proposition or another. And indeed in some cases the equivalence holds and was known to hold to ancient logicians. As I indicated above, Aristotle does not seem to have made a study of prosleptic premises or prosleptic syllogisms, but he knew that the proposition 'every b is a' was equivalent to the one which says that

\[(16) \quad \text{of whatever entity } b \text{ is predicated, } a \text{ is predicated of it} \] 25

This is a prosleptic premiss in the sense given to the expression by Theophrastus but its antecedent and consequent are both indefinite or perhaps singular propositions. Thus if we put \('U a b'\) to stand for the indefinite or singular 'a is b' then we can express (16) as follows:

\[(16') \quad \Pi \times C U \times b U \times a \]

Similarly, the proposition 'no a is c' appears to have been regarded by Aristotle as equivalent to the proposition which says that

\[(17) \quad \text{to whatever entity } a \text{ belongs, } c \text{ does not belong to it} \] 26
which, with 'Y a b' standing for the indefinite or singular 'a is not b', lends itself to the following symbolic translation:

\[(17') \Pi x C U x a Y x c\]

From the point of view of intuitiveness we can have no objections to the equivalences presupposed by Aristotle, always bearing in mind that in Aristotelian logic empty or fictitious noun expressions were not in the range of nominal variables. The weakness of the equivalences consisted in that they involved indefinite propositions, which in logic have no status of importance.\(^{27}\) In a proposition like (16) or (17) it would be the most natural thing to interpret both the antecedent and the consequent as singular propositions but singular propositions were shunned by ancient logicians.\(^{28}\) On the other hand if the antecedent and the consequent in a proposition like (16) or (17) were interpreted as particular propositions, in accordance with the practice of Aristotle in other contexts, then the equivalences would lose some of their intuitiveness. Now, Theophrastus appears to have noticed that the indefinite propositions embedded in (16) could be replaced by the corresponding universal propositions, without affecting the truth value of the whole. In his treatise On Assertion he held, so Alexander reports, that the proposition 'of whatever (entity) b (is predicated), a (is predicated of it)' was equivalent to the proposition

\[(18) \text{ of whatever (entity) } b \text{ is predicated universally, } a \text{ (is predicated of it universally)}\]

In terms of our symbolic language we can say that according to Theophrastus (16') was equivalent to

\[(18') \Pi x C A x b A x a\]

The next step was to equate (18) with the corresponding proposition 'every b is a'. That this step was in fact made by Theophrastus is amply attested by Alexander and by the Brandis scholium, which adds that in Theophrastus' view the proposition 'a (is predicated) of no b' was equivalent to the proposition 'of whatever (entity) b (is predicated) in every instance, a (is predicated of it in no instance').

To sum up we can credit Theophrastus with establishing three interesting and important equivalences, which with 'Q a β' standing for 'a if and only if β' can be given the following symbolic form:

\[(19) Q \Pi x C U x b U x a \Pi x C A x b A x a\]
\[(20) Q A b a \Pi x C A x b A x a\]
\[(21) Q E b a \Pi x C A x b E x a\]

Since the range of nominal variables in Aristotelian logic is restricted to shared names these equivalences are logically unassailable. We can only regret that our meagre sources do not tell us about any other equivalences that Theophrastus may have established between categorical and prosleptic premisses.
10. A different and, as far as I can judge, somewhat erroneous evaluation of Theophrastus contributions discussed in the preceding section has been given by Father Bocheński. According to Father Bocheński Theophrastus, as reported by Alexander, wrongly assumed the equivalence between

$$\text{(22)} \quad C \phi \land \psi$$

and

$$\text{(23)} \quad C \Pi \phi \land \Pi \psi$$

when he maintained that $\kappa a\theta' \overline{\psi} \varphi \odot \beta$, $\odot \beta \theta$ was equivalent to $\kappa a\theta' \overline{\psi} \varphi \odot \beta$, $\kappa a\theta' \overline{\psi} \varphi \odot \beta \theta$. For while (22) implies (23), argues Father Bocheński, (23) does not imply (22), which was well established by Aristotle.\(^{31}\)

Now, it is quite correct to say that (23) does not imply (22), but it is also correct to say that (22) does not imply (23) as can easily be shown by giving the variables an appropriate interpretation. The point is that neither (22) nor (23) appear to be the right translations of what Theophrastus is reported to have said. The language of the Functional Calculus is not perhaps the most suitable for translating expressions of Aristotelian logic, but if we were to use this language then the Theophrastian $\kappa a\theta' \overline{\psi} \varphi \odot \beta$, $\odot \beta \theta$ would have to be rendered with the aid of

$$\text{(24)} \quad \Pi \theta C \Sigma x K \theta \land \phi \land \Sigma x K \theta \land \psi x \quad ^{32}$$

or

$$\text{(25)} \quad \Pi x C \phi \land \psi x$$

depending on whether we wanted to interpret indefinite propositions as particular propositions or as singular ones. The translation of the proposition $\kappa a\theta' \overline{\psi} \varphi \odot \beta$, $\kappa a\theta' \overline{\psi} \varphi \odot \beta \theta$ is even more complicated. For as Prior has pointed out in his *Formal Logic* it has to have the following form:

$$\text{(26)} \quad \Pi \theta C \Pi x C \theta \land \phi \land \Pi x C \theta \land \psi x \quad ^{33}$$

It is fairly obvious that (24), (25), and (26) are all equivalent which shows again that Theophrastus was right. Father Bocheński’s criticism of the Greek logician is based on what appears to be a mistaken symbolic translation. Consider, for instance, (23). It says that if everything is $\phi$ then everything is $\psi$ (or rather if everything $\phi$’s then everything $\psi$’s where ‘$\phi$’s’ and ‘$\psi$’s’ are, as it were, verbs in the third person singular). Clearly, this is not what is conveyed by the Theophrastian $\kappa a\theta' \overline{\psi} \varphi \odot \beta$, $\kappa a\theta' \overline{\psi} \varphi \odot \beta \theta$.

Father Bocheński’s interpretation of what Aristotle says in the Prior Analytics, I 41, 49b14-16 seems to suffer from a similar defect. In his *La logique de Théophraste* Father Bocheński suggests that in this passage of the Analytics Aristotle denies the equivalence of propositions represented by formulae (22) and (23) respectively. In his *Ancient Formal Logic* Father Bocheński writes that the propositions examined by Aristotle in the passage under consideration can be interpreted by

$$\text{(27)} \quad Bx \subset (x)Ax$$

and

$$\text{(28)} \quad (x)Bx \subset (x)Ax \quad ^{35}$$
Now, if we turn to the text then we find that the two propositions involved can be translated as follows:

(29) to whatever (entity) b belongs, a belongs to it in every instance

and

(30) to whatever (entity) b belongs in every instance, a belongs to it in every instance

In our symbolism they can be expressed thus:

(29') \( \Pi \times C \times U \times b \times A \times a \) and (30') \( \Pi \times C \times A \times b \times A \times a \)

It is evident that (29) and (30) are prosleptic propositions, and so they are described by Alexander. Notice that proposition (29) has an indefinite antecedent. If we interpret it as a singular proposition then (29) and (30) turn out to be equivalent contrary to Aristotle's contention, but if we interpret the antecedent of (29) as a particular proposition, i.e., if we understand (29) as meaning the same as

(29'"") \( \Pi \times C \times I \times b \times A \times a \)

then we will easily see that (29) implies (30) while the converse implication does not hold. Thus if Aristotle's claim is to be upheld, we have to regard (29'"") as the correct interpretation of (29). It may be of interest to add that this is exactly how Alexander understood proposition (29). For in his commentary he equated proposition (29) with the one that says 'to whatever (entity) b belongs in some instance, a belongs to it in every instance'.

11. This seems to be all that could be gleaned from our sources for the purpose of reconstructing the Theophrastian theory of prosleptic premisses and prosleptic syllogisms. It is hoped that by now the meaning of the technical term 'prosleptic' has become a little clearer. Following our anonymous authority we can repeat that prosleptic premisses were called so because each of them contained an indefinite term, or a bound variable as we would say. Once this term has been made definite, i.e., once a constant noun expression has been substituted for the bound variable, the prosleptic premiss becomes an implication, which, granted its antecedent, yields its consequent as the conclusion in a valid inference of the modus ponens type. Inferences which originated from prosleptic premisses in this way were called prosleptic syllogisms.

Finally we ought to remember that in the terminology of the Stoic logicians the term πρόσληψις (or προσωλαμβάνειν) designated the minor premiss in their hypothetical inferences. This use of the term should be clearly distinguished from the one established by Theophrastus and discussed in the present paper.

NOTES

ON PROSLEPTIC SYLLOGISMS


4. Cf. Ammonius l.c. p. XII, 3: ἵστα ὑὰ ῥ καὶ τρίτον ἔδος συλλογιμοὶ μετὰ τὸ κατηγορικὸν καὶ ὑποθετικὸν ὅλον παρὰ Θεοφράστῳ καὶ τὰ πράσαληψιν, ὁ καὶ τὰ τρία σχῆματα πλέκεται οὕτως. Α ΣΣΗΜΑ. ὅ κατὰ παντὸς ἀνθρώπου, καὶ ἐκεῖνον παντὸς ὁσία. ζώον δὲ καὶ τὰ παντὸς ἀνθρώπου· καὶ ὁσία ἅρα καὶ τὰ παντὸς ἔσχον ... Β ΣΣΗΜΑ. ὅ κατὰ παντὸς ἀνθρώπου, τοῦτο καὶ τὰ παντὸς ἔσπον ... <Γ ΣΣΗΜΑ.> καὶ θυ παντὸς ζώον, καὶ τὰ τοῦτον <παντὸς?> καὶ λογικόν.

It appears that "παντὸς" is likely to have been omitted by the copyist. Wallies does not put it in in his edition.

Father Bocheński's reconstruction of the inference referred to under Β ΣΣΗΜΑ above seems to have been vitiated by typographical errors. It looks as if it should read thus: ὅ κατὰ παντὸς ἀνθρώπου, τοῦτο καὶ τὰ παντὸς ἔσπον. ζώον δὲ καὶ τὰ παντὸς ἀνθρώπου· καὶ ζώον ἅρα καὶ τὰ παντὸς ἔσπον. Cf. I. M. Bocheński, La logique de Théophraste, Fribourg 1947, p. 119.

Depending on the context the categorical propositions will be expressed in this essay as follows:

Universal affirmative: every a is b, b is predicated of a universally.

Universal negative: no a is b, b belongs to a in no instance.

Particular affirmative: some a is b, b belongs to a in some instance.

Particular negative: some a is not b, b does not belong to a in some instance.

Indefinite affirmative: a is b, b is predicated of a, b belongs to a.

Indefinite negative: a is not b, b does not belong to a.


It is interesting to note that the indication of quantity in the premiss illustrating the third figure seems to be missing here just as in the text quoted in note 4 above.

Γ', καὶ ἐκεῖνον τὸ Α· ἐν δὲ τῷ δευτέρῳ, ὅ κατὰ τοῦ Α, τοῦτο καὶ κατὰ τοῦ Β· ἐν δὲ τῷ Γ, καὶ τ' ἐκεῖνον τὸ Β.

8. Cf. Galenus I.c. p. 48, 1: ὅπωσι τῷ τὸ ἐδώς αὐτῶν (σε. τῶν κατὰ πρόσληψιν οὐσιολογομένων συλλογισμῶν), εἰρήσεται διὰ παραδειγμάτων δύον. ἐν μὲν ὦν ἐδόσ ἐστι τούτων "καθ' οὗ τόδε, καὶ τόδε· <ἀλλὰ τόδε κατὰ τοῦδε· καὶ τόδε> ἀρὰ κατὰ τοῦδε". καὶ ἐπὶ οὐνακῶν "ἐδ' οὗ δένδρουν, καὶ φυτών· δένδρον (δὲ) ἐπὶ πλατάνου· καὶ φυτών ἁρὰ ἐπὶ πλατάνου". προσταποκοῦσαι δὲ δηλονότι δὲ τῷ κατὰ τὸν λόγον τὸ "κατηγορεῖται" ἢ "λέγεται", ὥστε εἰναι τὸν ὀλκήρητον λόγον τούτων "καθ' οὗ δένδρουν κατηγορεῖται, κατὰ τοῦτον φυτῶν κατηγορεῖτα· δένδρον δὲ πλατάνου κατηγορεῖται· καὶ φυτῶν ἁρὰ πλατάνου κατηγορηθήσεται". ἕκρον δὲ ἐδώς συλλογισμῶν ἐκ τῶν κατὰ πρόσληψιν "οὗ κατὰ τοῦδε, καὶ κατὰ τοῦδε· <τόδε δὲ κατὰ τοῦδε· ὡστε καὶ κατὰ τοῦδε>". ἐπὶ οὐνακῶν "δὲ τὸ δένδρουν, καὶ <κατὰ> πλατάνου φυτῶν δὲ κατὰ τὸ δένδρουν· καὶ κατὰ πλατάνου ἁρὰ".


14. Cf. Ammonius l.c. p. XII, 12: καὶ ἐκ δύο γὰρ ἀποφασικῶν συνάγουσι (sc. οἱ κατὰ πρόσληψιν συλλογισμοὶ) καὶ ἐκ δύο μερικῶν καὶ ἔξ ὁμοοσχήμων ἐν δευτέρῳ σχήματι· καὶ ἄλλα πάντα ἰδια. p. 69, 33: ἀλλὰ συνάγεται νῦν καὶ ἐν δευτέρῳ κατασκίαν καὶ ἐν τρίτῳ καθόλου, καὶ ἐκ δύο ἀποφασικῶν ἐν πάσιν, καὶ τῷ ὑπάρχειν ἡ ανυπαρξία συνάγεται. Commenting on the premiss which says that τὸ Ἄ μηδενὶ ὑπάρχει, τὸ B παντὶ ὑπάρχει, the anonymous scholiast in C. A. Brandis l.c. p. 190, makes the following remark: ἔστι δὲ αὕτη ἐν τῇ ρήτῃ προτάσει ἐν τρίτῳ σχήματι· τὸν γὰρ μεσὸν καὶ ἀόριστοι ὑποκείμενον ἔχει τοῖς δύο, καὶ ποὺ ήδεικνύει πλεονέκτημα τὸ ἐν τρίτῳ συνάγειν καθόλου συμπέρασμα. οὐ μόνον δὲ τοῦτο ἄλλα καὶ ἐξ ἀποφασίσκης κατασκίαν καὶ ἐκ δύο μερικῶν συνάγει συμπέρασμα, ὡς ἐξής δείξωμεν.

15. It is fairly obvious that our inference schemata I" - 3" are special cases of a more general inference schema, whose validity is based on the following logical law:

\[ C \Pi x C \phi x \psi x C \phi x \psi x \]

Cf. I. M. Bocheński l.c. p. 110.


19. Cf. An. pr. 58\textsuperscript{a}26: εἶ δὲ ὅτι τὸ Β ὑπάρχει τῷ Γ δεῖ συμπεράνωθαι, οὐκ' ὅμως ἀναστρεπτὸν τὸ ΑΒ (ἢ γὰρ αὕτῃ πρόσατοι, τὸ Β μηδενὶ τῷ Α καὶ τῷ Α μηδενὶ τῷ Β ὑπάρχειν), ἀλλὰ λαγέτων, ὃ τῷ Α μηδενὶ ὑπάρχει, τῷ Β παντὶ ὑπάρχειν. Ὑπόκειντο τῷ Α μηδενὶ τῷ Γ ὑπάρχειν, ὁπερ ἦν τῷ συμπέρασμα· ὃ δὲ τῷ Α μηδενὶ, τῷ Β ἐλλήθων παντὶ ὑπάρχειν· ἀναγκὰ οὖν τῷ Β παντὶ τῷ Γ ὑπάρχειν.

H. Maier gives the following paraphrase of the Aristotelian inference:

kein A is B = alles, was nach seinem ganzen Umfang nicht A ist, ist B
kein C is A = alles C is ein solches, das nach seinem ganzen Umfang

alles C ist B

Cf. H. Maier, Die Syllogistik des Aristoteles, Zweite Teil, Erste Halfte, Tübingen 1900, p. 334. In the first premiss the sign of equation is meant apparently to indicate the transformation suggested by Aristotle in his theory of the circular and reciprocal proof. In the second premiss
it indicates equivalence. Clearly, Maier’s inference, valid as it cer-
tainly is, has a different logical structure from the inference proposed
by Aristotle. For Maier’s inference seems to have the form of a syllo-
gism in Barbara:

\begin{align*}
&\text{(i) every non-}A \text{ is } B \\
&\text{now, } (\text{ii) every } C \text{ is non-}A \\
&\text{therefore, (iii) every } C \text{ is } B
\end{align*}

W. D. Ross renders Aristotle’s inference as follows:

All of that, none of which is A, is B.

No C is A

\therefore All C is B

Cf. W. D. Ross, Aristotle’s Prior and Posterior Analytics, A revised


ἀναπτάρχη τὸ AB ὡσπερ κατ’ τῶν καθόλου, ... οἷον ὃ τὸ A πι ني μὴ
ὑπάρχει, τὸ B πι nei ὑπάρχειν.

22. It is obvious that in accordance with Aristotle’s intentions inference
(15) could, for instance, be formulated as follows: τὸ τὸ A πι nei μὴ
ὑπάρχει, τὸ B πι nei ὑπάρχειν ὡστ’ ἐκ τὸ A πι nei Γ μὴ ὑπάρχειν ἀνάγκη
οἷον τὸ B πι nei Γ ὑπάρχειν. Now, Maier has, in this connection, the
following inference:

kein A ist B = alles, was teilweise nicht A ist, ist teilw. B

einiges C ist nicht A = C ist ein solches, das teilw. nicht A ist

C ist teilweise B = einiges C ist B.

Cf. H. Maier l.c. p. 335. And Ross interprets Aristotle’s argument by
proposing an inference which runs thus:

Some of that, some of which is not A, is B.

Some C is not A.

\therefore Some C is B

Cf. W. D. Ross l.c. p. 439. I fail to see how this inference can be con-
strued as valid although I have no such difficulty if I consider the
original Aristotelian premisses and their conclusion. Nor can I agree
with Ross when he says that ‘all the reciprocal proofs fall into one or
other of two forms: If no X is Y, all X is Z, No X is Y, Therefore all
X is Z, or If some X is not Y, some X is Z, Some X is not Y, Therefore
some X is Z’ (cf. l.c., p. 440). For should this be the case then all
the reciprocal proofs would be instances or mere modus ponens. Clear-
ly they are more than that.

23. Cf. e.g. H. Maier l.c., p. 335 and W. D. Ross l.c. p. 441.

25. Cf. *An. pr.* 32b29: Τὸ δὲ καθ' οὗ τὸ Ἄ, τὸ Β ἐνδεχόμεθα ἣν παντὶ τῷ Β τὸ Α ἐνδεχόμεθα οὗ δὲν διαφέρει. Here the equivalence we are interested in is embedded in a modal context. Similar equivalence appears to be presupposed by certain turns of expression to be found in the Prior Analytics II, 22.

26. Cf. *An. pr.* 68a1: ὃ δὲ τὸ Ἄ, τὸ Γ οὐ ὑπάρχει which appears to be used as equivalent to τὸ δὲ τῷ Α οὐδὲν ὑπάρχει. In connection with the equivalences now under discussion see Aristotle’s definition of universal propositions in *An. pr.* 24b26: τὸ δὲ ἐν ὑπὸ ἐστιν ἑπόρη κἂν τὸ κατὰ παντὸς κατηγορεσθαι διαφέρειν. Λέγουμεν δὲ τὸ κατὰ παντὸς κατηγορεσθαι όταν μηδὲν ἦ λαβεῖν [τὸν ὑποκειμένου] καθ' οὗ θάρεται οὐ διαφέρεται κἂν τὸ κατὰ μηδενὸς ὑπάρχει.


29. Cf. Alexander l.c. p. 379, 9 ad *An. pr.* 49b30: ὁ μετὸι Θεόφραστος ἐν τῷ Περὶ καταφάσεως ἡν "καθ’ οὗ τὸ Β, τὸ Α ἡς Ἰσον δυναμεὺς λαμβάνει τῇ "καθ’ οὗ παντὸς τὸ Β, καὶ ’ἐκεῖνον παντὸς τὸ Α.

30. Cf. e.g. Alexander l.c. p. 378, 18; cf. A. C. Brandis l.c. p. 189b43 sq. ad *An. pr.* 58a21: λέγει δὲ ὁ Θεόφραστος ὅτι δυνάμει ἑσοτη ἔστι (οὐκα τὰ μέρη πρόσληψει διαφέρειν) τῇ κατηγορικῇ, οὗ μὴν διαφέρειν τὸ λέγειν "τὸ Α καὶ οὗ δενὸς τῷ Β" τοῦ λέγειν "καθ’ οὗ τῷ Β παντὸς, καὶ οὗ δενὸς ἐκείνου τῷ Α" τῷ πάλιν τὸ λέγειν "τὸ Α κατὰ παντὸς τῷ Β" τοῦ λέγειν "καθ’ οὗ τῷ Β παντὸς, καὶ ’ἐκεῖνοι καὶ τῷ Α παντὸς".


In (24) 'Σx' reads 'for some x', and expressions of the type 'Καβ' stand for the corresponding expressions of the type 'α and β'.


34. Cf. I. M. Bocheński l.c. p. 50.


36. Cf. *An. pr.* 49b14-16: Οὐκ ἐστὶ δὲ παύνυν οὐ' 'ἐ' εἶναι οὐ' 'ἐ' εἰπεῖν, ὅπ' ὃ τῷ Β ὑπάρχει, τοῦ ὑπάρχει τῷ Α ὑπάρχει, καὶ τὸ ἐπείν τῷ Ψ παντὶ τῷ Β ὑπάρχει, καὶ τῷ Α παντὶ ὑπάρχει.


38. Cf. Alexander l.c. p. 375, 17: καὶ γίνεται τῷ ἀδιπρόσωπῳ λεγόμενον ἰσον τῷ "Ψ τοῦ τῷ Β ὑπάρχει, τοῦ ὑπάρχει τῷ Α".

39. In his *La logique de Théophraste* pp. 109 sq. and 116 sq. Father Bocheński talks about 'syllogisms ἔστι προσλήψεως. This terminology
seems to be based on a wrong interpretation of the following passage in Alexander’s commentary to the Prior Analytics: λέγοι δ’ ἂν (sc. ὃ Ἀριστοτέλης) τοὺς τε διὰ συνεχούς, ὃ καὶ συνημμένον λέγειν, καὶ ἡς προσληψεως ὑποθετικοὺς καὶ τοὺς διὰ τοῦ διαρετικοῦ τε καὶ διεξενγμένου ἃ καὶ τοὺς διὰ ἀποφασικὴς συμπλοκῆς. Alexander l.c. p. 390, 3 ad An. pr. 50a39. Clearly, in this text the term πρόσληψις refers to the minor premiss (cf. note 41 below), and οἱ διὰ συνεχοὺς καὶ ἡς προσληψεως ὑποθετικοὶ (sc. συλλογισμοὶ) are nothing else but instances of the modus ponens.

40. Cf. Diogenes Laertius VII, 76: Λόγος δὲ ἐστιν ὃς οἱ περὶ τὸν Κρίνιν φασί, τὸ συνεπικός ἐκ λήμματος καὶ προσληψεως καὶ ἐπιφορᾶς οὗν ὁ ποιοῦτος "εἰ ἡμέρα ἐστιν, φῶς ἐστιν· ἡμέρα δὲ ἐστιν· φῶς ἀρα ἐστιν". λήμμα μὲν γὰρ ἐστὶν τὸ "εἰ ἡμέρα ἐστιν, φῶς ἐστιν". πρόσληψις τὸ "ἡμέρα δὲ ἐστιν." ἐπιφορὰ δὲ τὸ "φῶς ἀρα ἐστιν."