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FREE LOGIC AND THE CONCEPT OF EXISTENCE*

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The first part of this essay deals with one source of motivation underlying the development of *free logic*. The second part is an informal description of a logical system free of existence assumptions with respect to its terms, both general and singular. Part three presents a necessary and sufficient condition for statements of the form 'So and so exists', and the final section shows the use of this criterion as a means of testing some conceptions of the predicate 'exists'.

Part I

I shall discuss, in a rather rough and ready fashion, two squares of opposition, amendment of which generate, respectively, standard text-book quantification theory and free quantification theory.

The first square of opposition is as follows:

All such and such are so and so.	No such and such are so and so.
There are some such and such which are so and so.	There are some such and such which are not so and so.

'Such and such' and 'so and so' are *general term* placeholders. General terms are terms which purport to "refer" to any of a group of objects: they fail to refer if there *is* nothing of which they are true. For example, the expressions 'man' and 'runs' are referential general terms because they are true of some actual objects: the expression 'unicorn' is a

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nonreferential general term because though purporting to refer, it is, in fact, true of no actual object.

Suppose a nonreferential general term,—say, the term 'unicorn'—is put in the place of at least 'such and such' in the present square. Suppose further that 'all' and 'there is' statements are analyzed after the fashion of standard predicate logic. Then we run into trouble of a well known sort. It turns out that there are instances of the statement forms at the top of the square which are not contraries, instances of the statement forms at the bottom of the square which are not sub-contraries, and instances of the statement forms, respectively, at the top left and bottom left, and at the top right and bottom right which do not stand in the relation of subalternation. In fact, the only relation which does hold universally is that on the diagonnals, that is, the relation of contradiction.

One way of meeting the present difficulty, of "rehabilitating" the present square, to use Copi's[1] helpful expression, is to limit the placeholders 'such and such' and 'so and so' to *referential* general terms. Indeed, this manouvre does rehabilitate the square. It has, however, at least three rather closely connected adverse results.

First, the range of application of the square is severely restricted. It cannot be applied to arguments containing statements with nonreferential general terms. For example, it cannot be applied to arguments containing the statement 'All bodies upon which there are no external forces acting move uniformly in a straight line'.

Secondly, the restricted interpretation of the square blurs distinctions; it cannot distinguish between arguments whose validity ordinarily requires an existence assumption, and those which do not. For example, the validity of the argument from 'All men are animals' to 'There are men who are animals' requires the assumption that men exist, but the argument from "All men are animals' to 'It is false that there are men who are not animals' does not require such an assumption.

Thirdly, indeed, given the restriction method for saving the square, there is no satisfactory way of analyzing the concept of general existence,—the concept of existence expressed in the statement form 'Such and such exist',—because all cases of this form would be trivially true, and there negates would be contradictory.

I think it is fair to say that most contemporary philosophers of logic would feel that the cost of saving the present square is not worth the effort. It would be far less drastic simply to reject the validity of the inferences licensed by the square and to *amend* the square accordingly. The procedure, in fact, followed in most contemporary text-book presentations of quantification theory is perhaps most easily described as follows: insert between all pairs of statement forms around the outside of the present square, the statement form 'such and such exist', and allow *unrestricted* substitution into general term placeholders. This alternative method avoids the adverse results of the restriction method, for these adverse results were occasioned just by restricting the general term placeholders to referential general terms.

Indeed, it might be argued that the amendment method expresses what is tacit in the restriction method, but simply avoids its abuses. The tacit assumption is, of course, that the logical relations around the square hold only on the condition that a statement of the form 'Such and such exist' is true. It is not the motive of the restriction method which is at fault, it is rather its harshness of treatment; it is like giving a patient pneumonia in order to cure a cold!

Please do not misunderstand me. I do not claim that, as I have pictured the present square, anyone ever espoused the restriction method, though someone may well have. I am not peddling a straw man!; rather I am trying to make a point! It is this: there is another square of opposition where the restriction method is practically the law of the land—at least, it is the implicit law followed by most text-books in symbolic logic.

The square of opposition in question looks like this:

Everything is (a) so and so.	Such and such is not (a) so and so.
Such and such is (a) so and so.	There is something which is not (a) so and so.

In this square, 'so and so' is a general term placeholder, but 'such and such' is now a *singular term* placeholder. A singular term is one which purports to refer to exactly one object; it fails to refer, if there is no such object purported to be referred to. For example, 'Johnson' is a referential singular term because there is such an object,—that is, the president of the U.S.—which it purports to refer to, but 'Ichabod Crane' is a nonreferential singular term because there is no such object which is its purported referent.

The present square is implicit in most standard logic texts largely in virtue of the rules of *Universal Instantiation* and *Existential Generalization*, or in virtue of the principles of *Specification* and *Particularization*. I shall deal with the principles. The principle of *Specification* reads: if everything is so and so then such and such is so and so; the principle of *Particularization* says: if such and such is so and so then there is something which is so and so.

Suppose that 'so and so' is replaced by the general term 'existent' and that 'such and such' is 'Ichabod Crane'. Then, we run into trouble in the present square,—trouble precisely of the same variety occassioned by nonreferential general terms in the previous square. The relations around the square simply do not hold universally. For example, the relation of subalternation fails to hold between the statements 'Ichabod Crane is not an existent' and 'There is something which is not an existent'. And what is the remedy usually prescribed? Restrict the singular term placeholders to *referential* singular terms! The results of this remedy would appear to be precisely as before; the range of application of the square is restricted, formal distinctions are blurred and instances of the singular statement form 'So and so exists' are trivially true, thus precluding a satisfactory analysis of singular existence.

Standard text-book logic thus appears to the beset by a kind of theoretical schizophrenia. Well, maybe so. But whereas there is not much plausibility behind the restriction method in the case of the first square, there is in the case of the second square. What lends credibility to the method of restriction in the case of the second square is that the adverse results which it supposedly entails are avoidable. The defense of this claim is something like the following.

Consider the alledged counterexample to the inference licensed by, say, subalternation, on the right side of the second square. It would read: 'Ichabod Crane is not an existent; therefore, there is something which is not an existent'. Though it is the case that the premise of this inference is true and its conclusion false, it is not the case that it has the *form* of the inference licensed by the right hand side of the present square. What enables one to hold this position? The answer is short and sweet: Russell's theory of descriptions--with an assist from Quine. According to Russell's theory, sentences containing definite descriptions do not have the form of 'Such and such is (is not (a)) so and so'; they are really disguised general statements. If one supposes, as Russell did, that sentences with nonreferential grammatically proper names are latent descriptions, then they too do not really have the form of 'Such and such is (is not (a)) so and so'. But then 'Ichabod Crane is not an existent' does not have the form of 'Such and such is not (a) so and so' and the counter-examples to the present square are wiped out.

This ingenious manouvre obviates the criticism that the restriction method implies a narrowed range of application for the second square, because that criticism presumes that expressions like 'Ichabod Crane' are possible substituends of 'such and such'. The situation is similar to pointing out that the law 'all behavior is motivated' is not falsified by finding unmotivated rocks *because* rocks simply are not the sort of thing which could falsify the behavioral law.

Exactly the same goes for the argument that the restriction method, applied to the second square, blurs formal distinctions and cannot deal with the concept of singular existence. For these alleged adverse results presume that the sentence 'Ichabod Crane is not an existent' has the form of 'Such and such is not (a) so and so'. For example, though it is true that any instance of 'Such and such is a nonexistent' is self contradictory, this does not conflict with the noncontradictory character of 'Ichabod Crane is not an existent'. The latter does not have the form of the former, and further is adequately analyzed by Russell's theory of descriptions.

One more point. It has been objected, by Henry Leonard*, for example, that the present defense makes matters of form await factual answers, that to know whether, say, 'Ike is a man' has the form of 'Such and such is so and so' we have to know whether 'Ike exists'. Enter Quine! Turn all names into descriptions. Then, you see, no singular sentence has the form of 'Such and such is (is not (a)) so and so'. Accordingly, we simply disallow any singular term other than singular term placeholders (variables), to re-

*See note 2 at the end of this article.

place 'Such and such' in the above square. So when we say, *loosely speaking*, that only referential expressions like 'Ike' may replace 'such and such' in the second square, this is merely a practical expedient to avoid the complexities of paraphrase and proof via the Russell-Quine route.

To review, what makes the restriction method credible in the case of the second square, even as merely a practical expedient, is the availability of material—Russell's theory of descriptions plus Quine's artifice—for treating statements like 'Ichabod Crane is a nonexistent' in a manner which views them as not having the form of 'Such and such is (is not) so and so'; what makes the restriction method incredible in the case of the first square is the *lack* of any materials for treating statements like 'All unicorns are unicorns' in any alternative way.

Nevertheless, I have three objections to this defense of the restriction method as applied to the second square.

First, it places undue weight both on Russell's controversial theory of descriptions as the correct analysis of definite descriptions and on the validity of Quine's elimination of grammatically proper names. In short, it prejudges the philosophical issue of how to analyze singular terms. For example, there are other equally respectable description theories inconsistent with Russell's. One would like the logic of terms to be of use in helping to decide which theory is the best philosophical theory without prejudging the issue.

Secondly, the fact of the matter is that there is a certain inconsistency of attitude in standard text-book logic toward the question of logical form. Does the mere fact that we are dealing with singular terms rather than with general terms alone make that much of a difference? If this is so, why shouldn't we expect formal differences to be occassioned by the difference between general denotative terms like 'man' and general attributive terms like 'pretty'? Yet standard text-book logic tells us they are to be treated formally in the same way. When viewed from this angle, standard text-book logic's treatment of singular and general inference has a faint odor of adhocness about it.

Finally, there is another objection which is best put in the form of an analogy, an analogy which is really a very old chestnut. I refer to the oft cited distinction between the epicyclic and heliocentric theories of the solar system. It is often said that the two theories are equally adequate *explanations* of the facts. But it does not follow from this that both theories are equally *true* to the facts. The Coperincan view that the sun is the center of the solar system is so much simpler than its competitor that it is the chosen view. To me standard text-book logic's defense of the second square requires additions to elementary logic very much like the adding of epicycles to the Ptolemaic theory of the solar system. To be sure one can consistently hold that inferences like that from 'Santa Claus lives at the the North Pole' to 'There is something living at the North Pole' do not have the form of 'Such and such is so and so; therefore, there is something which is so and so', and still take care of these inferences via the torturous route of the Russell-Quine method. But look at what has happened. In order to defend the concept of validity in standard quantificational logic, first descriptions were disallowed the position of singular term placeholders and a complex theory dealing with them had to be constructed. That was the first epicycle. Then nonreferential names were denied placeholder positions and were handled via Russell. That is the second epicycle. Then Quine, that greatest epicyclist of them all, to borrow my colleague Robert Meyer's lovely phrase, denies all singular terms access to placeholder position and treats inferences containing them after Russell. Here is another epicycle. In short, theoretical justification or rejection of inferences containing names has become an extraordinarily complex affair.

To me it is simpler to give up the idea that the inferences licensed by the second square are valid and to amend the square accordingly. Insofar as the theory to be described is simpler, I maintain that it is truer to the facts about the nature of validity.

The emendation takes the same form as in the case of the first square. Insert between all pairs of expressions around the outside of the second square the statement form 'Such and such exists'. This is the major change which generates what is now called *free logic*. Free logic is simply a formulation which makes no assumptions about the existence of the purported designata of its terms, general or singular.

It is clear that the amendment method applied to the second square does not restrict its scope of application, nor does it blur certain formal distinctions. But it is not yet clear how the statement form 'So and so exists' is to be analyzed. It is to this task that I shall return after presenting a brief axiomatic rendition of free quantification theory based on the amendment of the second square.[2]

Part II

Let me start this part with a rough sketch of a familiar axiomatic version of standard first order quantification theory with identity. The axioms are

A_1: $(x)(Fx \supset Gx) \supset . (x)Fx \supset (x)Gx$
A_2: $(x)Fx . \supset Fy$

Here 'F' and 'G' are general term placeholders, 'y' is a singular term placeholder, '(x)' is read as 'every' and '(Ex)' as 'there is'. I assume also any sufficient set of axioms for the classical logic of truth functions and the usual identity axioms '$y = y$' and '$x = y \supset . Fx \supset Fy$'. The rules of inferences are: Universal Generalization, Detachment, and Sentential, General Term and Singular Term substitution. Further, let us assume that the quantifier '$(x)Fx$' is defined as '$\neg (Ex) \neg Fx$'.

In this theory, the following formulas are theorems.

T_1: $\neg Fy \supset (Ex) \neg Fx$
T_2: $(x)Fx \equiv \neg (Ex) \neg Fx$
T_3: $Fy \equiv \neg\neg Fy$

Along with the second axiom, these three theorems are among the important ones in the justification of the claim that standard quantificational logic (with identity) supports the second square of opposition. In other words, for every inference licensed by the second square of opposition there is a theorem in standard quantification logic justifying that inference. For example, the inference from 'John is not tall' to 'There is something which is not tall' is licensed by subalternation on the right side of the second square of opposition; it is justified by theorem T_1 in standard quantificational logic.

Recall, if you will, that the second square of opposition was amended earlier by placing the existence statement form 'Such and such exists' between each pair of statement forms around the square. I shall borrow Russell's sign, 'E!', and abbreviate the singular existence statement form, 'Such and such exists' as 'E!y'. Now the amendments to the second square of opposition occasion amendments in two of the theorems of standard quantificational logic. We must replace A_2 and T_1, respectively, by

A_2': $(x)Fx \;.\supset \mathrm{E!}y \supset Fy$
T_1': $\neg\, Fy \supset .\ \mathrm{E!}y \supset (\mathrm{E}x)\ \neg\, Fx$

No change is needed in T_2 and T_3. When the changes above are made, standard quantificational logic is turned into a free (though incomplete) quantificational logic. The correspondence between free quantification logic and the amended second square of opposition is just like that between standard quantificational logic and the unamended second square. Thus, for example, the inferences licensed by the right side of the amended second square are justified by T_1'.

I have just said that merely by changing axiom A_2 of standard quantificational logic to A_2' is not sufficient to produce a deductively complete system of free quantificational logic. I shall now present a complete axiom set for free quantificational logic with identity, where 'E!' is listed among the primitive terms. The completeness of the axiom set to be presented follows from the fact that it entails, and is entailed by, an axiom set recently proved to be semantically complete by R. K. Meyer and me.[3] We appealed essentially to the set of axioms for free logic that appear in my 1963 paper in the *Notre Dame Journal of Formal Logic* entitled 'Existential Import Revisited'.[4]

The quantificational axioms of this new formulation are as follows:

FA_1: $(\mathrm{E}x)Fx \supset (\mathrm{E}x)(\mathrm{E!}x\ .\ Fx)$
FA_2: $Fy \supset .\ \mathrm{E!}y \supset (\mathrm{E}x)Fx$

FA_2 is amended *Particularization*. Following a suggestion of Lejewski,[5] FA_1 may be interpreted as making explicit the existential interpretation of the quantifier '$(\mathrm{E}x)$'. The axioms for classical truth functional logic and those for identity are the same as before. The rules of inference are the same except that Universal Generalization is replaced by the Hilbert-Ackermann version of same; it reads: From '$A \supset B$', if the variable x is not free in A, infer '$A \supset (x)B$'. Finally, '$(x)Fx$' is defined as before.

Some theorems important for the developments in the next section are:

FT_1: $(x)(E!x \supset Fx) \supset (x)Fx$
FT_2: $(x)(E!x) \equiv (x)(E!x \supset E!x)$
FT_3: $(x)E!x$
FT_4: $(x)Fx \supset . E!y \supset Fy$
FT_5: $(x)((y)Fy \supset Fx)$
FT_6: $(x)(p \supset Fx) \supset . p \supset (x)Fx$
FT_7: $(x)(Fx \supset Gx) \supset . (x)Fx \supset (x)Gx$
FT_8: $(x)(Fx \supset Gx) \supset . (Ex)Fx \supset (Ex)Gx$

Earlier I said that the major problem confronting the amendment method of the second square of opposition was a satisfactory analysis of singular existence, that is, the major problem is a satisfactory analysis of the statement form 'E!*y*'. It is, therefore, the major problem confronting free logic. —It is clear, I hope, that free logic does not have a restricted range of application, at least with respect to its terms, and that it also discriminates between inferences licensed by the second square requiring an existence assumption and those that don't. See, for example, FT^4 (and T_2 which carries over into free logic). —The problem then is how to analyze the concept of singular existence. Does the present formulation of free logic give us any clues?

This problem can be given a definite ontological cast in the following way. Notice that 'E!' is undefined in the present system. So FA_1 and FA_2 can be taken as Carnapian "meaning postulates" for singular existence. In fact, one may regard free logic, as presented above, literally as a theory about singular existence, in the sense that it lays down certain minimum conditions for that concept. Construed in this philosophical way, then, the question before us is this; what does this theory of existence amount to? Does it yield a necessary and sufficient condition for existence statements?

Part III

In the system of free quantification theory with identity I have just presented, the statement form 'there is something with which such and such is identical'—or, in symbols, '$(Ex)(x = y)$'—is not valid; nor, indeed, is it provable. Now Jaakko Hintikka has pointed out that this statement form (or formula) is a very close formal analogue of what Quine means by being the value of a bound variable.[6] In fact, he argues, one of the advantages of free quantification theory with identity is that it allows formal expression of certain ontological dicta which have only trivially true analogues in standard quantification theory. It is an irony that Quine's famous aphorism that to be is to the value of a variable seems to have no adequate formal expression in the system of logic he so vigorously espouses—*unless* we complicate that theory with definite descriptions, etc. Now one of the claims to be established in this part is this; not only is Quine's criterion *expressible* in free quantification theory with identity, it is *provable*. I think the implications of this result are very far reaching, and I shall return to them in a minute. What is to be proved in this part then is the following biconditional.

Such and such exists if and only if there is something with which such and such is identical.

In symbols this is:

QC: $\mathrm{E}!y \equiv (\mathrm{E}x)(x = y)$.

The proof of QC proceeds by showing the impossibility of a counter-example. There are two cases to consider in virtue of the biconditional.

Case (A):	*Proof of* $\mathrm{E}!y \supset (\mathrm{E}x)(x = y)$	
(1)	$y = y \supset . \mathrm{E}!y \supset (\mathrm{E}x)(x = y)$	FA_2
(2)	$\mathrm{E}!y \supset (\mathrm{E}x)(x = y)$	(1) and Axiom of Reflexivity of identity Q.E.D.
Case (B):	*Proof* of $(\mathrm{E}x)(x = y) \supset \mathrm{E}!y$	
(1)	$\neg \mathrm{E}!y \supset . x = y \supset \neg \mathrm{E}!x$	FA_2 and Truth Functions
(2)	$\neg \mathrm{E}!y \supset . (x)(x = y \supset \mathrm{E}!x)$	(1) and UG
(3)	$\neg \mathrm{E}!y \supset . (\mathrm{E}x) x = y \supset (\mathrm{E}x) \neg \mathrm{E}!x$	(2) and FT_8
(4)	$\neg \mathrm{E}!y \supset \neg (\mathrm{E}x)(x = y)$	(3), FT_3 and Truth Functions
(5)	$(\mathrm{E}x)(x = y) \supset \mathrm{E}!y$	(4), Truth Functions Q.E.D.

To the formal question, does free quantification theory with identity suggest an analysis of 'Such and such exists'?, the answer is, "Yes".... treat it as: 'There is something with which such and such is identical'. Now in fact this is the way I treated it in my earlier paper in the *Notre Dame Journal of Formal Logic*.[8] Since, the system there, but with 'E' taken as primitive and supplemented by the axiom '$\mathrm{E}!y \neg (\mathrm{E}x)(x = y)$', is equivalent to the formulation of free logic in Part II, we may take the present proof as justifying the definition of 'Such and such exists' in my earlier formulation.

To the ontological question, what does the theory of existence reflected in the axioms FA_1 and FA_2 amount to?, the answer is: "Quine's dictum; namely, that to be is to be the value of a bound variable." It would be inconsistent, therefore, to accept the account of validity manifested in free quantification theory with identity but to reject Quine's dictum. It follows therefore that a rejection of Quine's dictum requires rejection of free logic and thus a certain view about validity. Insofar as the present analysis draws out this implication of Quine's famous dictum I believe free logic has made a definite contribution to ontology.

Part IV

The theorem QC proved earlier can be put to use in various ways. If we are interested in formal economy, it provides a basis for eliminating 'exists' from among the primitive signs of the language. Then paraphrase of natural language statements like 'John exists' would proceed in exactly the same way as does the paraphrase of the general statement 'Men exist'. Thus 'John exists' would be rendered 'There is something which is John'.

Another more philosophical use of QC would be as a measure of the adequacy of traditional conceptions of singular existence, *given acceptance of the account of valid inference in free logic*. In this case we take E!(xists) as primitive. Then QC can be viewed as providing a necessary and sufficient condition for singular existence. Accordingly, we can draw out the implications of a proposed *definition* of existence simply by replacing 'E!y' in the criterion by the proposed definiens.

Note that QC has a certain neutrality about it. Though it declares that the existing objects are the values of the bound variables it does not say what it takes to be a value of a bound variable, nor *which* things (for example, physical objects, abstract objects, ideal objects, persons, and so on,) *are*, or *must* be, the values of the bound variables; apparently it prejudges no philosophical account of *what* there is.

I shall consider three answers to the question; what does it mean to exist?—or equivalently—what does it take to be a value of a variable? The first answer is one proposed by Salmon and Nahknikian a few years ago in the *Philosophical Review*.[9] Their definition of existence was geared for a standard logic, but I think it is instructive to see how it fares when confronted by QC.

Their definition of existence was simply 'E!$y = df\ y = y$' - or 'To be is to be self-identical'. When we replace 'E!y' in QC by '$y = y$', we obtain, with the aid of the axiom of self-identity, $(Ex)(x = y)$, that is: There is something with which such and such is identical. In short, *any* singular term would designate a value of a bound variable. Therefore, unactualized possibles like Pegasus and unactualized impossibles like the round square would be values of the bound variables. In fact anything mentionable would be the value of a bound variable! Furthermore, since '$(Ex)(x = y)$' is provable, given the Salmon-Nahknikian definition of 'E!y', it would be contradictory to deny that any mentionable thing is the value of a bound variable.

A less obvious but equally unsuitable characterization of existence is one which is often attributed to Descartes, namely, that to be is to have a property. If we formulate this definition thusly: E!$y = df\ (EF)Fy$, then we can bring QC to bear upon it as follows. Replacing 'E!y' by '$(EF)Fy$' in QC we get: $(EF)(Fy) \equiv (Ex)(x = y)$. In words, 'Such and such is a value of a bound variable if and only if such and such has at least one property'. But if we count self-identity as a property, as any thoroughgoing Platonist would we can again derive the unsuitable conclusion that any mentionable thing is the value of a bound variable. But, I suppose, even Platonists would fell uneasy at the admission of unactualized impossibles among the furniture of the universe. This result follows because it is easy to derive, from the present characterization of what it takes to be the value of a bound variable, the statement that is self-identity is a property then if such and such is identical with such and such, such and such is a value of a bound variable.

The last definition I wish to consider is essentially a repair of the Cartesian characterization by Henry Leonard.[10] Leonard's characterization of existence is that to be is to have at least one contingent property. In symbols this is: '$E!y = df(EF)(Fy . \Diamond \neg Fy)$'. That is, 'Such and such exists' means essentially 'there is a property which such and such has but it is not necessarily the case that it has that property'. Replacing '$E!y$' in QC by Leonard's definiens we obtain: $(EF)(Fy . \Diamond \neg Fy) \equiv (Ex)(x = y)$.

There are at least three consequences of this definition which, without further restrictions, would make it unacceptable. The first is that if an object is not a value of a bound variable, then all of its properties are necessary. Thus, for example, given that Pegasus is not the value of a bound variable, not only his being a winged horse but also his being different from, say, Johnson is a necessary attribute. Secondly, given that being nonidentical with Johnson, or even being a nonexistent, is a contingent attribute we can infer that Pegasus for example, is after all the value of a bound variable. Finally, if both existence and nonexistence are contingent attributes, we can infer again from Leonard's definition that any mentionable object is the value of a bound variable.

Though I have illustrated why one might be discontent with each of the preceding definitions of existence I am really less concerned with their adequacy than with the fact that acceptance or rejection of their consequences will influence our views about the nature of validity. It seems to me that this is the important moral to be drawn from the proof of Quine's criterion in free logic. And if it is the case that the existence of an intimate relationship between ontology and logic is a leading principle in the school of logic called the Warsaw school, I am happy to have shown that a line of thought in a different tradition seems to support this insight.[12]

NOTES

1. Copi; *Introduction to Logic*, MacMillan (2nd Edition), p. 156.

2. H.S. Leonard, in his paper "The Logic of Existence" (*Philoso. Studies*, June: 1956), begins with an examination of the traditional square of opposition. He did not, however, develop the point that free logic stands to the second square as standard quantification theory stands to the traditional square.

3. Forthcoming in a paper entitled "Universally Free Logic" by R. K. Meyer and Karel Lambert. The completeness proof in the Meyer-Lambert paper is partly based on a translation of free logic into standard predicate logic, a discovery the key part of which is due to Meyer. In addition, it must be mentioned that the first published completeness proof of a system of free logic, closely related to the system in the body of this essay, is due to Bas van Fraassen. Van Fraassen's proof may be found in the *Zeitschr. für Math. Logik un Grundlagen d. Math.*, Bd. 12, S. 219-234 (1966). Further, in their forthcoming essay "Completeness theorems for some presupposition-free logics", H. Leblanc and R. Thomason have yet another completeness proof for (essentially) the system reported in the next footnote, a system equivalent to the one in the body of the present essay.

4. Karel Lambert; "Existential import revisited", *Notre Dame Journal of Formal Logic*, October: 1963, pp. 288-292.

5. C. Lejewski; "Logic and existence", *British Journal for the Philosophy of Science*, Vol. V: No. 18, 1954, pp. 1-16.

6. K. J. J. Hintikka; "Existential presuppositions and existential commitments", *Journal of Philosophy*, January: 1959.

7. Hintikka suggested to me the possibility of this proof at a recent meeting in May of 1965. QC is provable in his own rules formulation of free quantification theory with identity. (See Hintikka's excellent "Studies in the logic of existence and necessity", *The Monist*, January, 1966. Hintikka's proof of QC (essentially) occurs there and still other implications of QC are noted by him. I did not become aware of Hintikka's published essay until after I had written and submitted the present work to this journal.)

8. Op. cit., 'Existential import revisited'.

9. George Nakhnikian and Wesley C. Salmon, '"Exists" as a predicate', *Philoso. Rev.*, Vol. 66: 1957, pp. 535-542.

10. See note 2.

11. Leonard is perfectly well aware that other restrictions are needed to block some of the conclusions drawn in this paragraph from his definition of existence. For example, to block the conclusion that "Pegasus is, after all, the value of a bound variable" he disallows that both the predicate 'exists' and its complement denote properties. However, my purpose as I said above, is not so much to contest the acceptability of any of the definitions of existence as it is to show that acceptability or rejection of a given definition, other things being equal, directly influences our beliefs about the nature of validity.

12. C. Lejewski; "Leśniewski's Ontology" *Ratio*, Dec., 1958, pp. 150-176.

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