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S4.6 IS S4.9

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The system S4.4 is S4 + $\mathfrak{C}pCMLpLp$. In [2], the system S4.9, which may be formulated as S4.4 plus

(1) $ACLMqMLqCMLpLp$

and the system S4.6, which may be formulated as S4.4 plus

(2) $\mathfrak{C}LMpCLMqCMKpqLMKpq$

are discussed; S4.6 is shown to be contained in S4.9; the question of whether the containment is proper is left open. It has been communicated to me that K. Fine has solved this problem negatively; S4.6 is S4.9. The following is based upon but a bit briefer than Fine's proof:

(3) $\mathfrak{C}LMCpqCLMCpNqCMKCpqCpNqLMKCpqCpNq$ (2), P/Cpq, $q/CpNq$

The formula $KCpqCpNq$ is strictly equivalent to the simple Np; thus (3) gives

(4) $\mathfrak{C}LMCpqCLMCpNqCMNpLMNp$
(5) $\mathfrak{C}KLMCpqLMCpNqCMLpLp$ (4), S1°

Even in S1° the formulas $\mathfrak{C}qCpq$ and $\mathfrak{C}NqCpNq$ are theses; by the semi-substitutivity of strict implication (which holds in S2°), then, we may replace Cpq and $CpNq$ in formula (5) by q and Nq respectively:

(6) $\mathfrak{C}KLMqLMNqCMLpLp$

In S1° this last formula converts to (1); we have (2) giving (1), then, in the field of S2°, and S4.9 is S4.6 and so also the system Z9 of [1].

REFERENCES

[1] Sobociński, B., "A new class of modal systems," *Notre Dame Journal of Formal Logic*, vol. XII (1971), pp. 371-377.

[2] Zeman, J. Jay, "A study of some systems in the neighborhood of S4.4," *Notre Dame Journal of Formal Logic*, vol. XII (1971), pp. 341-357.

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