Notre Dame Journal of Formal Logic
Volume XII, Number 4, October 1971
NDJFAM

A MISTAKE IN COPI'S DISCUSSION OF COMPLETENESS

W. C. WILCOX

Professor I. M. Copi, in his informal discussion of completeness, says:[1]

> The notion of *deductive completeness* is a very important one. . . . In the least precise sense of the term we can say that a deductive system is complete if all the *desired* formulas can be proved within it. . . .
>
> There is another conception of completeness which can be explained as follows. . . . In general, the totality of formulas constructed on the base of a given system can be divided into three groups: first, all formulas which are provable as theorems within the system; second, all formulas whose negations are provable within the system; and third, all formulas such that neither they nor their negations are provable within the system. . . . Any system whose third group is empty, containing no formulas at all, is said to be *deductively complete.* An alternative way of phrasing this sense of completeness is to say that every formula of the system is such that either it or its negation is provable as a theorem.
>
> Another definition of 'completeness', roughly equivalent to the preceding one, is that a deductive system is complete when every formula constructed on its base is either a theorem or else its addition as an axiom would make the system inconsistent.

The second and third senses of completeness above are not, even roughly, equivalent. Consider a propositional calculus with axioms and substitution, such as that of *Principia Mathematica.* Such a system will be complete in the third sense but not in the second. As regards such a propositional calculus, the three groups will be: first, tautologies; second, contradictions; and third, contingent formulas. For a calculus to be complete in this sense, the third group to be empty, it would be necessary that there be no contingent formulas constructable upon its base. I can assign no other meaning to Professor Copi's words. I am inclined to say that no system which can plausibly be interpreted as a propositional calculus is

1. Irving M. Copi, *Symbolic Logic,* 3rd ed. (New York, 1967), pp. 188-189. This passage has remained unchanged from the first edition of 1954.

Received May 1, 1969

complete in this sense, but at least the propositional logic of *Principia Mathematica* is not, for in that system neither p nor $\sim p$ is provable. This, of course, does not show the non-equivalence of senses two and three, but that does follow from the demonstrable completeness of the propositional calculus of the *Principia Mathematica* in Copi's third sense.

That there is such an error in Copi's fine book is surprising; that it has gone unremarked for fifteen years is incredible.

University of Missouri at Columbia
Columbia, Missouri