Notre Dame Journal of Formal Logic
Volume XI, Number 2, April 1970

CORRIGENDUM TO OUR PAPER:

"A THEOREM ON n-TUPLES WHICH IS EQUIVALENT TO THE WELL-ORDERING THEOREM"

H. RUBIN and J. E. RUBIN

In the original paper[1] the last part of the proof of Theorem 1 was incorrect. (The error was called to our attention by J. D. Halpern. See p. 49, lines 2-9). We correct it as follows: If k_γ is not a subset of any element of T_γ, let t'_γ be the smallest element s of T such that $k_\gamma \subseteq s$, but for all $\beta < \gamma$, $k_\beta \nsubseteq s$, and for all $\beta > \gamma$, if there is an $r \,\varepsilon\, T$ such that $k_\beta \subseteq r$ then $k_\beta \nsubseteq s$.

An example of such an s is $s = k_\gamma \cup \{u_1, \ldots, u_{n-k}\}$ where the u_i's are distinct elements of $y = x \sim (\bigcup T_\gamma \cup k_\gamma)$. The set y is infinite because $\bigcup T_\gamma \cup k_\gamma < \omega_\alpha \approx x$. Then, clearly $k_\gamma \subseteq s$, and if either $\beta < \gamma$, or if $\beta > \gamma$ and $k_\beta \subseteq r \,\varepsilon\, T_\gamma$, then $k_\beta \subseteq \bigcup T_\gamma$ so $k_\beta \nsubseteq s$.

Now, if k_γ is not a subset of any element of T_γ, define $T_{\gamma+1} = T_\gamma \cup \{t'_\gamma\}$. If γ is a limit ordinal define $T_\gamma = \bigcup_{\beta<\gamma} T_\beta$. Then $N = \bigcup_{\gamma<\omega_\alpha} T_\gamma$ is the required set.

Purdue University
Lafayette, Indiana

[1]The paper was published in this Journal, Vol. VIII (1967), pp. 48-50.

Received July 2, 1969