# SOME NOTES ON SYLLOGISTIC IN THE SIXTEENTH AND SEVENTEENTH CENTURIES. ${ }^{1}$ 

E. J. ASHWORTH

Although a number of different schools of logic flourished in the sixteenth and seventeenth centuries, ${ }^{2}$ they seem to have shared a lack of interest in formal logic which expressed itself in a greater concern for the soundness than for the validity of arguments. An example of this tendency is the emphasis placed upon the Topics, or the ways of dealing with and classifying precisely those arguments which were not thought to be susceptible of formal treatment, since they depended for their effectiveness upon the meaning of the terms involved. ${ }^{3}$ It is true, of course, that the Humanists and, later, the Ramists, devoted considerably more space to the Topics and to the "invention" of arguments than did the scholastics, the Aristotelians, the Philippists or followers of Melancthon, or even the eclectics; but this was balanced by the greater devotion of the other schools to the categories, the predicables, the pre-, post-, and even extra-predicaments. ${ }^{4}$ However, there was one subject which was both formal in inspiration and common to all text-books, namely, the syllogism; and as a result it provides a very good test of how much interest and competence in purely formal matters was retained during these centuries of logical decline.

Before any texts are actually examined, it will be useful to provide some framework for discussion by outlining a small part of syllogistic logic. Every categorical syllogism contains three terms, major, minor, and middle, arranged in three propositions, a major and a minor premiss, and a conclusion. The major term ( P ) can be defined as the predicate of the conclusion, and it appears in the major premiss together with the middle term (M). The minor term (S) can be defined as the subject of the conclusion, and it appears in the minor premiss together with the middle term. Four types of proposition are involved, universal affirmative (A), universal negative (E), particular affirmative (I), and particular negative (O). Since a syllogism contains three propositions, there are $4^{3}$, i.e. 64 , possible modes or arrangements of A, E, I and O propositions. There are also said to be four figures from the four possible positions of the middle term, and hence there are 256 possible syllogisms, only 24 of which are classically valid. The following table of figures should be born in mind:

| M P | M P | P M | P M | M P | M P | P M | P M | S M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S M | S M | S M | S M | M S | M S | M S | M S | M P |
| S P | P S | S P | P S | S P | P S | S P | P S | S P |
| 1st | Indi- <br> rect | 2nd | Indi- <br> rect | 3rd | Indi- <br> rect | 4th | Indirect | False <br> 4th (1st) |

The hard core of traditional syllogistic consisted in a presentation of four direct modes of the first figure, Barbara, Celarent, Darii and Ferio; four of the second, Cesare, Camestres, Festino and Baroco; and six of the third, Darapti, Disamis, Datisi, Felapton, Bocardo and Ferison; together with some general rules concerning validity and methods of reduction. A number of the logicians whom I have examined stopped here; ${ }^{5}$ and many others merely added the five indirect modes of the first figure, Baralipton, Celantes, Dabitis, Fapesmo, and Frisesomorum. ${ }^{6}$ Since to go this far a writer needed only to copy his predecessors, from Aristotle down through the Middle Ages, I shall largely ignore the basic doctrines in order to concentrate upon those matters, such as the fourth figure of the syllogism, which provide a genuine test of logical competence. Although modal and oblique syllogisms were occasionally treated of by scholastics like Fonseca, ${ }^{7}$ I shall not discuss them here, because of the wider issues they raise.

The first question to be considered is that of the possible number of modes. A combinatorial method, similar to that mentioned earlier, of determining the number of syllogistic modes, was known to mediaeval logicians, apparently as a result of Arab influence. ${ }^{8}$ Albert the Great, for instance, gave an account of how 16 possible modes may be produced for each figure by taking just the two premisses into consideration. ${ }^{9}$ In the period we are concerned with, Caramuel, Ormazius, and the Carmelites of Alcalá recognized threefigures each with 16 modes; ${ }^{10}$ Ducius and Trutvetter recognized four figures each with 16 modes; ${ }^{11}$ and Hunnaeus, Arnauld and Aldrich recognized the full 64 modes for each figure, although only the latter two accepted four figures. ${ }^{12}$ Aldrich is the one logician among those I have examined who not only computed the full number of possible syllogisms but also gave a complete list of the 24 valid direct modes, including the five so-called subalternate syllogisms, Barbari, Celaront, Cesaro, Camestrop and Calemop. ${ }^{13}$ It may be noted here that the subalternate modes were not usually mentioned, although Trutvetter gave three, together with some duplicates and two invalid modes. ${ }^{14}$

A less orthodox approach is to be found in Maiolus, Crellius, and, probably following Crellius, Alsted. ${ }^{15}$ Instead of accepting the division of propositions into A, E, I and O propositions, these writers preferred to adopt an alternative classification in terms of quality and quantity taken separately. Traditionally propositions are said to display two kinds of quality, negative and affirmative, and four kinds of quantity, universal, particular, indefinite and singular. An indefinite proposition is one like "Cretans are liars", which has to be redeemed from ambiguity by the
provision of a sign of quantity, namely "some". ${ }^{16}$ A singular proposition is characterized by the presence of a singular term, whether a name, like "Caesar", a description, like "The teacher of Alexander", or a demonstrative phrase, like "This dog". Maiolus, Crellius and Alsted derived 36 possible modes for each figure by combining the two qualities of the premisses with three of the quantities, universal, particular, and indefinite. Hospinianus went a step further and derived 512 modes by combining the two qualities and four quantities of all three propositions. ${ }^{17}$

However, despite their novel method of computing the number of syllogistic modes, these logicians were not adding any new kinds of mode to those already known. It seems that indefinite propositions were always taken to have the force of particular propositions; and indeed Maiolus remarked that, since this is the case, perhaps one ought after all to speak of 16 rather than 36 possible modes. ${ }^{18}$ Similarly, singular propositions were treated as either universal or particular when they appeared within a syllogism.

Nor was their appearance unusual. The so-called expository syllogism, which has singular premisses, was well-known to mediaeval logicians like Ockham and Duns Scotus; ${ }^{19}$ and was one of the most common variants of the standard syllogism to be treated of in our period. Melancthon, for instance, defined it as a syllogism whose middle term is singular and which belongs to the third figure. ${ }^{20}$ It was also discussed by Hunnaeus, Fonseca, Villalpandeus, Polanus, Burgersdijck and C. Martini, who remarked that it was not a real syllogism, "cùm ad sensum potius quam ad rationem referatur". ${ }^{21}$ Many writers added an examination of the so-called special or singular syllogisms which have a singular conclusion as well as singular premisses, and are said to belong to the first and second figures. ${ }^{22}$ Typical examples are: "Plato was not ugly; Socrates was ugly; therefore Socrates was not Plato"; ${ }^{23}$ and "Aristotle was not an Athenian; the teacher of Alexander was Aristotle; therefore the teacher of Alexander was not an Athenian'". ${ }^{24}$ Piscator even supplied mnemonic names on the mediaeval modes for the two singular modes of the first three figures, namely Burburu, Cylurynt; Cysury, Cumystrys; Durupti and Fylupton. ${ }^{25}$ He explained that ' $u$ ' stood for an affirmative singular proposition and ' $y$ ' for a negative singular proposition. Crakanthorpe struck the only really unusual note in the discussion when he offered an example of a singular syllogism in the fourth figure. He argued that, where the premisses were singular, this figure was as natural and acceptable as the first, since it did not matter which term was the predicate. ${ }^{26}$

Mediaeval logicians had recognized the problem of the logical status of singular propositions in relation to the syllogism, and had offered varying solutions. Petrus Hispanus obviously regarded the singular proposition as a particular proposition, for he called the sophism "Every man is every man; Socrates is a man; therefore Socrates is every man'" a syllogism in tertio primae, that is, in Darii. ${ }^{27}$ Ockham, on the other hand, saw the singular as a universal, since "just as the subject of a universal proposition actually stands for all its significates, so too the singular subject
stands for all its significates, since it only has one"; ${ }^{28}$ and according to Prior, post-Renaissance writers normally classified singular propositions as universal. ${ }^{29}$ This judgment is born out by most of the examples given by the writers mentioned, but there is one group of exceptions. Gorscius and Willichius both gave as an example of Dimrai, a mode of the false fourth figure, the following: "Clodius is a traitor; all traitors are justly slain; therefore Clodius was justly slain', ${ }^{30}$ Jungius had the same syllogism, with reversed premisses, as an example of Darii. ${ }^{31}$ It would seem, then, that the singular propositions are taken as particular. In the case of Jungius this is unfortunate, for he had other examples with only singular propositions, which must be treated as universal if the syllogistic law, that every valid syllogism must have at least one universal premiss, is to be taken seriously. Dietericus fell into the same trap. ${ }^{32}$ In modern logic, the problem need not arise, for inferences containing singular terms can be handled by a calculus with identity and descriptions, rather than being forced into the strait-jacket of the categorical syllogism.

Apart from Agricola, who did not intend to give an extensive account of syllogistic, the only eminent logician who focussed upon syllogisms exhibiting singular terms almost to the exclusion of the true syllogism was Pierre de la Ramée, whose logic texts enjoyed an undeservedly high reputation. In his early Dialecticae Institutiones, he gave a standard account of the three figures and 14 modes, although without using any of the usual terminology; and syllogisms with singular terms were discussed in the section following. ${ }^{33}$ But by the time he produced the last edition of the Dialecticae libri duo a radical transformation had taken place. ${ }^{34}$ Here the simple, or categorical, syllogism is divided into the contracted and the explicit, and he gives two types of explicit syllogism. The first seems to correspond to Aristotle's second figure, and he lists six modes, four of which are but variations on the first two. These have only general or universal propositions and are obviously Cesare and Camestres. The two corresponding special syllogisms are said to have one general premiss, although in the examples he gives both the major and minor terms are singular. The two proper syllogisms are said to have no general premisses; and they in fact differ from the special syllogism by having only singular terms. No particular propositions appear anywhere. La Ramée's second type seems to correspond to Aristotle's first figure and, with Barbara and Celarent as the basis, another six modes are listed. In an earlier edition of the same work, we find that the two types are reversed, and that the example of the affirmative special syllogism of the first type contains particular rather than singular propositions. ${ }^{35}$ This is not the place for a detailed examination of la Ramée's logic; but it seems quite clear that there was no formal basis for his so-called reforms. His only apparent principle was one of random simplification.

Another common variant on the standard syllogism was the indirect syllogism, in which the minor term is said to be predicated of the major term in the conclusion. This immediately raises the question of how the major and minor terms are to be defined, for if the major term is merely
the predicate of the conclusion, it does not make sense to speak of an indirect mode as being more than a parasite upon a direct mode. According to the Kneales, "all mediaeval and later writers adopted the expedient put forward by John Philoponus, that the major term be defined as the predicate of the conclusion'"; ${ }^{36}$ but in fact Petrus Hispanus defined the major term as that appearing with the middle term in the major, or first, premiss; ${ }^{37}$ and according to Bocheński this was the usual method. ${ }^{38}$ No extensive discussion is to be found in the texts I am dealing with, but all possibilities seem to be covered, although their relevance to the problem of the indirect syllogism is nowhere mentioned, except by Arnauld, who felt that one cannot properly say that the conclusion is ever reversed. ${ }^{39}$ Hunnaeus, Crakanthorpe and Caramuel followed Petrus Hispanus; ${ }^{40}$ whereas Keckermann, Jungius, Arnauld, Wallis and Aldrich followed Philoponus; ${ }^{41}$ and Melancthon and Setonus combined the two definitions. ${ }^{42}$ Burgersdijck is the only one who seems to echo Aristotle by saying that in the first figure the major term is that which is predicated of the middle term and the minor term is that which stands as subject to the middle term; but that in the second and third figures the major and minor terms are those which appear in the first and second premisses respectively. ${ }^{43}$ Fonseca, too, differs from the others, for he says that the major term is that which has the preeminent place in the syllogism, and the major premiss is that which contains it. ${ }^{44}$

Most logicians do at least mention the indirect modes of the first figure, but only Wallis discussed their relationship to the five direct modes of the fourth figure, Cadere, Fedibo, Digami, Fegani and Balani, as he called them. ${ }^{45}$ He pointed out that only one of these two groups ought to be accepted, and said that it should be the latter since it is not sensible to speak of the interchange of premisses having any effect when they are distinguished not by position, but by the presence of the major and minor terms respectively. A few logicians went on to give indirect modes for the other figures also. Burgersdijck and Ormazius give two for the second figure, Faresmo and Firesmo, and two for the third, Fapemo and Frisemo (or Fipemo); ${ }^{46}$ whereas Piccolomineus gives only three in all; ${ }^{47}$ and Caramuel gives an extra one, Dacoccor, for the third figure. ${ }^{48}$ In fact, as Crakanthorpe and Wallis realized, for every valid direct mode there is an indirect mode obtained by reversing the premisses and interchanging $P$ and $S$ throughout; ${ }^{49}$ but in the case of the second and third figures the modes obtained are very uninteresting. Since the position of the middle term does not change, the figure remains the same, and there is no genuine difference between a direct and an indirect mode save for the order of premisses, as can be seen from the examples below:

| Firesmo: (indirect 2nd) | Some $P$ is $M$ No $S$ is $M$ | Some A is B No C is B |
| :---: | :---: | :---: |
|  | Some P is not $S$ | Some A is not C |
| Festino: | No $P$ is M | No C is B |
| (direct 2nd) | Some S is M | Some A is B |
|  | Some $S$ is not $P$ | Some A is not C |

The indirect modes of the fourth figure are derived from the first figure in the same way that the indirect modes of the first figure are derived from the fourth, by the reversal of premisses and the interchange of $S$ and $P$; but it is not entirely clear whether their existence was recognized. Trutvetter listed nine modes of the fourth figure, four indirect called Bambara, Camerent, Dimari and Fimero, and five direct, Bamalipton, Camentes, Dimatis, Fesmapo, and Fremsisomorum; ${ }^{50}$ but he gave no examples and offered no discussion other than the remark that a conclusion was said to be derived indirectly when the premisses were transposed. This suggests both that he defined the major term as that which appeared in the first premiss, and that he was speaking of the false fourth figure (see below). Campanella also listed nine modes, four direct called Balnama, Carmente, Dinami and Fimeo, and five indirect, called Balamiptu, Cangemops, Diramis, Fesarpo and Fesistro. ${ }^{51}$ He explained that the direct modes were indirect with respect to the first figure, since the minor term was predicated of the major, and that a reduction to the first figure involved the transposition of premisses. One of his examples reduced to schematic form is the following:

Barbara

| All A is B | All C is A Balnama |
| :--- | :--- |
| All C is A | All A is B |
|  | All C is B |

Since Balnama is said to have an indirect conclusion in relation to Barbara, we are dealing with the following pattern:

| Barbara | All $M$ is $P$ | All $P$ is $M$ Balnama |
| :--- | :--- | :--- |
|  | All $S$ is $M$ | All $M$ is $S$ |
| $S$ is $P$ | All $P$ is $S$ |  |

Hence we may conclude that Campanella's so-called direct modes are in fact the indirect modes of the genuine fourth figure, and his indirect modes are the standard direct modes of the same figure.

This brings us to a question which was discussed fairly frequently and which was a relative novelty in the period, namely, the question of the status of the fourth figure of the syllogism. Although a complete and satisfactory account had already been given in the thirteenth century by the Jewish philosopher Albalag, his writings were not known to the West; ${ }^{52}$ and it was Averroes's remarks on the figure he attributed to Galen that chiefly stimulated discussion, especially after his commentaries were published at Venice in $1552 .{ }^{53}$ As some Arabic logicians had realized, there are two ways of approaching the problem of the number of syllogistic figures. ${ }^{54}$ If one considers the position of the middle term in two undifferentiated premisses, there are only three possible figures, but if one draws an initial distinction between the major and minor premisses, there are four possible figures. In the West, the figures of the syllogism were always defined with reference to the position of the middle term in two differentiated premisses, but most logicians contented themselves with three of the four possible combinations. As Ormazius, following tradition, put it: "Sub prae
prima; secunda bis prae; tertia, bis sub". ${ }^{55}$ That is, in the first figure the middle term is subject in the major premiss and predicate in the minor premiss; in the second figure it is predicate in both premisses; and in the third, it is subject in both. The possibility of a figure in which the middle term was predicate in the major premiss and subject in the minor was recognized by a number of logicians, but very few saw clearly what was involved. Probably part of the trouble was caused by the frequent definition of the major premiss as the first premiss and the minor as the second, for some, ignoring the status of the terms in the conclusion, seemed to think that a new figure could be produced simply by reversing the order of premisses. This false fourth figure was presented by a number of mediaeval logicians; ${ }^{56}$ and in our period Acerbus, Javellus, Gorscius, Willichius, and probably Trutvetter (see above), did the same. Acerbus and Javellus offered a single example, ${ }^{57}$ but the other two listed four modes, Bamana, Camene, Frimeno and Dimrai, with identical examples. ${ }^{58}$ Gorscius remarked that this figure, if it were a separate figure, could be reduced to the first by an interchange of premisses. Caramuel gave no examples, but he too may have had the false fourth figure in mind when he argued that the fourth figure ought to be accepted. ${ }^{59}$ If, he said, the only objection to it was that a difference in the order of premisses was not sufficient to make a distinction between syllogisms, one could reply that it was indeed sufficient. After all, Cesare and Camestres, Datisi and Disamis were distinct modes. He completely overlooked the question of the structure of these syllogisms; as Zabarella pointed out (see below). Crakanthorpe, who quoted Zabarella several times, also picked up this point and said that it was a crass error to think that the transposition of premisses could alter figure, for it is the disposition of terms that is the crucial factor. ${ }^{60}$

The false fourth figure may have seemed acceptable, because innocuous, but the consensus among those who discussed the genuine fourth figure was that it ought to be rejected. The reasons given were all taken from or related to those of Averroes; although only a few referred to Averroes directly. ${ }^{61}$ In the first place, the fourth figure was dismissed as being unnatural. Averroes himself had said that people just do not argue like that, and Niphus repeated the claim. ${ }^{62}$ Alsted said that the fourth figure was ' $a$ destruction and perversion of the first", ${ }^{63}$ and Dietericus called it "unnecessary, obscure, contrary to nature and useless in disputation'". ${ }^{64}$ Jungius granted that if the premisses were true the conclusion was true, but added paradoxically that the conclusion did not necessarily follow from the premisses, since the premisses, on account of the inept and unnatural disposition of terms, did not arouse the mind to arrive at a conclusion without the addition of some intermediary. ${ }^{65}$ Crakanthorpe was less abusive but argued that although a fourth figure must be recognized, and must be accepted as valid, it may be omitted from consideration, since it is both unnatural and not commonly used. ${ }^{66}$ All of these arguments are extralogical and irrelevant, for how people do in fact argue has no bearing on the validity of a given syllogism.

The rejection of the fourth figure as unnatural was usually either linked with or explained in terms of the further objection, again drawn from Averroes, that the subject of the conclusion becomes a predicate in the premisses and the predicate of the conclusion becomes a subject. ${ }^{67}$ When this is the case, the middle term must be predicate of the predicate sought and subject to the subject sought, so that it is both greater than the major term (majus majori) and smaller than the minor term (minus minori). ${ }^{68}$ Furthermore, it turns out to be predicated of itself, which involves the absurd claim that it is both greater than itself and contained in itself. ${ }^{69}$

These undesirable consequences are said to arise from the violation of two logical principles, dictum de omni, and the first rule of the Antepredicaments. As formulated by Jungius, dictum de omni was the principle that whatever is truly said of all, that is, of some universal subject, is truly said of something contained in that subject, or of which that subject is affirmatively predicated. ${ }^{70}$ Alsted, Horstius and C. Martini interpreted it as meaning that what is a predicate in the premisses should be a predicate in the conclusion, and what is a subject in the premisses should be a subject in the conclusion. ${ }^{71}$ It is true that the fourth figure does not conform to this principle, but neither do the second and third figures, a point which was mentioned only by Zabarella.

The first rule of the Antepredicaments as stated by Javellus is that when one thing is predicated of another, whatever is a predicate of that predicate, is a predicate of the subject. ${ }^{72}$ That is, if $A$ is predicated of $B$ and $C$ of $A$, then $C$ is predicated of $B$. According to Javellus, this is sufficient to justify the false fourth figure, and to render its explicit addition to syllogistic unnecessary; but the important point is how this principle applies to the true fourth figure. $S$ is predicated of $M$ and $M$ of $P$ in the premisses, so it does indeed follow that M is predicated of M . The horror with which this situation was greeted rested upon an uncritical acceptance of the Aristotelian view that the subject was contained in and was less general than the predicate. Logicians did not realize that such an interpretation of the relationship between terms raises difficulties for syllogisms of all figures, and not just the fourth. ${ }^{73}$ For instance, one could take as a syllogism in Darii "All men are mortal; some physical objects are men; therefore some physical objects are mortal'. Here it makes sense to speak of the class of mortal things and the class of physical objects as having some members in common; but there are obviously no grounds for any speculation about the relative size of the two classes, and equally obviously the subject class is in fact larger or more general than the predicate class. Nor is it only particular propositions that cause trouble, for the two classes referred to in the universal proposition "All living things are mortal" are of equal generality.

Keckermann and du Moulin had one more point to make about the fourth figure. ${ }^{74}$ They felt that people had been beguiled into an acceptance of it through the problem of reduction. It is true, they claimed, that the two negative syllogisms cannot be reduced to syllogisms of the first figure; but they can easily be reduced to the third, and it is only because of the
implicit force of the third figure that a conclusion is validly reached. The example they give is Fesapo:

No $P$ is $M$<br>All M is S<br>Some $S$ is not $P$

They convert it to Felapton by converting the major premiss to "No M is p'. This is a valid move; but they both overlooked the fact that the laws of reduction were perfectly adequate to deal with Fesapo in the standard manner. As Dietericus pointed out, the ' $F$ ' in Fesapo shows that it is to be reduced to Ferio; ' $S$ ' shows that the E proposition is to be converted simply; and ' P ' shows that the A proposition is to be converted accidentally. ${ }^{75}$ This gives us:

## No M is P <br> Some S is M <br> Some $S$ is not $P$

which is Ferio, a first figure syllogism. Presumably the difficulty which Keckermann and du Moulin experienced was due to the fact that they felt that reduction from the fourth to first figures should begin with reduction to an indirect mode of the first figure.

The only extensive, carefully reasoned treatment of the whole problem is to be found in Zabarella's treatise, De Quarta Syllogismorum Figura; ${ }^{76}$ and I shall accordingly summarize his main points. As will become apparent, he has nothing essentially new to add to the arguments I have already examined, but his presentation is clearer, and he often throws light upon the comments of other writers by placing them in a context.

The basis for much of his argument is to be found in the distinction he draws between natural logic, or the ways in which people actually do argue; and artificial logic. There are two types of artificial logic, one of which is merely the formalization of natural logic in terms of rules, and one of which is opposed to natural logic, through its defiance of the principles of dictum de omni and dictum de nullo. He later adds that there are two ways in which a syllogism can be said to be natural. Firstly it is natural if it conforms to dictum de omni; and in this sense the syllogisms of the second and third figures are only potentially natural, in so far as they may be reduced to the first figure. Secondly, it is natural if it is naturally used, and here all the first three figures have equal standing. Only the fourth figure is excluded, because it is artificial in the sense of being opposed to natural logic in all possible ways. It is repugnant to dictum de omni; and people never do argue like that.

He claimed that some people felt that a further distinction should be drawn between a syllogism which was natural with respect to its matter and one which was natural with respect to its form; but this was an unprofitable line of argument. The subject-matter of propositions had not been Aristotle's concern in his syllogistic; and moreover, any proposition which is natural in the sense that the predicate is appropriate to the subject will
immediately become unnatural when it is converted. Those who interpreted Averroes as speaking of material unnaturalness rather than the formal unnaturalness which results from a failure to conform either to dictum de omni or to natural modes of reasoning, were wrong.

Equally wrong were those followers of Galen who argued that if the fourth figure is to be called unnatural, so must the second and third figures. They take as examples the modes Cesare and Camestres, Datisi and Disamis, and point out that the modes in question differ only with respect to the order of premisses and to the order of terms in the conclusion. For instance:

Cesare

| No A is B | All C is B |
| :--- | :--- |
| All C is B | No A is B |
| No C is A | No A is C |

Yet it is in precisely these ways that the modes of the fourth figure differ from the modes of the first figure. If both Cesare and Camestres can be accepted, why not both the first and fourth figures? Zabarella points out that the examples given are not compelling, because the position of the middle term, and hence the figure involved, remains the same. His main argument, however, concerns the extension of the major and minor terms. In the first figure the major term is greatest in extent and the minor term least in extent; but in the second and third figures the major and minor terms are equivalent in extent. Either they both include the middle term, or they are both included in the middle term; and as a result it does not matter which is to be called major and which minor. An arbitrary decision can be made simply on the basis of which appears first, and it is for these reasons that Cesare and Camestres, Datisi and Disamis are equally acceptable. No such considerations can be applied to the first figure; and so the fourth figure cannot be supported by the drawing of analogies with the second and third figures.

Zabarella strongly approved of Averroes's first argument to the effect that the fourth figure was unnatural, but he did not find the second argument, that the predicate in the fourth figure turns out to be predicated of itself, thus rendering the figure "nugatoria, inutilis \& absurda", quite so cogent. ${ }^{77}$ This does not prevent him from supporting it against the attacks of the medici or followers of Galen. If we take the premisses "All A is B" and "All B is C", we naturally derive the conclusion "All A is C", but in the fourth figure we get "Some C is A" instead, and the combination of conclusions gives us in turn "Some C is C". Those who support the fourth figure claim that we do not naturally derive "All A is C", which is wrong; that something can be predicated of itself when it is viewed as being different from itself, which is not only absurd, but leads to a fourth term; and finally, that the same consequences will follow from the conversion of propositions appearing in syllogisms of other figures, so that the inconvenience is one of matter rather than form. Zabarella replies firstly that form is useless unless it is apt for the material; and secondly that awkward results drawn from the first three figures are the fault of the user, but
here they are inevitable. Conversion, after all, is a matter of choice, but recognizing the natural conclusion "All A is C" is not.

As Zabarella sees it, the Galenists have four main arguments for their position, none of which can be satisfactorily upheld in the light of his discussion. Firstly, they argue that the middle term can be arranged in four different ways, giving us four figures. But the fourth arrangement, Zabarella feels, cannot be accepted because it implies a contradiction. To claim that the middle term is not between the extreme terms, but outside them, being greater than the major term and less than the minor, is ridiculous. Secondly, they say that the conclusion follows naturally from the premisses, which is false. Thirdly, they say that the fourth figure is acceptable because it can be reduced to the first figure. However, Zabarella answers, not all those arguments which can be reduced to the first figure are acceptable, and he gives the indirect modes as an example. Finally, the Galenists claim that the fourth figure is acceptable because Aristotle rejected only syllogisms which were useless in the sense of not leading to a specific conclusion; but Zabarella's answer to this is that, although the fourth figure gives us conclusions, it is useless in the sense that it is unnatural. The fourth figure, therefore, must be rejected.

Zabarella spoke as if there were a number of champions of the fourth figure, but among the text-books I have consulted, I have found only three, all later than Zabarella. One of the most vocal was Campanella, who felt that the science of the syllogism would be incomplete if only three figures were acknowledged, when the terms could be arranged in four ways. ${ }^{78} \mathrm{He}$ also felt that despite the claims of Averroes, the conclusion of a fourth figure syllogism followed necessarily and naturally. The contention of some Aristotelians that the first rule of the Antepredicaments was an adequate substitute for the first figure was quite mistaken; and Campanella accused his opponents of being animated by superstition, and an unwillingness to admit that anything new could have been discovered. As has already been mentioned, Campanella went on to give nine modes, four indirect, and five direct. He omitted Camestres, but included Cangemops (Calemop), the subalternate mode. Another champion was Arnauld, who pointed out that a fourth figure must be accepted if the major and minor terms are defined with reference to the conclusion and figure is defined with reference to the disposition of terms. ${ }^{79} \mathrm{He}$ too listed five modes. Aldrich, who gave all six, did not bother to discuss whether the fourth figure ought to be accepted, although in a later chapter of general comments on the syllogism he remarked that it was weaker (deterior) than the other three figures, principally because of the relative position of the middle term. ${ }^{80}$ Although his contemporary Wallis also gave an adequate account of the fourth figure, and preferred it to the indirect first figure (see above), he did not in fact accept either as natural or desirable, ${ }^{81}$ so that Aldrich remains the most acute of the English logicians.

In the light of this discussion, I find myself driven to the reluctant conclusion that genuine competence in formal logic was not often to be found in this period, at least where syllogistic was concerned. One dis-
tressing feature is the lack of discussion of issues like the definition of the major and minor terms or the status of singular propositions. Frequently one is left to guess differences in meta-theory from differences in usage. And even where there is discussion, it is not always adequate. For instance, a doctrine of the relationship between terms was used to exclude the fourth figure without any realization that this doctrine could not properly be applied to the first, second or third figures. Another characteristic of logicians of this period was a random introduction of new modes. What reason could be given for listing only two indirect modes of the second figure, or for allowing singular terms to appear only in third figure syllogisms? Finally, many logicians introduced frankly extra-logical considerations into their discussions. What was natural, what was fitting, what people tended to say, were all thought to be relevant issues. Only Arnauld and Alsted and, to a lesser extent, Campanella, present the right doctrines for the right reasons, unencumbered by extraneous material.

## NOTES

1. This study is based on an examination of printed texts in the British Museum, the Cambridge University Library, and the Bodleian. I do not mention Leibniz because he was not a writer of logical textbooks.
2. For a comprehensive account of the various schools, see Wilhelm Risse, Die Logik der Neuzeit. I Band. 1500-1640 (Stuttgart-Bad Cannstatt, 1964).
3. The situation is rather different today. For instance, much of the material discussed under the Topic of genus and species could be dealt with by set theory, and much of that discussed under the Topic of part and whole could be formalized by the methods of S . Lesniewski. The Topics, as treated by Boethius, Abelard, and Peter of Spain, are discussed by Otto Bird, in his article 'The Formalizing of the Topics in Mediaeval Logic,', Notre Dame Journal of Formal Logic, vol. 1 (1960), pp. 138-149.
4. For a typical account of these matters see Joachim Jungius, Logica Hamburgensis, edited by R. W. Meyer (Hamburg, 1957), Book I.
5. See Franciscus Byrseus, Dialecticarum Praeceptionum libri duo (Coloniae, 1565); Jacobus Carpentarius, Compendium in Universam Dialecticam (Parisiis, 1551); Johannes Clauberg, Logica Vetus \& Nova (Amstelaedami, 1658); Thomas Erastus, Ratio Formandorum Syllogismorum (Basileae, 1565); J. T. Freigius, Logica ad Usum Rudiorum in Epitomen Redacta (---, 1590); F. Hotomannus, Dialecticae Institutiones libri IIII (---, 1573); Chasparus Rhodolphus, Dialectica (Moguntiae, 1550); Johannes Sturmius, Partitionum Dialecticarum libri quatuor (Argentorati, 1582); M. Petrus Vincentius, Compendium Dialectices (Wratoslaviae, 1597).
6. See J. F. Burana, Aristotelis Priora Resolutoria (Parisiis, 1539); Johannes Caesarius, Dialectica (Coloniae, 1559); Ludovicus Carbo, Introductiones in Logicam (Venetiis, 1597); P. Gassendi, Institutio Logica in Opera I (Lugduni, 1658), pp. 91-124; Lodovicus Lemosius, Paradoxorum Dialecticorum (Salmanticae, 1558); Georgius Trapezontius, Dialectica Brevis (Coloniae, 1526); Joannes

Setonus, Dialectica . . . Annotationes Petri Carteri (Londini, 1574); Joannes Sandersonus, Institutionum Dialecticarum libvi quatuor (Antverpiae, 1589).
7. See Petrus Fonseca, Institutionum Dialecticarum libri octo (Conimbricae, 1590), Book VI, chapters 29 and 30.
8. See I. M. Bochenski, History of Formal Logic, translated and edited by Ivo Thomas (Notre Dame, Indiana, 1961), p. 219.
9. Albert the Great, quoted by Bochenski, loc. cit.
10. Joannes Caramuel, Rationalis et Realis Philosophia (Lovanni, 1642), p. 43; Matthaeus Doniensis Ormazius, De Instrumento Instrumentorum sive de Dialectica libri sex (Venetiis, 1569), p. 60; Collegii Complutensis Sancti Cyrilli . . . Disputationes in Aristotelis Dialecticam et Phylosophiam Naturalem (Lugduni, 1668), p. 30.
11. Laurentius Ducius, De Inventione Medii liber unus (Lucae, 1550), quoted by Risse, op. cit., p. 250, n. 246, n. 247. Jodocus Trutvetter, Summulae Totius Logicae, (Erphurdie, 1501) [no pagination].
12. Augustinus Hunnaeus, Dialectica seu Generalia Logices Praecepta Omnia (Antverpiae, 1585), p. 195; A. Arnauld and P. Nicole, The Port-Royal Logic, translated by T. S. Baynes (London, 1854), pp. 190-191; Henry Aldrich, Artis Logicae Compendium (Oxford, 1692), pp. 16-17.
13. Aldrich, op. cit., pp. 18-19.
14. Trutvetter, op. cit. He gave: 1st, Celaro and Celantos; 2nd, Cesaro and Camestros; 3rd (invalid) Fasesmo and Frisesmo; false 4th, Camero and Camento.
15. Laurentius Maiolus, Epiphyllides in Dialecticis (Venetiis, 1497), chapter nine [no pagination]; Fortunatus Crellius, Isagoge Logica (Neustadii, 1590), pp. 186-187; J. H. Alsted, Logicae Systema Harmonicum (Herbonae Nassoviorum, 1614), p. 396.
16. Jungius, op. cit., p. 78.
17. Johannes Hospinianus Steinanus, Non esse tantum triginta sex bonos malosque categorici syllogismi modos, ut Aristoteles cum interpretibus docuisse, videtur, sed quingentos et duodecim, quorum quidem probentur triginta sex . . (Basileae, 1560), quoted by Risse, op. cit., pp. 556-557.
18. Maiolus, loc. cit.
19. For references, see Risse, op. cit., p. 133, n. 44.
20. Philip Melancthon, Erotemata Dialectices 1540 ? p. 140.
21. Hunnaeus, op. cit., pp. 211-215; Fonseca, op. cit., pp. 67-72; Gasparus Cardillus Villalpandeus, Summa Summae Summularum (Madrid, 1615), pp. 257-259; Amandus Polanus, Logicae libri duo (Basileae, 1599); Franco Burgersdijck, Institutionum Logicarum libvi duo (Lugduni Batavorum, 1634), pp. 259-261; C. Martini, Commentatiorum.Logicorum adversus Ramistas (Helmaestadi, 1623), Book III, chapter 7, especially p. 300. Cf. Burgersdijck, op. cit., p. 259, who says that a singular term is not naturally affirmed of anything.
22. Rudolph Agricola, De Inventione Dialectica libri tres (Coloniae, 1538), p. 454; Laurentius Valla, Dialectice libri tres (---, 1509), Book III, chapter 48; Bartholomaeus Keckermann, Systema Logicae (Hanoviae, 1600), pp. 420-422; Conrad Dietericus, Institutiones Dialecticae (Giessae Hassorum, 1655), pp. 279-289; Jacobus Gorscius, Commentatiorum Artis Dialecticae libri decem (Lipsiae, 1563), p. 891; Jodocus Willichius, Erotematum Dialectices libri tres (Argentorati, 1540), p. 183; David Wasius, Rudimenta Dialecticae (Swinfurti, 1608); Burgersdijck, loc. cit.; Jungius, op. cit., pp. 129-136; Fonseca, loc. cit.
23. Jungius, op. cit., p. 133.
24. Burgersdijck, op. cit., p. 260.
25. Johannes Piscator, Animadversiones in Dialecticam P. Rami (Francofurti, 1586), pp. 184-185.
26. R. Crakanthorpe, Logicae libri quinque (Londini, 1622), pp. 291-292. "Hic vero observandum est, quartam figuram in syllogismis singularibus aeque usitatum est \& aeque naturalem, ac primam: \& ratio est quia cum praemissae ambae sunt singulares, non omnino refert, quae alteri praeponatur, neutro enim modo invertitur ordo naturalis."
27. Petrus Hispanus, Summulae Logicales, edited by I. M. Bochenski (Turin, 1947), p. 119.
28. Quoted by Bochenski, op. cit., p. 232.
29. A. N. Prior, Formal Logic (Oxford, 1955), p. 160.
30. Gorscius, loc. cit.; Willichius, loc. cit. The same example appeared in Agricola, loc. cit., but he did not specify a figure or mode.
31. Jungius, op. cit., p. 130.
32. Dietericus, op. cit., pp. 279-280.
33. Pierre de la Ramee, Dialecticae Institutiones (Parisiis, 1543), pp. 20-24vo.
34. Pierre de la Ramee, Dialecticae libri duo. Postrema Editio post Mortem Autoris (Lutetiae, 1574), pp. 59-67.
35. P. Rami Dialectica Audomari Talaei Praelectionibus illustrata (Coloniae Agrippinae, 1573), pp. 286-317. The text is interwoven with commentary.
36. W. and M. Kneale, The Development of Logic (Oxford, 1962), p. 71.
37. Petrus Hispanus, op. cıt., p. 37. ''Major extremitas est terminus sumptus in maiore propositione cum medio''.
38. Bochenski, op. cit., p. 216, note.
39. Arnauld, op. cit., p. 205.
40. Hunnaeus, op. cit., p. 193; Crakanthorpe, op. cit., p. 270; Caramuel, op. cit., p. 42.
41. Keckermann, op. cit., p. 407; Jungius, op. cit., p. 125; Arnauld, loc. cit.; J. Wallis, Istitutio Logicae (Oxonii, 1715), p. 148; Aldrich, op. cit., p. 14.
42. Melancthon, op. cit., p. 120; Setonus, op. cit., Book III.
43. Burgersdijck, op. cit., p. 257. Cf. the discussion of Zabarella below.
44. Fonseca, op. cit., p. 38. "Maius extremum est quod praestantiorem habet in syllogismo locum''.
45. Wallis, op. cit., pp. 180-183.
46. Burgersdijck, op. cit., pp. 261-263; Ormazius, op. cit., p. 60vo.
47. Franciscus Piccolomineus, Discursus ad Universam Logicam Attinens (Marpurgi, 1606), p. 31.
48. Caramuel, Praecursor Logicus Complectens Grammaticam Audacem (Francofurti, 1654), pp. 70-71.
49. Wallis, op. cit., p. 177. Crakanthorpe, op. cit., p. 292, said of singular syllogisms: "Ceterum in quavis figura possunt syllogismi indirecte concludere. Nam si mutes terminos conclusionis, hi omnes syllogismi indirecte concludunt." He was not so explicit about ordinary syllogisms, but listed the five indirect modes of the first figure, to which he added "exemplum unum in secunda, altera in tertia figura.', (Ibid., p. 275.) The two in question were Cesares and Dasisi. The mediaeval logician, Peter of Mantua, apparently gave a complete list of all the indirect modes. See Bochenski, op. cit., p. 215.
50. Trutvetter, op. cit., Book III.
51. Thomas Campanella, Logicorum libri tres in Philosophiae Rationalis partes quinque (Parisiis, 1638), p. 391.
52. Bochenski, op. cit., pp. 217-219.
53. Priorum Resolutiorum Liber Primus in Aristotelis Omnia Quae Extant Opera, Averrois Cordubensis Commentarii, Vol. I (Venetiis, 1552), p. 63vo. For a discussion of the attribution of the fourth figure to Galen, see J. W. Stakelum, "Why 'Galenian figure'?', The New Scholasticism, vol. 16 (1942), pp. 289-296, and N. Rescher, Galen and the Syllogism (University of Pittsburgh Press, 1966).
54. Rescher, op. cit., pp. 13-17, p. 19.
55. Ormazius, op. cit., p. 60.
56. E.g. Albert of Saxony, Peter Tartaret, Peter of Mantua. See Bochenski, op. cit., p. 216; and C. Prantl, Geschichte der Logik im Abendlande, Vol. IV (Leipzig, 1870), p. 179, n. 29; p. 206, n. 162; p. 245, n. 404; p. 287, n. 701.
57. Emilius Acerbus, Logicarum Quaestionum libri quattuor (Venetiis, 1596), p. 105vº Chrysostom Javellus, Compendium Logicae (Venetiis, 1569), p. 152.
58. Willichius, op. cit., pp. 182-183; Gorscius, op. cit., pp. 890-891. The names are the same as those given by Peter of Muntua (see note 56 ).
59. Caramuel, Praecursor, p. 295, col. 2.
60. Crakanthorpe, op. cit., p. 274.
61. Acerbus, op. cit., p. 103vo; Petrus Bertius, Logicae Peripateticae libvi sex (Lugduni Batavorum, 1604), p. 133; Gregor Horstius, Institutionum Logicarum libri duo (Witebergae, 1608), p. 268; C. Martini, op. cit., p. 132; Augustinus Niphus, Super Libros Priorum Aristotelis (Venetiis, 1554), p. 26vo; Collegii Complutensis, p. 29.
62. Niphus, loc. cit. ". . . nullus homo naturalis impetu cadit ad syllogizandum per quartam illam Galeni figuram, sed aut cadit ad syllogizandum per primam, aut per secundam, aut per tertiam''.
63. Alsted, op.cit., p. 394. "Ad summam, quarta figura est eversio \& perversio primae',
64. Dietericus, op.cit., p. 296. "Cüm itaque nulla sit huius figurae necessitas, consequentia ejusdem obscura, natura contraria, in disputando exigua, meritò ad primam reducitur \& supervacanea judicatur".
65. Jungius, op. cit., pp. 187-188. ". . . ut sumptione utrâque vera existente, vera etiam existat conclusio. . . Est autem syllogismus quidem similis haec ratiocinatio, non tamen syllogismus, quia Conclusio ex praemissis necessariò non consequitur, quod praemissae ob ineptam et minus naturalem terminorum dispositionem intellectum excitare nequeant ad eliciendam conclusionem, nisi intermedia alia propositio vel una vel altera interveniat'".
66. Crakanthorpe, op. cit., pp. 273-274. He said of Aristotle, p. 274, 'Significat nimirum se hanc quartam figuram omisisse, non ideo quia ratio concludendi in ea non est legitima \& syllogistica; sed quia non est naturalis \& usitata. Ita non reiecit hanc figuram sed omisit."
67. Acerbus, op. cit., p. 103vo; Alsted, op. cit., p. 394; Crellius, op. cit., p. 178; Horstius, loc. cit.; Bertius, op. cit., p. 134; Niphus, loc. cit.
68. Alsted, loc. cit.; Crellius, loc. cit.; Keckermann, op. cit., p. 423; Wallis, op. cit., p. 150.
69. Keckermann, loc. cit.; Pierre du Moulin, Introductio ad Logicam in Opera Philosophica (Amsterdam, 1645), p. 46.
70. Jungius, op. cit., p. 139.
71. Alsted, loc. cit.; Horstius, loc. cit.; C. Martini, op. cit., pp. 314-315.
72. Javellus, loc. cit.; Cf. Bertius, loc. cit.
73. See the discussion in Kneale, op. cit., pp. 68-69.
74. Keckermann, op. cit., pp. 423-424; du Moulin, op. cit., pp. 47-48.
75. Dietericus, op. cit., pp. 295-296. He reduces all five modes of the fourth figure according to the standard rules. Of those who rejected the fourth figure, only he and C. Martini listed five modes.
76. Giacomo Zabarella, De Quarta Syllogismorum Figura, in Opera Logica (Francofurti, 1623), columns 101-132.
77. Unlike his contemporaries, Zabarella does not refer to the middle term in this context.
78. Campanella, op. cit., p. 392. ''Defensio quartae figurae. Neque hanc dicant superfluam. Si secunda \& tertia non sunt frustrà. Nam quot modis configuratur medius terminus tot oportet esse figuras. Item saepissime apud philosophos, \& Theologos fiunt syllogismi in quarta bene concludentes sicut in aliis figuris: ita ut necessariò sequatur conclusio ex praemissis: \& naturaliter, quidquid dicat Averr: Et quamvis alii Peripatetici dicant sufficere regulam antepraedicamentalem loco huius figurae. Hoc dicunt ex superstitione, qua venerantur Aristo-
telem, \& nolunt plus posse inveniri, quam quòd ille. . . . ista regula . . . limitibus mille indiget, \& sufficit pro primo huius figurae tantum, non tamen pro caeteris . . .truncata esset scientia de syllogismi. Si tres termini quadrupliciter possunt configurari \& non nisi tres sunt figurae'".
79. Arnauld, op. cit., pp. 192-193.
80. Aldrich, op. cit., p. 22. "Adverte etiam quod figura quarta tribus caeteris deterior est; cum aliis de causis, tum ex hoc praesertim, quod medium dicat de majori, hunc de minori, minorem de medio, h.e. medium nugatorie de seipso."
81. Wallis, op. cit., p. 182. Cf. p. 150.

University of Manitoba
Winnipeg, Manitoba, Canada

