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THE LOGICAL CONCEPT OF EXISTENCE[1]

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§1. The concept of existence has long been a topic of philosophical discussion, but until the modern development of logical techniques, this discussion lacked clarity and precision. The use of logical techniques has sharpened the discussion, but there is not yet a universally accepted account of existence. In this paper, I will examine the logical concept of existence, by exploring the ways in which it is possible to say that an individual exists (or does not exist). This examination will be limited to the use of singular terms in existential statements of formalized languages—i.e. to the formalized counterparts of sentences like

Pegasus does not exist.
Santa Claus exists.

Customary formalized languages (or formal systems), employing individual variables and quantifiers, impose restrictions on statements of existence. I will argue that these restrictions constitute an existential presupposition of customary formalized languages. And I will present the outlines of a formalized language that avoids this presupposition.

For the discussion that follows, it will be helpful to consider a specific formalized language. The system of predicate calculus of Hilbert and Ackermann (presented in *Mathematical Logic*) will be used for this purpose. The axioms of this system, apart from the axioms common to propositional calculus, are

$$(x)f(x) \supset f(y)$$
$$f(y) \supset (\exists x)f(x).$$

This system of predicate calculus is not really a language in the ordinary sense, for its only constants are logical ones. But the system presents the bare bones (the logical structure) of a genuine language, and we can discuss the concept of existence with respect to this system.

The goal of this paper is to consider ways of saying that an individual does or does not exist. If such statements are to be possible, the system must be extended to allow the employment of proper names (or other

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singular terms).[2] One way to do this is to allow the substitution of proper names for free individual variables in universally valid formulas. Proper names can also be substituted for individual variables bound by an initial universal quantifier, if this quantifier is then dropped. However, there is a restriction on the proper names that can be added in this manner: any such name must be non-empty (it must be a name with reference).

If the system is extended in the way just outlined, it is possible to define a predicate 'Exists' which can be used with proper names.[3] If the system of predicate calculus is supplemented with axioms for identity, a convenient definition will be

$$\text{Exists } (x) =_{(def)} \quad x = x.$$

However, because of the limitation on the proper names that can be used, this definition of 'Exists' is trivial. For every individual that we can talk about exists. If '*a*' is a name that can be used with the system,

$$\text{Exists } (a)$$

is true. The denial that an individual exists is significant, but the denial will never be true. This trivial concept of existence is not a useful one. If we are to get clear about the concept of existence, we must find a way to say truly that some individuals exist and that others do not.

§2. In two important articles (references [1] and [2]), Professor Jaakko Hintikka has explained a non-trivial way of saying that an individual exists. He has done this by eliminating what he calls the existential presupposition of customary formalized languages, which presupposition is the characteristic that only non-empty names can be employed. In what follows, Hintikka's account of existence (or 'Exists') will be considered in some detail. I will argue that he has not succeeded in eliminating the most fundamental existential presupposition of customary formalized languages. But his discussion of existence is both clear and elegant—it provides a good starting point for the present discussion.

To construct a language completely analogous to the language presented by Hintikka (in [1]), it would be necessary to begin with a system which does not permit free occurrences of individual variables. This system would then be extended with axioms containing free individual variables. Both empty and non-empty names could be substituted for these free variables, but the names could *not* be substituted for bound variables. Hintikka's approach cannot be followed with the system of Hilbert and Ackermann, for this system already contains free individual variables. Only non-empty names can be substituted for these variables.

Let us consider how the system of predicate calculus must be modified in order to imitate Hintikka's treatment. In the system of predicate calculus, it is possible to proceed from a valid formula containing free individual variables to a generalized version of the formula where these individual variables are bound by initial universal quantifiers. We could modify the system to eliminate free occurrences of individual variables, and drop the rule of generalization. It would then be possible to reintroduce free individual variables by means of axioms; these free variables could not be generalized. The free individual variables would be chosen from the same alphabet as the bound variables, but a free variable would be entirely different from a bound variable. For the free variables take the place of proper names that may be empty or non-empty, while bound variables cannot be replaced by names.

We might signal the difference between the two kinds of individual variables by employing symbols from two different alphabets. But then there would be no need to modify the system of Hilbert and Ackermann by eliminating free individual variables and dropping the rule of generalization. If the symbols

$$a, b, c, a_1, b_1, c_2, a_2, \ldots$$

are used as variables for which empty and non-empty proper names can be substituted, the system of predicate calculus with identity can be extended with these axioms:

$$\begin{aligned} &a = a \\ &a = b \supset [f(a) \supset f(b)] \\ &a = x \supset [f(a) \supset f(x)] \\ &x = a \supset a = x. \end{aligned}$$

The original rules can be left unchanged (but the variables

$$a, b, c, \ldots$$

are not subject to quantification). There must be an additional rule allowing the substitution of one of the new variables for another. But the new variables cannot be substituted for the original individual variables.

The extended system of predicate calculus contains two kinds of individual variables; variables of either category can occur free. This system of predicate calculus satisfies Hintikka's criterion for a pre-supposition-less system. For empty names can be used, if these replace the variables

$$a, b, c, \ldots.$$

In this system it is possible to define a predicate 'Exists' which can significantly and truly be both affirmed and denied. Hintikka has shown that the most suitable definition is

$$\text{Exists}\ (a) =_{(def)} (\exists x)\ [x = a].$$

Now we can add a name for Pegasus, and the following will be true:

$$\sim\text{Exists (Pegasus)}.$$

Hintikka has called attention to what he claims is the existential presupposition of customary quantificational systems. And he has provided

a way to dispense with this presupposition. There are two major conclusions that he draws from this accomplishment. These are
(a) 'Exists' can be a predicate, but it is a predicate which is defined in terms of the existential quantifier.

> If the traditional denial that existence is a predicate is taken to mean that no predicate logically independent of the existential quantifier can express existence, it appears to be correct.[4]

(b) What he calls Quine's thesis: to be is to be the value of a bound variable.

> Even if the existential presuppositions on which usual systems of quantification are based are given up, sentences of the form $(\exists x)$ $(a = x)$ will necessarily have the logical force of '*a* exists.' I have thus proved that Quine's thesis is correct in a rather strong sense.[5]

The extended system of predicate calculus provides support for both of these conclusions. For the definition of the predicate 'Exists' requires (in the definiens) the existential quantifier. And the requirement that the existential quantifier be used to express existence seems to support the thesis that to be is to be the value of a bound variable.

> Quine's thesis may therefore be taken to mean that such formulae as $(\exists x)$ $(x = a)$ bring out the logical difference between what there is and what there is not.[6]

§3. With the extended system of predicate calculus, it is possible to employ both empty and non-empty names. But the definition of 'Exists' in this system does not clarify the logical concept of existence. For the extended system of predicate calculus is characterized by an existential presupposition, one it shares with the original system. Hintikka's conclusions are true with respect to the extended system of predicate calculus, but he has not established universal conditions for logical talk about existence.

Both the original and the extended systems of predicate calculus contain a category of individual variables which take individuals as values. *This* is what I consider the fundamental existential presupposition of these systems. To understand what it means to call this a presupposition, it is necessary to distinguish a variable which takes entities as values from one which is a schematic letter. Schematic letters do not take entities as values—they are replacements for expressions of a given category. In the extended system of predicate calculus, the variables

$$a, b, c, \ldots$$

are schematic letters. They are used in writing formulas valid for all names, empty or non-empty. Perhaps one could say that these variables take proper names as values, but this would be "taking values" in quite a different sense from that in which the variables

$$x, y, z, \ldots$$

take individuals as values.

It is commonly thought that quantifiers can only be used with variables that take entities as values. Hence the claim that to be is to be the value of a bound variable. It is clear that this does not require that variables taking entities as values only occur bound, for the system of Hilbert and Ackermann contains free individual variables which are not schematic letters. A formalized language can be constructed so that all and only bound variables take entities as values, while free variables are schematic letters; but this is by no means a universal condition on formalized languages.

Since the distinction between bound and free variables is distinct from that between taking entities as values and being a schematic letter, it makes sense to consider the possibility of quantifying schematic letters. Let us see what this would involve for the variables

$$a, b, c, \ldots.$$

It is clear enough that quantification over these variables could be allowed without collapsing the distinction between the two kinds of individual variables. What is needed is an explanation of how we are to understand formulas like

$$(a) f(a), \ (\exists a) f(a).$$

With variables that take entities as values, the quantifiers are used to talk about all or some entities. It is natural to interpret quantifiers with these schematic letters as statements about all or some names. A statement of the form

$$(a) f(a)$$

would mean that

$$f(a)$$

is true no matter what name is put in place of 'a'. The particular quantifier is then used to say that there is a name which makes the quantified expression true (or valid).

In both the original and the extended systems of predicate calculus, the use (the interpretation) of the individual variables

$$x, y, z, \ldots$$

determines how one will talk about existence. For these variables provide the basis for distinguishing what exists from what does not, regardless of whether the predicate 'Exists' is introduced. The use of these variables ties the concept of existence to a grammatical (symbolic) category of predicate calculus; and this constitutes a presupposition of talk about existence. We are simply stating this presupposition if we say that to be is to be the value of such a variable.

§4. To dispense with the existential presupposition of customary quantificational systems, it is necessary to avoid using variables which take entities as values. The only variables used must be schematic letters, and some of these must be subject to quantification. In this paper, the only variables that are quantified will be replacements for proper names. On the interpretation I am proposing, a statement of the form

$$(a)f(a)$$

does not mean that every individual is f (although this would be a consequence of the statement)—the statement asserts that

$$f(a)$$

is true no matter what name replaces '*a*.' And

$$(\exists a)f(a)$$

means that there is a name which makes

$$f(a)$$

true.

Before constructing a language without existential presuppositions, it is necessary to consider two important objections to this proposed interpretation. The first objection is that on this interpretation, a quantified statement is about language instead of the non-linguistic world. If the sentence

All blue objects are colored.

is translated into

$$(a)\ [\mathrm{B}(a) \supset \mathrm{C}(a)],$$

the translation seems to have a different meaning from the sentence it translates. For the original sentence is used to make a claim about blue and colored objects, while the quantified statement says something about all names. This objection is based on a misconception. It is derived from a view which regards sentences of the formalized language as mere abbreviations for sentences of ordinary language. But the symbolic counterpart to an ordinary sentence need not be a literal translation of this sentence. Nor is there a requirement that symbolic expressions be exactly translatable into ordinary-language sentences. We have a sufficient basis for using a formalized language if we can explain in ordinary language how the symbolic expressions are to be understood, how they are to be used. The demand for an exact translation is too strong. Both of the sentences

All blue objects are colored.
For any name which replaces '*a*,' if *a* is blue then *a* is colored.

would be rewritten

$$(a)[B(a) \supset C(a)].$$

There is no difference in the formalized language which corresponds to the difference between the two English sentences. However, only the second English sentence provides an acceptable reading of the quantified formula, because only this sentence brings out the character of the variables being used.

The second objection to the interpretation I have proposed is based on the possibility of talking about things without names. How does the formula

$$(a)\ [B(a) \supset C(a)]$$

apply to those blue objects which have not been named? This objection calls attention to a requirement imposed on a language which quantifies schematic letters. Such a language must possess a device for introducing names for whatever individuals can be identified. The language might have rules for definitions, or a description operator could be incorporated into the language (where descriptions count as names). This requirement for introducing names is a reasonable one; it *should* be possible to name whatever can be identified. Quantified statements will not apply only to those names that have been formed at a given time. The universal quantifier will be understood as applying to any name that *can* be formed; and the particular quantifier will be used to assert that a name *can* be formed so that, etc.[7] The formula

$$(a)\ [B(a) \supset C(a)]$$

claims that for any name '*a*' which can be formed, if *a* is blue then *a* is colored. This claim surely encompasses all blue objects, for it scarcely makes sense to argue that there is a blue object which cannot be named.

§5. It is not difficult to formulate a system without existential presuppositions. The system of predicate calculus with identity provides the basis for such a system. But now the individual variables must be understood as belonging to the same category as proper names *which may or may not designate existing individuals*. By reinterpreting the individual variables of the system of predicate calculus, it is possible to take over the whole formal apparatus of this system. The formulas of the system are given new meanings, but none of the formal results are affected.

Because the existential presuppositions of normal systems have been eliminated, the reinterpreted system of predicate calculus does not possess the resources for distinguishing what exists from what does not. A statement of the form

$$x = y$$

only means that '*x*' and '*y*' name the same thing, if they name anything at all. It is possible to distinguish an empty from a non-empty domain, because of the extensional nature of predicate calculus. In an empty domain, it will be true that

$$(x)\,(y)\,[x = y];$$

while in a non-empty domain, the following will be true:

$$(\exists x)\,(\exists y) \sim [x = y].$$

But being able to distinguish an empty from a non-empty domain does not enable us to take a name, say '*a*,' and write a symbolic expression claiming that *a* exists.

The general logical framework of talk about individuals does not require the concept of existence. This seems intuitively satisfactory, because it is not a logical matter to determine what individuals exist. But we want to investigate the logical relations between statements about individuals that have been determined to be existing individuals, as well as the relation between the concept of existence and other concepts which apply to individuals. To accomplish these goals, it is necessary to extend the reinterpreted system of predicate calculus in such a way that we can distinguish what exists from what does not.

Any language that is suitable for informative discourse (about individuals) must possess a device for making this distinction. For the difference between existing and not existing is not a difference between individuals that we encounter. It is a linguistic difference. A condition of the significance of most talk about individuals is that they be existing individuals (this is presupposed). Talk about nonexisting individuals, except for statements denying that they exist, is clearly derivative from talk about existing individuals. And many things that we can say about existing individuals cannot be said of fictional or mythical individuals.[8] To say that an individual exists, then, is to endorse it (or its name) as a candidate for certain kinds of statement.

The distinction between what exists and what does not is fundamental to the significant use of language. Because it is so fundamental, this concept cannot be explained by more basic concepts. A term for distinguishing what exists from what does not must simply be introduced into the reinterpreted system of predicate calculus—its sense cannot be defined or characterized with respect to expressions already present. However, axioms may be required to preserve the extensional nature of the system of predicate calculus. If the term introduced is 'Exists,' the following axiom is needed:

$$(1)\quad \sim\text{Exists}\,(x)\ \&\ \sim\text{Exists}\,(y) \supset x = y.$$

All empty names are regarded as names of the same individual.[9] It is also desirable to introduce a standard empty name by an axiom. If 'Λ' is this empty name, a suitable axiom is

$$(2)\quad x = \Lambda \supset \sim\text{Exists}\,(x).$$

An alternative procedure to adopting axioms (1) and (2) would be to introduce '$\wedge$' (perhaps by an axiom like $\wedge = \wedge$), and then to define 'Exists' as follows:

$$\text{Exists } (x) =_{(def)} \sim[x = \wedge].$$

The name '$\wedge$' does not designate an extra-linguistic individual. Yet we can talk about the non-existing individual—to do this is simply to use '$\wedge$.' On the interpretation of variables and quantifiers that I am defending, the treatment of '$\wedge$' is perfectly intelligible. A statement of form

$$f(\wedge)$$

justifies

$$(\exists x) f(x),$$

because individual variables are replacements for names, and quantifiers are used to talk about all or some names.[10] The presence of '$\wedge$' serves to eliminate a shortcoming of the reinterpreted system of predicate calculus. In this system it is possible to prove

$$(3) \quad (\exists x)\ [f(x) \vee \sim f(x)],$$

but before the introduction of '$\wedge$' there was no name answering to the quantified variable. Now we can prove

$$f(\wedge) \vee \sim f(\wedge),$$

so there can be no objection to (3).

In the reinterpreted system of predicate calculus, it is not possible to prove that anything exists. But we can prove

$$(\exists x) \sim \text{Exists } (x),$$

for we can prove

$$\sim \text{Exists } (\wedge).$$

The reinterpreted system is thus valid in an empty domain. This validity is a consequence of eliminating variables which take individuals as values; but the consequence is an incidental one, for validity in an empty domain can be achieved for systems containing the customary sort of individual variables.[11]

The reinterpreted system of predicate calculus enables us to reach a satisfactory understanding of the relation between the concept of existence and other concepts which apply to individuals. In this system, we can distinguish strong and weak predicates; these are defined

$$f^{+}(x) =_{(def)} \text{Exists } (x)\ \&\ f(x)$$
$$f^{-}(x) =_{(def)} \text{Exists } (x) \supset f(x).$$

Because the concept of existence is a fundamental one, the predicates used will already be strong or weak (in a two-valued system, intermediate cases are ruled out), but the symbols '+' and '-' can be used to make the force of the predicates explicit.

The negation of a strong predicate has a different sense from the negation of a weak predicate, for

$$\sim f^{+}(x) \equiv \sim \text{Exists } (x) \vee \sim f(x)$$
$$\sim f^{-}(x) \equiv \text{Exists } (x) \,\&\, \sim f(x).$$

If 'a' is an abbreviation for 'the present king of France,' then

$$\sim \text{Wise}^{+} (a)$$

means that either there is no king or he is not wise, while

$$\sim \text{Wise}^{-} (a)$$

means that there is a king who is not wise. The predicate 'Exists' is a strong predicate—i.e.,

$$\text{Exists}^{+} (x) \equiv \text{Exists } (x).$$

But in contrast to other strong predicates, 'Exists' can go wrong in only one way. For clearly,

$$\sim \text{Exists } (x) \equiv \sim \text{Exists } (x).$$

This peculiarity of the predicate 'Exists' is due to the fact that the predicate plays a defining role with respect to strong and weak predicates. The distinction between what exists and what does not is fundamental to everything else we want to say about individuals.

§6. The formulation of a system without existential presuppositions is an achievement of some significance, although it is not accompanied by new formal results. This system shows that the customary logical treatment of existence is by no means a necessary one. In fact, the use of variables taking individuals as values constitutes a presupposition of talk about existence. And the restriction of quantifiers to non-schematic-letter variables appears as an arbitrary restriction.

The customary quantificational systems obscure the concept of existence. The formula 'To be is to be the value of a bound variable' has an impressive sound, and seems to convey a profound ontological message. But this formula is really an announcement that the concept of existence is being built-in to the category of quantified variables. In the reinterpreted system of predicate calculus, the concept of existence is introduced explicitly, either by 'Exists' or '$\wedge$.' For this concept does not depend upon a particular logical symbolism. In fact, the most general logical framework for talk about individuals (which includes the apparatus of quantification) does not enable us to distinguish what exists from what does not. But the concept of existence is fundamental to significant statements about individuals. It is the basic concept of an interpreted system (for talking about individuals), and it cannot be reduced to more fundamental concepts.

The reinterpreted system of predicate calculus constitutes evidence that logic provides no support for the claim that 'Exists' is not a predicate. This system also undermines the claim that "no predicate logically independent of the existential quantifier can express existence." The reinterpreted system of predicate calculus enables us to get clear about the concept of existence, and shows that 'Exists' is a predicate that can stand on its own feet.

NOTES

1. I would like to acknowledge the helpful comments I received from Professor Jaakko Hintikka on an earlier draft of this paper.

2. Instead of using proper names, it is possible to employ predicates that apply to (are true of) at most one individual. But the goal of this paper is to explore the use of proper names in existential statements.

3. This possibility was demonstrated by Nakhnikian and Salmon, in "Exists as a Predicate," *Philosophical Review*, 66 (1957).

4. [2], p. 66.

5. [2], p. 74.

6. [1], p. 133.

7. Someone who is willing to admit every sort of abstract entity might be willing to recognize names that no one has ever used. Abstract entities are often introduced in order to avoid the problems connected with potentiality and possibility.

8. For example, we cannot describe Hamlet's childhood, because he had none.

9. This axiom is only valid in a thoroughly extensional language. It would also be possible to recognize different sorts of existence, and to admit predicates corresponding to them. For example, 'Exists' might be used for straightforward existence, while 'Exists*' was used for fictional and mythical existence. Then it would be true that

$$\sim \text{Exists}(\text{Pegasus}),$$

but also

$$\text{Exists}^* \ (\text{Pegasus}).$$

And it would be false that

$$\text{Hamlet} = \text{Pegasus}.$$

10. In [4], R. M. Martin has proposed that we use a language that is formally similar to the reinterpreted system of predicate calculus (with 'Λ'), but he regards the "null individual" as one value of individual variables. This is a "convenient technical fiction" which is justified by its useful results. But Martin's treatment is unintelligible, because we no longer know what it means to be the value of a variable. The present interpretation of variables and quantifiers is required to

make sense of the formal procedures and statements that Martin defends. And the useful results that he achieves are exactly the same as the results of using the reinterpreted system of predicate calculus.

11. The feasibility of a customary system valid in an empty domain was established by Andrzej Mostowski, in "On the Rules of Proof in the Pure Functional Calculus of the First Order," *Journal of Symbolic Logic*, 16 (1951).

The problem of using empty names seems to have been confused with the problem of achieving validity in an empty domain by Czesław Lejewski, in [3]. Lejewski argues for an interpretation of variables and quantifiers that is similar to the one I have proposed, on the grounds that such an interpretation is necessary to achieve validity in an empty domain. But the interpretation is not required for this purpose. The real advantage of the proposed interpretation is that it enables us to dispense with existential presuppositions.

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