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SEMANTICS FOR CONTINGENT IDENTITY SYSTEMS

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In [2], it was shown that the semantics developed by Hughes and Cresswell in [1], pp. 198-199, for the contingent identity systems T + CI, S4 + CI, and S5 + CI is inadequate in that none of these systems is sound with respect to the corresponding notion of validity. The purpose of this note is to present a semantics which is adequate. We restrict our attention to T + CI; extending this semantics to S4 + CI and S5 + CI is straightforward.

A *model structure* is an ordered quadruple $\langle W, R, D, I\rangle$ such that W and D are nonempty sets, R is a binary reflexive relation on W, and I is a nonempty subset of the set of functions from W into D. A *value assignment* V on a model structure $\langle W, R, D, I\rangle$ is a function which assigns each variable $\mathbf{a}$ a value $V(\mathbf{a})$ in I and each n-place predicate letter φ a value $V(\varphi)$ in the set of functions from W into the power set of the n'th Cartesian product of D with itself. Let V be a value assignment on $\langle W, R, D, I\rangle$ and let $i \in I$. Then $V\begin{bmatrix}\mathbf{a}\\ i\end{bmatrix}$ is defined to be that value assignment on $\langle W, R, D, I\rangle$ which assigns i to $\mathbf{a}$ and elsewhere agrees with V. A *model* is an ordered quintuple $\langle W, R, D, I, V\rangle$ such that $\langle W, R, D, I\rangle$ is a model structure and V is a value assignment on $\langle W, R, D, I\rangle$. Let $\mathfrak{M} = \langle W, R, D, I, V\rangle$ be a model and $i \in I$. Then $\mathfrak{M}\begin{bmatrix}\mathbf{a}\\ i\end{bmatrix}$ is defined to be $\left\langle W, R, D, I, V\begin{bmatrix}\mathbf{a}\\ i\end{bmatrix}\right\rangle$. Let $\mathfrak{M} = \langle W, R, D, I, V\rangle$ be a model and $w \in W$. Then we define *truth* of a formula at w in $\mathfrak{M}$ (read '$\mathfrak{M}, w \models \alpha$' as '$\alpha$ is true at w in $\mathfrak{M}$') inductively as follows:

(i) $\mathfrak{M}, w \models \varphi\, \mathbf{a}_1 \ldots \mathbf{a}_n$ iff $\langle V(\mathbf{a}_1)(w), \ldots, V(\mathbf{a}_n)(w)\rangle \in V(\varphi)(w)$,
(ii) $\mathfrak{M}, w \models \mathbf{a} = \mathbf{b}$ iff $V(\mathbf{a})(w) = V(\mathbf{b})(w)$,
(iii) $\mathfrak{M}, w \models \sim\alpha$ iff it is not the case that $\mathfrak{M}, w \models \alpha$,
(iv) $\mathfrak{M}, w \models (\alpha \vee \beta)$ iff either $\mathfrak{M}, w \models \alpha$ or $\mathfrak{M}, w \models \beta$,
(v) $\mathfrak{M}, w \models L\alpha$ iff for all $w' \in W$ such that wRw', $\mathfrak{M}, w' \models \alpha$,
(vi) $\mathfrak{M}, w \models (\mathbf{a})\alpha$ iff for each $i \in I$, $\mathfrak{M}\begin{bmatrix}\mathbf{a}\\ i\end{bmatrix}, w \models \alpha$.

A formula α is *valid* iff for each model $\mathfrak{M}$ and w in $\mathfrak{M}$, $\mathfrak{M}, w \models \alpha$. Proof that

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T + CI is both sound and complete with respect to this semantics will not be given here. Such a proof can be easily constructed following the proof that the system B1 is sound and complete given in [3].

REFERENCES

[1] Hughes, G. E., and M. J. Cresswell, *An Introduction to Modal Logic*, London (1968).

[2] Parks, Zane, and Terry L. Smith, "The inadequacy of Hughes and Cresswell's semantics for the CI systems," *Notre Dame Journal of Formal Logic*, vol. XV (1974), pp. 331-332.

[3] Parks, Zane, "Investigations into quantified modal logic," in preparation.

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