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## ON REFERENTIALITY AND ITS CONDITIONS

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1.\* In the second chapter of his book Word and Object, Quine has put forward his thesis of indeterminacy of translation based on the considerations that two different and incompatible conceptual translations in a home language of a native expression in the target language are always possible. This indicates that the meaning of a term is never fully determinate in a language but can receive different conceptual identifications in different translations, reproducible even within one language. In this manner the meaning of an expression in a given language is only to be relatively determined in another language. Since there could be different ways of characterizing the meaning of a native expression in a language as Quine strongly urges in the case of translating "gavagai" in terms of "rabbit," "rabbithood," "unattached rabbit part" or "rabbit stage," one might observe, first that different ways of characterizing the meaning of a native expression are differently meaningful only in the translating language, but not in the translated language. Thus they are incompatible to each other only relative to the translating language, because in the translating language they have different uses and different conceptual statuses. But relative to the translated language, these uses and conceptual identifications are irrelevant or extraneous, and therefore different ways of characterizing the meaning of the native expression form an equivalence class relative to the meaning of the given expression and because of this there is no reason to regard these different ways of characterization as internally

Second, that the so-called internal meaning must be clarified in terms

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<sup>1.</sup> See Willard V. Quine, Word and Object, MIT Press and John Wiley and Sons (1969), pp. 26-29.

of Quine's notion of overt behavior on appropriate occasions of use of language. For Quine different translations of the same expression are equally acceptable because they equally preserve the given set of verbal behaviors or dispositions of verbal behaviors in the native community. As he puts it, "There seem bound to be systematically many different choices, all of which do justice to all dispositions to verbal behavior on the part of all concerned." Thus the so-called internal meaning of an expression is founded on the observable behavior of the speakers or users, of the language. There is incompatibility of translations of a native expression, if there are incompatible behavioral bases of the translations within the translating language and for the translating community.

2. Now in regard to this thesis of conceptual indeterminacy of translation as suggested by Quine, we must raise the question of objectivity and determinateness of the verbal behavior of the community. We must ask whether this behavior has an expression of optimum explicitness or definiteness? And whether an internally structured behavior, when fully expressed, might not admit a single model and a single translation or representation? It seems evident that from the native's point of view, and in terms of the native language, even though there could be many ways of describing the same behavior, the native could hold that there is one correct description of a behavior. The correctness of description consists in the conventionality of the use of an expression under given circumstances. This is particularly conspicuous in the native's own formulation of the grammar of the language either outside the given language or within the same language. Different linguists may formulate different grammars for the same language, on the basis of finitely accessible data. But this does not entail that given an adequate basis, they may not come to agreement on a correct formulation of the native grammar.

From the above there ensure two important points. (1) The grammar of a language can be regarded as representing *some* internal structure of a language, and this internal structure can be formalized within a given language. (2) There is present in each language a rationale of correctness for the formulation of its grammar, and for that matter, a correct interpretation of the meaning of an expression in the language. Given these

<sup>2.</sup> See his article "Ontological Relativity" in the *Journal of Philosophy* (1968), pp. 185-212, especially p. 190.

<sup>3.</sup> Here I use the term "observable" in the sense that an observable behavior is one which can be described by what Quine calls observation sentences in Word and Object. Quine says that "Occasion sentences whose stimulus meanings vary none under the influence of collateral information may naturally be called observation sentences, and their stimulus meanings may without fear of contradiction be said to do full justice to their meanings." (Ibid., p. 42.) As the notion of observationality is relative to length of stimulation, the notion of observable behavior must be also relativized to a system of stimulations.

two observations about language, it makes sense to say that a translation of a language can be theoretically uniquely determinate, and this is brought out in the case of translation of grammatical statements within a language, for such statements are *intended* to characterize some *unique* internal structure of the native language.

More specifically, from an internal point of view there is no relativity of conceptual indeterminacy, and all translations of an expression can be regarded as partial or incomplete, which can be gradually improved and rectified to correction. In this sense the given language is a reality to be characterized gradually toward perfection, even though this perfect characterization may not be actually attainable, or may even necessitate changes in the translating language. Yet it can be indefinitely approximated on the basis of empirical investigation of observing more verbal behavior in a longer period of time. This point certainly is borne out by the field experience of empirical linguists and this is further strengthened by the analogy to a scientific inquiry rooted in the method of hypothetical construction-confirmation-revision-and-re-construction. Following from this is the consequence that what Quine has called "analytical hypotheses" need not be considered analytical in his sense of "analytical."

Before we advance to the relation of the doctrine of conceptual indeterminacy to Quine's doctrine of relativity of ontology, we have to observe that conceptual indeterminacy as described by Quine can be said in a certain sense to be rooted in the theoretical nature of language itself. As a language is not fully formalized nor finitely formalizable, there is no proof in the language itself that the language has given a unique correct theoretical account of reality or everything in reality. Thus with regard to what a given language is intended to refer to or to say about, it is always relatively determined. But what is that to which it is relatively determined? Like a scientific theory, a language is relatively determined by its practical value trained to the complicated needs of man. What is important is that there is no empirically given unique language with regard to reality which can do everything and represent everything. This fact of inherent referential relativity is indeed the basis for positing conceptual indeterminacy. This basis can be more formally formulated in the proposition that we cannot show in our language that the language formulates a unique theory for a given set of experiences or the proposition that we cannot show that there is only one model for the given set of experiences of the language (or theory). This directly reveals the ontological basis for the thesis of conceptual indeterminacy. But in view of this understanding, we should point out what Quine seems to fail to point out, namely, a distinction

<sup>4.</sup> See Quine's Word and Object, 68ff. In fact Quine has not given specific reasons for calling "analytic hypotheses" "analytical." However, they are analytical in the sense in that they are a priori determinations regarding the equivalences between native utterances of a translated language and words and phrases of the translating language by a linguist.

between two cases of conceptual indeterminacy. One case involves construction of actual isomorphic models of a language (or theory); another involves construction of actual non-isomorphic models of a language or theory. The former admits systematic transformations preserving a commonly shared behavioral basis, but does not guarantee that all models (actual and possible) of the language or theory are isomorphic in being such. The latter does not present systematic transformations even though preserving a commonly shared behavioral basis. Ramifying this distinction, one can say that there are various degrees and many kinds of conceptual indeterminacy, and that furthermore there is a sense in saying that given an adequate behavioral exposure, a translation (or interpretation) of a given language (or theory) is more adequate than another as has been in fact the case with dictionary compiling between natural languages.

4. Perhaps we should consider some of the examples which Quine has considered for more illumination on the reality of conceptual indeterminacy. One of these examples is related to Japanese use of classifiers, which according to Quine can be explained in either of two ways.<sup>5</sup> In one way they can be explained as contributing to forming compound numerals of distinctive styles to be used for different general terms. In another way they can be explained as contributing to various mass terms to produce composite individuative terms. Now as the expression in which a classifier occurs always has this form: numeral/classifier/ thing-noun, it is clear that the two explanations of the classifier depend on the two possible ways of relating a classifier to its surrounding elements. In the first explanation, the classifier is related to the numeral and in the second explanation, it is related to the thing-noun. Since the Japanese classifier and its use in actual expression are based on Chinese which is full of classifiers, it could be more interesting to review some Chinese examples such as "san-chih-yang" (three sheep or three headed individuals of sheep), "wu-tiao-yu" (five fish or five string-shaped individuals of fish), "pa-to-hua" (eight flowers or eight flower-like individuals of flowers), "liang-ke-tao" (two peaches or two discrete individuals of peach), etc. From these examples it is clear that the thing-nouns can be logically construed as mass terms as well as terms of divided reference. But if we examine how classifiers are actually formed in the language, it is also clear that they are derived from the individuative forces of the thing-nouns and are therefore expressions, not reasons, for the individuation of things denoted by the thing-nouns. In the light of this consideration, we can see that it would be more correct or accurate to regard classifiers as styles of numeral expressions of individual things than otherwise, i.e., after knowing more about the Chinese or Japanese language, one can see that the second explanation is more acceptable than the first one.

Another reason for the appropriateness of taking the second explanation

<sup>5.</sup> See his article "Ontological Relativity," 192ff.

as the correct one is that in the classical written Chinese classifiers are optional items, and there need be no classifier to indicate the individuative forces of thing-nouns. Thus in regard to our earlier examples, one can speak of "san-yang" (three sheep), "wu-yu" (five fish), "pa-hua" (eight flowers) and "liang-tao" (two peaches), etc. To say this is not to say that there does not exist genuine mass terms in Chinese or for that matter in Japanese. In fact there are mass terms which also have their own classifiers which means again that classifiers can be regarded as individuating in many cases. We should note that it is only when many different classifiers can fit with a thing-noun that the thing-noun is a mass term; that is, a mass term would be a term which has no distinctive styles of individuation, but depends upon different classifiers as means for individuation. For example, "shui" (water) can be individuated in terms of classifiers "peng" (basin), "tung" (basket), "wan" (bowl), or "pei" (glass). Thus three peng of water, three tung of water, three wan of water and three pei of water could be equally correct usages depending upon appropriate occasions of uses.

The upshot of this discussion is that the internal structure of a language can be uniquely and correctly determined and there need not be indeterminacy when an adequate basis of translation is made available. Perhaps we can formulate our point in the following way: given any time t, there is a more or less correct determinate meaningful translation of a native expression satisfying all empirical data and pragmatical principles of simplicity and usefulness. This again is very similar to the case of formulating scientific truth.

5. The "rabbit" and the relative determinate "classifier" examples, 6 lead to the consideration that verbal behavior of an individual or a community can be expressed in terms of two or more meaningful expressions of either the native or translating language. This reveals the relation of meaning to reference in translation. No consideration of meaning of a native expression is self-sufficient without involving considerations of possible behavioral references of the expression on the basis of which translation can be made.

In the light of this we observe that first a given behavioral basis can not be considered to be exhaustively and uniquely characterized by the native expression so that other expressions are not possible. This is the characteristic of re-interpretability of the referential basis of an expression. Second, a composite expression which is meaningful and relevant for characterizing a given situation may have components which have an internal structure which can be alternatively interpreted in regard to its referential significance. These alternative interpretations may be incompatible with each other in contexts other than contexts in which the

<sup>6.</sup> Quine also suggests the example of the referential parity between formulae of a formal system and their corresponding Gödel numbers.

composite expression achieves its meaningfulness. They form an equivalence class within the given context of use of the composite expression.

On the basis of these two points, we may elaborate on two aspects of referential indeterminacy of conceptual translation. First, it is possible to re-interpret a given state of affairs as denoted or defined by a native term. That is, for a given term "F," the state of affairs associated referentially with "F" is open to a set of other interpretations of "F"; these may include "G," "H," "I," etc., each of which has a potential semantic structure defined or determined in a specific language. Thus there is an openness of the meaningfulness of "F" and an "openness" of the associated reality of "F." Second, it is possible to make alternative interpretations of referential meanings (potential or manifest) of the components of "F" as a term. The components of "F," say, represented by "a, b, c, d," can receive various interpretations in another language. In fact another language always brings out the flexible identity of "a, b, c, d" relative to a determinate meaningfulness of "F." In other words, the meaningfulness of "F" permits different ontological commitments resulting from various interpretations of its components.7

At this point I conclude that these two aspects constitute what Quine calls inscrutability of reference<sup>8</sup> from which conceptual indeterminacy of meaning should follow as a consequence. What I have done above is to show how conceptual indeterminacy is actually related to referential opacity or inscrutability in the light of an intended correct translation of an internal structure in a language. Such an inscrutability of reference is seen to be twofold or two aspected, each aspect having its referential meaning.

What is inscrutable about reference is that there need not be a unique way of representing the meaningfulness of a state of affairs, and that relative to a given representation, various ontological identifications can be made, which are independently incompatible in a translating language, but which forms an equivalence class in the translated language.

6. Quine has himself recognized the relation of indeterminate meaning to indeterminate reference. He says that "Of two predicates which are alike in extension, it has never been clear when to say that they are alike in meaning and when not; it is the old matter of featherless bipeds and rational animals, or of equiangular and equilateral triangles. Reference, extension, has been the firm thing; meaning, intension, the inferior. The indeterminacy of translation now confronting us, however, cuts across extension and intension alike. The terms 'rabbit,' 'undetached rabbit part,' and 'rabbit stage' differ not only in meanings, they are true of

<sup>7.</sup> This is of course actually represented by the indeterminacy of translation of identity and other individuative apparatus.

<sup>8.</sup> See his article "Ontological Relativity," 193ff.

different things. Reference itself proves behaviorally inscrutable." The last sentence is suggestive. From behavior alone one can never tell which reference or ontological commitment is made by the use of an expression. The extension of a term can be said to be some observable behavior, but the intension of a term is intended to characterize something in, or beyond, but not unrelated to, the observable behavior—it is a matter of ontological interpretation. Because of the very possibility of re-interpretation of behavior, the indeterminacy of meaning derives its significance from indeterminacy of reference to the behavior. Thus, we see that the problem of conceptual indeterminacy is reduced to that of referential indeterminacy and this constitutes a reason for us to turn to Quine's theory of relativity of ontology, which I also consider as resulting from his thesis of inscrutability of reference.

- 7. Independently of Quine, an important reason for relating meaning to reference as well as for relating indeterminacy of meaning to indeterminacy of reference is that there is a certain tension between reality and language. As language cannot be considered a full unique formulation of reality and as reality can be conceived basically indeterminate and susceptible of different ways of characterization, the meaningfulness of an expression in a language could vary in the course of time, simply because different aspects or different interpretations of reality may force a change in the semantic meaningfulness of reality. On the other hand, the given meaningfulness of an expression because of its basic referential involvements may introduce an understanding of reality not previously otherwise noted in a language. This interplay and interaction between language and reality is thus not a stable one. From this one can see that no reference is fully determined in a language without considerations of meaning and no meaning is fully determined without considerations of reference. 11 This perhaps also explains the basic non-distinguishability between synthetic statements and analytical statements, because synthetic statements can be made relatively analytical by divorcing them from referential considerations, and analytical statements can be made relatively synthetic by relating them to referential considerations.
- 8. If we regard language as a theoretical formulation of reality which is basically open to alternative interpretations, there is no reason not to expect that a given language is both conceptually and referentially indeterminate, i.e., has a meaning and a reference which are not fully and uniquely determined. In fact inscrutability of reference seems to be able to

<sup>9.</sup> Quine, Ibid., p. 191.

<sup>10.</sup> This point is also brought out in Quine's distinction between *direct ostension* and *deferred ostension*. See Quine, *ibid.*, 194ff.

<sup>11.</sup> This is so even in the case of pure ostension as recognized by Quine. See Quine, ibid., p. 194.

explain both the external indeterminacy of translation and the internal indeterminacy of meaning and reference. In fact, the former can be regarded as a consequence of the latter and the latter can be regarded as a condition of the former.

This consideration of the relation of internal indeterminacy of meaning to reference leads to what Quine calls relativity of ontology. Now Quine's notion of this relativity of ontology seems to have many meanings, but they seem not to be differentiated in Quine's recent article on "ontological relativity."

In the first place, there is the relative empirical determination of meaning and reference in regard to the preservation of verbal behavior of co-speakers in a language. In the second place, there is inscrutability of reference in a given language on both levels of inscrutability: the meaningfulness of an expression is open to new interpretation, and the components of an expression can be given alternative ontological identifications (or alternative imputed ontologies), while preserving meaningfulness of an expression. Finally, there is an essential sense of relativity of ontology which Quine wishes to bring out: the explicit specification of an ontology (or reference) for a language is to be carried out in a background language previously accepted, and thus relative to it. This latter background language need not be different from the given language facing the problem of determination of ontology. This last sense of relative ontology, as I take it, has to do with the explication or interpretation of the meaning-fulness of a given language.

Now the distinction between object language and meta-language can be crucial for analyzing the relativity of ontology of a language. A language can be used to refer to things and talk about things it refers to. The singular-terms and general terms, plus the individuative devices of identity and quantification constitute the referential apparatus of the language. On the other hand, we could introduce a meta-linguistic framework to talk about the apparatus of reference of the language. In this framework we perhaps need a primitive term "refer-to" to indicate what objects are referred to in the language by what terms. With this meta-linguistic framework we can list all the references of the referring terms in the given object language in the form "A' refers to A," "B' refers to B," "C' refers to C," etc.

As made clear by Quine, under the principle of preserving meaningfulness of verbal behavior and speech dispositions, there could be an automorphism of the set of thing-nouns with respect to the "referring to" relations. Thus, given a body of verbal behavior, there is no difference between speaking of an ontology of "A" referring to A, "B" referring to B... and speaking of an ontology of "A" referring to B, "B" referring to A... For relative to this body of verbal behavior, the difference in ontologies becomes mere difference of ways of talking. This shows the

<sup>12.</sup> Such as represented by what Quine calls homophonic translation. See Quine, *ibid.*, p. 199.

relativity of ontology in terms of a given chosen or accepted way of speaking as made clear by earlier explanation. What determines a chosen or accepted way of speaking according to Quine is a background language, that is, the background language specifies the ontology for the given language in such a way that "A" refers to A—rather than that "A" refers to B. In this way one may say that a background language is a specific meta-linguistic framework in which the ontology of the object language is specified. One may consider the background language as providing a listing of objects described in the background language to be identified as references of the object language. Different background languages will provide different listings of objects described in the background languages to be identified as references of the object language.

An object language, conceived in terms of a body of verbal behavior or a set of speech dispositions, is, according to Quine, a starting point for various background languages to specify its ontologies. If there are various background languages for a given language, there must be therefore many ontologies for the same given language. Relative to this given language, these ontologies will form an *equivalence class* of ontologies of the given language. But relative to each background language as a meta-linguistic framework of reference, each ontology is different from and indeed incompatible with the other. Thus we may present a thesis corresponding to the thesis of conceptual indeterminacy or indeterminacy of meaning: the thesis on ontological relativity. This thesis says that ontological relativity can be thus best explained in terms of ontological indeterminacy (openness to ontological specification) and existence of equivalence class of ontologies provided by various possible background languages.

9. In the light of what we have said in section 4, we can make a subtle distinction between what a language says what its own ontology is, and what another language says what its ontology is. We have seen that a language can formulate its own referential involvements and thus contains a background language. Unlike the case of grammar or syntax, this grammar of reference may be both equivocal and unformulated. Nevertheless it may also be univocally determined in terms of its own rationale of correctness. Given this condition, the correctness of what another language says about its ontology must be judged by reference to this intended sense of ontology. Of course, on the other hand, what another language says about the ontology of the given language may still differ from the internal ontology in the light of the full exposure of behavior for that language. This is because the very condition of reference of a language is that a reference (ontology) of the language cannot be proved to be the sole reference of the language. For there is no sense for the word "sole reference."

<sup>13.</sup> Even in the case of scientific language, this proof cannot be effected. But in the case of pure logic, a logical system can be said to be categorical in the sense that it has a sole type of model (or reference) of a certain structure.

As a language can be considered a theory, the basic question for Quine's thesis of ontological relativity then is to determine what the objects of a theory are. In order to answer this question, one can see that we have to re-interpet one theory in terms of another, or technically speaking, reduce one to another. In so far as the reduction does not make any difference to the confirming basis of the given theory, the ontology of the reduced language can be specified relative to the reducing language and there is no sense of saying what the objects for a theory absolutely are. <sup>14</sup> A clear reason for this is that the universe of a theory need not be considered fully specified or interpreted. As Quine noted, there is only full interpretation relative to the home theory. But we cannot prove the existence of the full interpretation of the universe of our language in the language itself. On the basis of this understanding, one can always provide a specification or interpretation of the universe of the language as a theory. Different theories will naturally specify different interpretations.

- 10. We have four observations regarding the  $r_2$  interpretation of a theory in another.
- (1) The reducing ontology need not be a meta-linguistic theory for the reduced ontology, for both can belong to the same order of language. But the reduction or interpretation takes place in a meta-linguistic framework which can be said to contain both the language  $S_1$  of the reduced ontology and the reduction in its simplest way is a matter of mapping the ontology of  $S_1$  onto or into that of  $S_2$ . This meta-linguistic framework for this purpose of course must be of a higher order than both  $S_1$  and  $S_2$ . Through this meta-linguistic mapping operation,  $S_1$  we can see that the ontology of  $S_1$  received a new interpretation relative to  $S_2$ .
- (2) It is clear that the reducing ontology can be further reduced to the ontology of another language, say that of  $S_3$ , and that of  $S_3$  can be further reduced to that  $S_4$ , etc. Since the reducing ontology need not be one of the higher order language than that of the reduced ontology, the infinite regress of ontological specifications in terms of background languages or theories need not be considered an infinite regress of meta-languages and meta-

<sup>14.</sup> Referring by ostension is not 'saying' and furthermore involves ambiguity. M.C. Bradley has recently criticized Quine's thesis of ontological relativity as involving an undesirable infinite regress and therefore suggests the view of regarding meaning as a determinate psychic experience. Now I think that the relativizability of the ontology of a language for Quine does not entail the unknowability of that ontology of the language, but only entails its knowability relative to another language under a certain interpretation. Bradley's criticism therefore seems to be non sequitur. See Bradley, "How Never to Know What You Mean," in the Journal of Philosophy (1969), pp. 119-124.

<sup>15.</sup> This also requires consideration of what Quine calls "proxy function." See Quine's article "Ontological Reduction and the World of Numbers," Journal of Philosophy (1964), pp. 209-215. Reprinted also in Quine's The Ways of Paradox and Other Essays, New York: Random House (1966).

theories of higher and higher orders. This infinite regress of ontological specifications reflects openness and indeterminateness of ontological specification, due to lack of *objective* and provable formalization or complete interpretation of ontology in a language or a theory. More will be said on this point. What I wish to stress here is that the "infinite regress" into background languages should not suggest the existence of a hierarchy of languages.

- (3) On the basis of this clarification of ontological specification in terms of theoretical re-interpretability with regard to an internal ontology of a given language, we may raise the question of the transitivity of reference. It is clear that in the hierarchy of meta-languages the relation of reference cannot be transitive, for the reference of "A" to A and the reference of ""A"" to "A" does not entail the reference of ""A"" to A. It is also clear that in a sequence of background languages the relation of reference can be transitive, for the reference of "A" to A in  $S_1$  and the identity of A with B and the reference of "B" to B in  $S_2$  and the identity of Bwith C as the reference of "C" in  $S_3$  entails reference of "A" to C under a comprehensive theory relating A to B and B to C. Of course here one need recognize the relevance of the question of ontological specification for the comprehensive theory of two subordinate theories: the reducing theory and the reducible theory. As Quine notes, the ontological specification of this comprehending theory is a matter to be determined relatively but remains essentially indeterminate. What is important to recognize here is the existence of two senses of background language: background language in the sense that reference can be transitive, and background language in the sense that reference cannot be transitive.
- (4) In the light of the above, one can make more clear the relativity of ontology of a language. As Quine himself does not make the distinction between infinite regress of background languages in the sense of a hierarchy of meta-linguistic frameworks and an infinite sequence of ontological specifications of a language, he seems to suggest the association of relativity of ontology with infinite regress of background languages in the sense of a hierarchy of meta-linguistic frameworks and speaks of ending with one's mother tongue as a solution. But in fact, in regard to the infinite regress of background languages in the sense of an infinite sequence of ontological specifications of the same order, we may suggest symmetry of relativity as a logically better solution in some cases. That is, it could happen that the ontology of "A" in  $S_1$  is specifiable and interpretable relative to "B" in  $S_2$  in some way, whereas the ontology of "B" in  $S_2$  is specifiable and interpretable relative to "A" in  $S_1$  in some other way, both specification and interpretation being done in the framework of a comprehensive theory which, however, need not be formulated. Within this general framework "A" and "B" need not be isomorphic to each other, for the reduction functions in the two directions involved can be very different. Given this view, we may indeed regard the question of what is an "A" as being answered by saying that an "A" is a "B" or by saying

that a "B" is an "A." The classical case of reducing numbers to sets and sets to numbers suggest this symmetry of relativity. The ontological specification and interpretation of what numbers are is brought out by either reducing numbers to sets in set theory or by reducing sets to numbers in number theory.

- 11. We now can say that the relativity of ontology of a language is inherent in the notion of referentiality of a language considered as a theory of reality. The so-called referentiality can be characterized in terms of the following conditions in general derived from the above discussion.
- (1) A language or theory is not fully referentially interpreted except with respect to its own terms and the existence such full interpretation cannot be proved in the language or theory.
- (2) The reference of a theory is always referentially or re-interpretable in another theory and language.
- (3) There are many kinds of referential interpretation or re-interpretation of a theory or language and there is no end to such process of interpretation or re-interpretation.
- (4) The ontology of language can be interpreted in terms of another language or in terms of the ontology of another language being interpreted in the given language.

It seems to me that all these conditions can be explained on the basis of the inscrutability of reference in our interpretation, that is, the basic indeterminacy or openness of the reference of a language.

12. The indeterminacy or openness of the reference of a language can be furthermore reconstrued as a matter of non-formalizability of a language or theory. By introducing the idea of non-formalizability we believe that we are able to specify in logical terms the conditions (and consequences) of admitting a re-interpretable ontology of a language—a state we have referred to as the referentiality of a language.

The concept of formalization has been discussed to a certain extent by Wang Hao in his writings on the axiomatic method and formalization.<sup>17</sup> According to Wang Hao, there are different degrees of formalization and these apparently consist in the degrees of explicitness and adequacy of definability and deducibility of a system of concepts and propositions. Briefly speaking, and without a full discussion on formalization, to formalize is to make explicit the meaning of a concept in terms of other concepts and to make justifiable the validity of an assertion in terms of other assertions in the system. The former procedure of course is a procedure of definability in a system and the latter one is one of deducibility.

<sup>16.</sup> Quine says: "A question of the form 'what is an F?' can be answered only by recourse to a further term: 'An F is a G.' The answer makes only relative sense: sense relative to an uncritical acceptance of G." See Quine's "Ontological Relativity," p. 204.

See Wang Hao, A Survey of Mathematical Logic, Amsterdam and Peking (1963), Chapters I and III.

A second point about formalization is that making explicit the meaning of a concept and the justification of validity of an assertion must be completely expressed in explicit symbolism of the system or must be capable of being so expressed. The axiomatic system which evolved from Euclid provides a means for making explicit meanings of concepts and for justifying validity of assertions in the system. The formulation of an axiom system depends upon the adequacy of its axioms in such a way that they must express all the relevant properties of the undefined technical terms so that it should be possible to perform deductions even if we treat the technical terms as meaningless words. Furthermore, we should make explicit the principles which determine the meanings of the logical and nonlogical words in the system. In this way we are able to recognize axioms, proofs and theorems by looking at the symbolized patterns.

Thus, Wang Hao says: "We shall speak of formal or axiomatic systems only when the systems satisfy the following criterion: There is a mechanical procedure to determine whether a given notational pattern is a symbol occurring in the system, whether a combination of these symbols is a well-formed formula (meaningful sentence) or an axiom or a proof of the system. Thus the formation rules, i.e., rules for specifying well-formed formulas are entirely explicit in the sense that theoretically a machine can be constructed to pick out all well-formed formulas of the system if we use suitable physical representation of the basic symbols. The axioms and rules of inference are also entirely explicit. Every proof in each of these systems, when written out completely, consists of a finite sequence of lines such that each line is either an axiom or follows from some previous lines in the sequence by a definite rule of inference. Therefore, given any proposed proof, presented in conformity with the formal requirements for proofs in these systems, we can check its correctness mechanically." <sup>18</sup>

In short we can take full and complete formalization as a state fulfilling the condition that the content of the formalized system in the explicit form of symbolism can be recognized or checked by a mechanical procedure. It is in this sense that a completely formalized system is a decidable theory with a proof procedure and a decision procedure. Now what is the significance of this notion of complete formalization for the referentiality of a system? The answer is that if a system is fully formalized, then the meaningfulness of the expression in the system is completely explicit and mechanically decidable, and therefore the referentiality of the system is also explicitly represented and mechanically decidable. In this sense we can say that there is no problem of referentiality of the system, because there is no sense of speaking of reference to an ontology, as reference to an ontology is something not to be expressed

<sup>18.</sup> See Wang Hao, *ibid*., pp. 3-4.

in the system and the reference relation is basically not completely unformalizable. 19

Take for example the propositional calculus: It is well known that the propositional calculus is a decidable and complete system, that is, there exists a decision and proof procedure for the system. As such it is fully formalizable and fully abstractable from any meaningful interpretation. Therefore it does not have to be understood in regard to any ontology nor is there anything to prevent it from being applicable to many universes of objects. The very essence of formalizability for the propositional calculus is precisely that one need not go out of the system to find a justification of truth claims and there is no irreducible relation of reference in the system. for a mechanical procedure could effectively single out true formulas and well-formed formulas on the basis of what has been identified as true or well-formed formulas. Furthermore, as a complete and decidable theory, it can have interpretations in such a way that it can be assigned a universe of objects, but any two of these interpretations would be essentially equivalent or isomorphic, i.e., the system would be categorical. In this light, we may explain the lack of referentiality of a completely formalizable system as one in which the formalization of the system and the ontology of the system are one and identical. That is, we can regard all its primitive terms as names of entities. And as such, they can be regarded as objects which can be named in a background language. But disregarding the latter possibility, propositional calculus as a system of names qua names makes no reference to any external objects and hence no ontological commitment as such. The loss of referentiality, therefore, is a result of the complete formalization or abstraction of meaning of a system, which is thereby characterized by completeness and decidability of the system.

As Quine noted,<sup>20</sup> even quantification theory with a finite universe of named objects will lose referentiality. For here we can expand quantification into finite conjunctions and alternations with the disappearance of variables and the values of variables as a result. The quantification theory hence is reduced to propositional calculus, and becomes complete and decidable. Hence Quine concludes,<sup>21</sup> "Ontology is internally indifferent also, I think, to any theory that is complete and decidable. Where we can always settle truth values mechanically, there is no evident internal reason for interest in the theory of quantifiers nor, therefore, in values of variables. These matters take on significance only as we think of the decidable theory as embedded in a richer background theory in which the variables and their values are serious business."

<sup>19.</sup> Since referentiality is conceived as unrestricted interpretability and reinterpretability of a system, any finitely represented system is inadequate to expressing this unrestricted interpretability and re-interpretability.

<sup>20.</sup> See Quine, ibid., p. 209.

<sup>21.</sup> See Quine, ibid.

We must observe that the loss of referentiality in complete and decidable systems does not mean that the system cannot be referential in any sense. There are at least two senses in which it can be still referential. First in Quine's sense of reinterpretation in a broader containing theory. That is, we can assign a universe to terms of the system as a subordinate portion of a language (or theory) which is not complete and not decidable. Secondly, in the sense we mentioned earlier, that is, all the real models of the system are isomorphic. These models make no difference to what reality or ontology is chosen. In both these senses, referentiality can be claimed of such systems, but the relativity of ontology of such systems becomes meaningless.

- 13. In the light of non-formalizability of a system as an equivalent condition for relativity of ontology, we can list several important conditions for relativity of ontology or referentiality, in terms of the conditions for non-formalizability of a system. These conditions are as follows:
- (1) The system in question is not categorical, i.e., no two models of the system can be proved to be isomorphic.
- (2) There is no unique intended model of the system. Or no model can be shown to be the fully intended interpretation of the system.<sup>22</sup>
- (3) The universe of the system is not fully and uniquely interpreted and there are alternative specifications of the universe.
- (4) The system is not logically complete. (It is not the case that for every sentence in the system, either it or its negation is a theorem.)
- (5) There is no truth decision procedure for the system.<sup>23</sup>

Each of these conditions is a condition for the attainment of referentiality of a system as each of them is a condition for the non-formalizability of a system. It is clear that conditions (1), (2), and (3) are logically equivalent. Then the question is how they are related to (5) and how (5) is related to (4). In the light of the theory that every categorical system is complete and in the light of Gödel's incompleteness theorem, we readily see that (4) entails (1), (2), and (3). From this we can see that categoricality is the basis for both completeness and truth decidability, while absence of truth decidability is the basis for both incompleteness and non-categoricality, while non-categoricality does not entail incompleteness or truth non-decidability. In the light of these logical relationships, we can distinguish between two cases of referentiality or relativity of ontology: A strong case and a weak case. A strong case of referentiality is where no truth decidability obtains, whereas a weak case of referentiality is where no categoricality obtains. It is then clear that the strong case entails the

<sup>22.</sup> This is based on the fact that one criterion of the adequacy of a formalization is that the theory of which the system is a formalization is the intended interpretation of the formalization.

<sup>23.</sup> But we note that there exist incomplete axiom systems for which there are decision procedures for provability. See Wang Hao, *ibid.*, p. 18.

weak case, whereas the weak one does not entail the strong one. Similarly, we may distinguish between two cases of non-referentiality or meaning-lessness of relativity of ontology. A strong case of non-referentiality is where categoricality obtains, and a weak sense of non-referentiality is where truth decision procedure exists. Again the strong case entails the weak one, but the weak one does not entail the strong one.

It seems clear that the strong case of referentiality is closely bounded with Gödel's incompleteness theorem. This theorem has the consequence of showing the importance of the distinction between truth and demonstrability. As demonstrability is a matter of formalizability, and truth a matter of interpretation, we can see that no formalization can always formalize the notion of truth or reference to an object in interpretation. Gödel's theorem and the related Tarski theorem (In a number system Z or any system which contains Z, we cannot define the notion of truth for the system itself), provide a framework for understanding the notion of referentiability of a system.<sup>24</sup>

What we would like to stress of course is that referentiality could be regarded formally indefinable, and philosophically fundamental. But in the light of the above we have related some meta-mathematical properties to this notion of referentiality and used them as criteria and conditions for referentiality of a language or a theory in a language. On the other hand, we could also regard these conditions and criteria (to be formulated as properties) as logical consequences of the notion of referentiality, to be explicated or formalized. But we can see that this cannot be done, for we have no idea of how to formalize the notion of referentiality and guarantee its relevance to these logical consequences. Furthermore, if it is ever formalizable, we would be able to decide on the properties of noncategoricality, non-decidability or non-completeness in a categorical, decidable and complete system, and this will eliminate a higher order of referentiality. But this is paradoxical. As it is, referentiality or nonreferentiality remains basically an intuitive notion which lurks behind all discussions of meta-methematical properties such as categoricality or non-categoricality, completeness or incompleteness, decidability or nondecidability. The terms "referentiality" and "non-referentiality" are themselves referential. Because of this, it seems to me more justifiable to consider these properties as conditions of referentiality rather than consequences of referentiality.

14. On the basis of our explanation of the conditions (or consequences) of referentiality, we see that existence of relativity of ontology to background language is equivalent to existence of meta-mathematical properties as listed above. It is clear in the proofs of Gödel's incompleteness theorems that it is only relative to an intuitive number theory that expressions are

<sup>24.</sup> Quine has himself noted the similarity of "regress in ontology" to "regress in truth, satisfaction and naming" in Tarski's system. See Quine, *ibid.*, p. 212.

specified as Gödel numbers. Furthermore, in the proof of non-categoricality of a system actual reference to models of the system must be made and these models must be different. Hence, relativity to background languages is essential for the statement of the referentiality of a system.

At this juncture we must bring in the challenge of substitutional interpretation of quantification which seems to force us to reduce quantification (a theory with an infinite universe) to the fate of a theory with finite universe, whereby relativity of ontology loses its meaning. This is certainly a challenge to the very strong condition of non-decidability of the quantification theory—a condition for referentiality of a system. In brief, this challenge is equivalent to contradicting (1) or (2) or (3) so that non-decidability of the theory will be eliminated or the question of such will not arise. In other words, by the device of substitutional interpretation quantification will lose its ontological significance and no explicit statements of reference to objects of existence is needed. According to that scheme, we can explain universal quantifications as true when true under all substitutions, and existential quantifications as true when true under some substitution.<sup>25</sup> Of course, in order to make this possible, one has to assume that each of the infinitely numerous objects of the universe has a name. In making a substitutional interpretation the force of quantification is explained away, as Quine remarks, "substitutional quantification, thus brings no way of distinguishing names from other vocabulary, nor any way of distinguishing between genuine referential or value-taking variables and other place-holders.",26

As there is no ontological distinction between reference and predication in the substitutional interpretation of quantification, there is no question of ontology of the system per se as under the substitutional interpretation, even though the original theory can still have an ontology by a referential interpretation of quantification in the theory. The question is then: Can we therefore eliminate or relativize the notion of referentiality by introducing the substitutional interpretation of the quantification? An adequate answer to this question depends upon a full examination of the nature of substitutional quantification. As substitutional quantification can be proved to have preserved many meta-mathematical properties such as entailment and completeness, there is no reason why the understanding of it may not already carry a referential significance.<sup>27</sup> But on the other hand, there is no reason why a thorough substitutional interpretation of quantifiers in a hierarchy of meta-languages is not possible. This only means that substitutional interpretation gives us an effective way of considering all sys-

<sup>25.</sup> Quine has cited Ruth Marcus as favoring such a course of interpretation. See her article "Interpreting Quantification," in *Inquiry*, vol. 3 (1962), pp. 252-259.

<sup>26.</sup> See Quine, ibid., pp. 209-210.

<sup>27.</sup> Cf., J. Michael Dunn and Nuel D. Belnap, Jr., "The Substitutional Interpretation of the Quantifiers," in Noûs (1968), pp. 177-185.

tems as basically completely formalizable or as fulfilling intended models so that talk of ontology, and for that matter, talk of relativity of ontology, becomes meaningless. But of course we need not forget that an infinite system can be given a referential standard interpretation and that the possible prevalence of substitutional interpretation does not prevent us from reinterpreting a whole system in a referential manner by relating the system to a broader theory, which however, need not be actually formulated or formalized. In this sense referentiality of a system is a primitive notion which is bound up with our fundamental sense of interpretability.

15. The question of substitutional interpretation of quantification leads Quine to search for a case of referential quantification which cannot be replaced by substitutional quantification. Furthermore, this case of referential quantification will not be considered relativizable to a background language. This certainly will establish the irreducibility of referentiality as a characteristic of formal systems together with the thesis that we need not specify the referentiality of a system in regard to a background language. Now, Quine has found  $\omega$ -inconsistent or numerically insegregative systems as providing a case of irreducible referential quantification, without being relativized to a background language. To quote Quine: "This is a case where an absolute decision can be reached in favor of referential quantification and against substitutional quantification, without ever retreating to a background theory."

A  $\omega$ -inconsistent system is one in which every result of substituting a name for the variable in a certain open sentence is true in the theory, but the universal quantification of the sentence can be proved false in the theory. To be more specific, an  $\omega$ -inconsistent system is one in which there is a formula Fx such that all substitutional instances of numerals 1, 2, 3, ... could be proved, but  $(Ex)(x \in N. \sim Fx)^{29}$  could be proved too. Given this understanding of  $\omega$ -inconsistent system, it is clear that besides all substitutional instances of an open sentence there are still nameless objects which will not be taken care of in the substitutional interpretation of quantification over natural numbers. Now the question is how the case of an  $\omega$ -inconsistent system affects the claim of relativity of ontology or Apparently, it does not change the significance of the referentiality. relativity principle, for the specification of a universe of objects can be an integral part of the total system. In this sense every system with a self-determined specification of universe will be referential not relative to a background language, but relative to itself. In the case of  $\omega$ -inconsistent system, it seems clear that the ontological specification of a system, which makes the system  $\omega$ -inconsistent is that a system of numbers N cannot be defined in such a way that the definition will enable

<sup>28.</sup> See Quine, ibid., p. 210.

<sup>29.</sup> See Quine's Set Theory and Its Logic, revised edition, Harvard University Press (1969), p. 305.

us to prove that N contains just 1, 2, 3, without nameless objects besides this. As Quine points out, <sup>30</sup> Henkin has shown that there is no contradiction between assuming the validity of

$$0 \in N$$
,  $1 \in N$ ,  $2 \in N$ , . . .

and the supposition that

$$x \in \mathbb{N}, x \neq 0, x \neq 1, x \neq 2, \ldots$$

From this, it seems clear that the independent reference to nameless objects in the system is made by a specification of the attempt to formalize the notion of number. This existence of  $\omega$ -inconsistency indicates that in formalizing the number concept, a universe must be implicitly chosen beforehand-this reflects the non-formalizability of number concept and the incompleteness, non-categoricality, as well as undecidability of the number system. This only shows that the system of numbers must be basically referential.<sup>31</sup> What definition and interpretation of number theory reflect is the relativity of ontology. Furthermore, the re-interpretation of the extras beyond numbers is again a matter relativized to a background language, i.e., the referentiality of referential quantification in the sense of referring to nameless objects can be proved or assumed relative to an intuitive background language. Not only  $\omega$ -inconsistent systems are referential in this sense, systems involving non-constructive existence theorems such as based on the axiom of choice are also referential in this sense, and this is not incompatible with relativity of ontology of the same system.

To conclude, the example of  $\omega$ -inconsistent systems does not harm nor detract the original sense of referentiality of a system, but has the merit of showing the irreducibility of referential quantification to substitutional quantification. Furthermore, as Quine noted, the existence of nameless objects in the  $\omega$ -inconsistency example provides a lesson for reinterpreting substitutional quantification, to the effect that one cannot make a distinction between referential and substitutional quantification within a theory, because one can always assume that nameless objects are inseparable from the named ones. This latter is conceivable because it is always philosophically possible to partially define an ontology of a system within a system or to make a reference to something to be defined. On the other hand, on the side of substitutional quantification, one may not neglect the possibility of reinterpreting referential quantification in  $\omega$ -inconsistent systems in terms of substitutional quantification. This is the case when the universe of nameless objects is reinterpreted in terms of objects named. But then

<sup>30.</sup> See Quine's Set Theory and Its Logic, p. 304.

<sup>31.</sup> Or numerically insegragative in Quine's sense as one cannot get rid of the extras in a numerically insegregative system by re-defining the concept of number. See Quine's article "On  $\omega$ -inconsistency," in *Journal of Symbolic Logic*, vol. 18 (1953), pp. 119-124. Also reprinted in Quine's *Selected Logic Papers*, New York: Random House (1966).

referentiality is introduced in the back door before it exits through the front door. This perhaps adds to the weight of the argument against substitutional interpretation, but not against relativity of ontology as explained earlier. For the nameless objects are not a reflection of the inscrutability of reference, but a result of relativity of ontology (as relativized to an intuitive background theory), in so far as we can talk about nameless objects at all. In other words, the statement that we do not have to regress to a background language to refer to nameless objects as separate from the named ones in the  $\omega$ -inconsistent system, does not contradict the statement that it is relative to a certain way of formulating the notion of number that reference to and existence of, certain nameless objects are possible and that it is also relative to a certain interpretation in the language that nameless objects will be able to be identified as such.

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