

## A NOTE ON A CONSISTENCY PROOF

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*Natural Deduction* by Anderson and Johnstone contains a mistake on page 295 in the proof of Lemma a, which is used to prove the consistency of their rules of PL.

In the *Basis* step on page 295, they want to show that if all **B'** lines (assumptions to be discharged later) possess 1 (have truth value T), then the line (\*) just before the innermost discharge of  $\supset I$  possesses 1. To that end, they consider the possible ways in which the line (\*) can occur. One of these is given in (ii) on page 295 when they say, "Or the line before discharge is a line possessing I" (a line possessing I is a tautology). The possibility they should have considered and fail to consider elsewhere is that the line before discharge is a **D'** line (zero premise conclusion). Of course a **D'** line possesses I, but they do not prove this until Lemma b, whose proof uses Lemma a. Therefore their consistency proof of the rules of PL is incorrect.

Their proof of Lemma a becomes correct if they merely prohibit **D'** lines in their deductions and require that the deduction for the **D'** line be given in place of the **D'** line, thereby eliminating the need to consider them.

## REFERENCE

- [1] Anderson, John M., and Henry W. Johnstone, *Natural Deduction*, Wadsworth Publishing Co., Belmont, California (1962), pp. 285-299.

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