

ANDREAS KESLER AND THE LATER THEORY OF CONSEQUENCE

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1 In another paper I examined the theory of consequence presented by a number of later fifteenth and early sixteenth century writers, ending with Javellus, an Italian who died in 1538.¹ For this earlier period, there was an abundance of material, containing much sophisticated discussion of semantical issues; but the next hundred years do not offer more than a few sources, and these are of limited value. The only really outstanding figure, so far as I can see, is that of Andreas Kesler. He was a Protestant theologian who was born at Coburg in 1595, educated at Jena and Wittenberg, and died in 1643 after a long career in education. In 1623 he published a book entitled *De Consequentia Tractatus Logicae* which is unique, both for its own time, and as compared to the products of the earlier period, in that it explicitly subsumes the whole of formal logic under the theory of consequence. The laws of opposition and conversion, the categorical and hypothetical syllogism, were all seen as different types of consequence. Moreover, no extraneous material was included. Instead of starting with the categories, like the Aristotelians, or with the invention of arguments, like the Ramists, he devoted his first chapter to the definition of consequence. Topics, informal fallacies and other such subjects found no place, whereas some rarely discussed matters like exclusive and reduplicative propositions and the modal syllogism did appear. Thus he stands out for his contents as well as for his organization. All his contemporaries devoted time and space to non-formal matters like the categories; and none of them devoted so much as a separate tract to the theory of consequence. Petrus Fonseca (1528-1599), a Jesuit who taught at Coimbra, treated the subject more thoroughly than most, but only as a brief prelude to the syllogism. Writers like Carbo and Gabriel of St. Vincent followed him in this, whereas others like Timplerus and Blundeville discussed it after the syllogism. Mercado only mentioned the matter in his tract on hypothetical propositions, while Caesarius introduced it in his tract on hypothetical syllogisms. At first sight, Kesler seems to have been the only rigorous formal logician of his time, the only man who saw precisely what kind of material he was handling, and how it should be organized.

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However, the picture is not so impressive when one considers what Kesler said about the theory of consequence in more detail. He began by defining a consequence as an *oratio* or statement in which one thing is derived from another thing,² but in this he was not only quoting Fonseca, but saying no more, and no less, than most other logicians of his time.³ Only Caesarius gave the definition which had been standard in the earlier period, namely, that a consequence is an *oratio* with an antecedent, a consequent, and a sign of illation;⁴ and only Regius and Timplerus discussed alternatives in detail.⁵ The former quoted Javellus's definition of a consequence as an *oratio* containing several propositions joined by a sign of illation, and compared it to the definition in terms of an illative connection between propositions; while the latter concluded that a consequence should be defined neither as an *oratio* nor as a *habitudo*, or connection, but as an illation that takes place in a statement and is founded on a connection. Both Oddus and John of St. Thomas raised the further question of whether, in the light of such definitions, a consequence could legitimately be called invalid.⁶ For a solution Oddus relied on the traditional remark that an invalid consequence is to a valid one as a painted man is to a living man; but John of St. Thomas pointed out that when a consequence is defined as a "statement in which from one given thing another follows", "it follows" means the same as "it is said to follow", and that the definition is therefore a neutral one with respect to validity. He was the only author to add that consequences are not propositions, since they do not affirm or deny, but rather connect propositions; and that they are therefore assessed as valid or invalid rather than as true or false.⁷

The first division of consequence was into good and bad, or valid and invalid; and here again what little Kesler had to say was derived from Fonseca. A valid consequence, he said, was a statement in which one proposition is inferred from another by a legitimate deduction, in such a way that the consequent is dependent on the antecedent. The main criterion is that it is impossible to have a true antecedent and a false consequent; and two further criteria are that a true antecedent leads to a true consequent, and that the contradictory opposite of the consequent is repugnant to the antecedent.⁸ He explained later that two propositions are repugnant when they cannot both be true.⁹ The elaborate discussions of validity in which earlier authors had indulged were ignored by Kesler, as they were by his source, Fonseca, and by all the other logicians of the period.¹⁰ The only alternative approach to be found is that of Melancthon, who defined a valid consequence as one which did not violate the precepts of dialectic.¹¹ He was followed in this by Vincentius who, like Melancthon, failed to realize that such a definition was valueless, given that the point of defining validity was to produce a standard for the assessment of the precepts of dialectic themselves.¹²

Having established the conditions for validity, Kesler went on to divide consequences into formal and material. Here he quoted two sources, Fonseca and Javellus, or rather, Regius's account of what Javellus had said.¹³ Fonseca had given the standard definition whereby a consequence is

formal if its validity depends upon the form of the propositions concerned, but material if its validity depends upon the terms appearing in the propositions. Javellus, however, had introduced psychological criteria when he said that a consequence is formal if the opposite of the consequent is manifestly repugnant to the antecedent, but material if the repugnance is present without being manifest.¹⁴ The example Javellus gave was "God does not exist and some man does exist" which seems quite consistent to an atheist, although the conjunction is in fact an impossible one.¹⁵ Regius said that Fonseca was clearer than Javellus; and Kesler explained that he too preferred Fonseca because of the analogy with syllogisms. The distinction between the form and matter of a syllogism has nothing to do with the obviousness of the inference.¹⁶ He added that one cannot call a consequence materially valid on the grounds that the consequent is true although some rule is violated, for there must also be a connection of matter or terms. He and Regius were the only two logicians who discussed this subject in any detail; for all the other sources gave the standard distinction without comment,¹⁷ though Carbo and Gabriel of St. Vincent added a few words about what constituted the formal elements of a proposition.¹⁸

In the earlier period, the distinction between formal and material consequence had led into a discussion of the paradoxes of strict implication, namely that from the impossible anything follows, and the necessary follows from anything. These rules were, however, given a completely new interpretation in the period with which I am concerned. It was carefully explained that by 'anything' was meant a necessary, contingent, or impossible proposition;¹⁹ and Fonseca's examples show that a close link between antecedent and consequent was assumed. For instance, he gave "Every stone subsists by itself, man is a stone, therefore man subsists by himself" as an example of how the necessary is derived from anything.²⁰ Only Regius referred to the original interpretation, whereby 'anything' meant some arbitrary proposition. He reported that Javellus and the doctors of Cologne had rejected this view on the grounds that in a case like "Man is an ass, therefore the stick is in the corner", there is no relation of dependence between the propositions, and the negation of the consequent is not repugnant to the antecedent.²¹ The only earlier parallel to Fonseca's view of the paradoxes is found in the Cologne commentary on Peter of Spain, which said that the impossible could lead to anything with respect to the genus of propositions, but not with respect to their species or number.²²

The last distinction, which Kesler again took from Fonseca, was that between necessary and probable consequences.²³ All formal consequences are necessary, but some material consequences, like "She is a mother, therefore she loves her son" are merely probable. The only analogue to this distinction found in the earlier period is that between illative and probative consequences, where a merely probative consequence was said to be persuasive, but without logical force.²⁴ Some formal consequences were illative and probative; others, like the paradoxes of strict implication, were merely illative.²⁵

2 *Rules.* In the following section, I shall use ‘ P ’, ‘ Q ’ and ‘ R ’ as metalinguistic variables ranging over propositions; ‘ \rightarrow ’ as a metalinguistic sign meaning ‘implies’ or ‘is a valid consequence’; and ‘ \vdash ’ as a meta-metalinguistic sign to indicate that from one valid consequence another can be formed. ‘ $=$ ’ is the sign of mutual implication. ‘ $P \circ Q$ ’ will be read as ‘ P stat cum Q ’ or ‘ P is consistent with Q ’; and its negation, ‘ $\neg(P \circ Q)$ ’ will be read ‘ P is repugnant to or inconsistent with Q ’. Otherwise my symbolism should need no explanation.

I. *General Rules.* Kesler gives a list of eight general rules:

$$1.1 \quad P \rightarrow Q, \mathbf{T}'P' \vdash \mathbf{T}'Q'$$

$$1.2 \quad P \rightarrow Q, \mathbf{T}'Q' \vdash \mathbf{T}'P' \vee \mathbf{F}'P'$$

Ex vero non nisi verum; verum autem tum ex vero tum ex falso colligitur.²⁶

$$2.1 \quad P \rightarrow Q, \mathbf{F}'P' \vdash \mathbf{T}'Q' \vee \mathbf{F}'Q'$$

$$2.2 \quad P \rightarrow Q, \mathbf{F}'Q' \vdash \mathbf{F}'P'$$

Ex falso falsum & verum, falsum autem non nisi ex falso concluditur.²⁷

$$3.1 \quad P \rightarrow Q, \neg\Diamond P \vdash \neg\Diamond Q$$

$$3.2 \quad P \rightarrow Q, \neg\Diamond Q \vdash \neg\Diamond P \vee \neg P \vee (\Diamond P \cdot \Diamond \neg P)$$

Ex necessario non nisi necessarium, necessarium autem ex quolibet scilicet necessario contingenti vel impossibili.²⁸

$$4.1 \quad P \rightarrow Q, \Diamond P \cdot \Diamond \neg P \vdash \neg\Diamond Q \vee (\Diamond Q \cdot \Diamond \neg Q)$$

$$4.2 \quad P \rightarrow Q, \Diamond Q \cdot \Diamond \neg Q \vdash \neg\Diamond P \vee (\Diamond P \cdot \Diamond \neg P)$$

Ex contingenti nunquam colligitur impossibile, sed vel necessarium vel contingens: contingens autem nunquam ex necessario sed vel ex contingenti vel impossibili concluditur.²⁹

$$5.1 \quad P \rightarrow Q, \neg\Diamond P \vdash \neg\Diamond Q \vee \neg\Diamond \neg Q \vee (\Diamond Q \cdot \Diamond \neg Q)$$

$$5.2 \quad P \rightarrow Q, \neg\Diamond Q \vdash \neg\Diamond P$$

Ex impossibili sequitur quodlibet, hoc est, necessarium, contingens et impossibile. Impossibile autem non nisi ex impossibili colligitur.³⁰

$$6 \quad P \rightarrow Q, \mathbf{r}'P \circ R' \vdash \mathbf{r}'Q \circ R'$$

Quicquid stat cum antecedente, stat etiam cum consequente, non contra.³¹

$$7 \quad P \rightarrow Q, \mathbf{r}'\neg(Q \circ R) \vdash \mathbf{r}'\neg(P \circ R)$$

Quicquid repugnat consequenti, repugnat etiam antecedenti, non tamen contra.³²

$$8.1 \quad P \rightarrow Q, R \rightarrow P \vdash R \rightarrow Q$$

$$8.2 \quad P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$$

Ex quo antecedens, ex eo etiam consequens, & quicquid sequitur ex consequente, sequitur etiam ex antecedente.³³

In addition to this list of rules which Kesler took almost word for word from Fonseca³⁴ he gave the following as a corollary to rule 1:

$$9 \quad P \rightarrow Q, P \vdash Q$$

In omni verâ consequentiâ posito antecedente, ponitur consequens.³⁵

By virtue of his reference to Fonseca's claim that the first type of consequence is when a proposition is inferred from itself or from an equivalent, one can also give the following rules³⁶:

$$10.1 \quad P \rightarrow P$$

$$10.2 \quad P, P = Q \vdash Q$$

Although Kesler's list of rules seems short and uninteresting when it is compared with those given by earlier authors, his contemporaries did no better, and only a handful of extra rules can be added to the above.

$$11.1 \quad \neg \diamond \neg P \vdash P$$

$$11.2 \quad \neg \diamond \neg \neg P \vdash \neg P$$

A propositionem, tam de non inesse, quam de inesse, de necessario ad propositionem de inesse simpliciter consequentia est formalis.³⁷

The author does not explain how this is compatible with the two following rules:

$$12.1 \quad P \vdash \diamond \neg P, \diamond P$$

$$12.2 \quad \neg P \vdash \diamond P, \diamond \neg P$$

A propositionem de inesse simpliciter ad propositionem de contingenti non inesse, & a propositionem de non inesse simpliciter, ad propositionem de contingenti inesse, consequentia est formalis.³⁸

$$13 \quad P \rightarrow Q, \diamond P \vdash \diamond Q$$

Si alicuius bonae consequentiae antecedens est possibile, & consequens similiter est possibile.³⁹

$$14 \quad P \rightarrow Q, \neg Q \vdash \neg P$$

Si ad alicuius bonae consequentiae datae antecedens sequitur consequens, ad oppositum consequentis sequitur oppositum antecedentis.⁴⁰

$$15 \quad P \rightarrow Q, \ulcorner P \cdot Q \urcorner \rightarrow R, Q \rightarrow R \vdash P \rightarrow R$$

This is my interpretation of Santolaria's words " & quicquid sequitur totum, & solùm consequens, sequitur antecedens",⁴¹ but as he gives no examples or further explanation, I may well be mistaken.

II. *Rules for Hypothetical Propositions.* In later chapters, Kesler added some rules for hypothetical propositions. Most of these were common-place, so I shall not bother to quote the Latin text, with two exceptions.

$$1.1 \quad \ulcorner P \cdot Q \urcorner \rightarrow P$$

$$1.2 \quad \ulcorner P \cdot Q \urcorner \rightarrow Q^{42}$$

$$2 \quad \ulcorner \neg(P \cdot Q) \urcorner = \ulcorner \neg P \vee \neg Q \urcorner$$

A totâ copulativâ negatâ ad disiunctivam & contrâ valet consequentia sub extremis negatis, ut Non & dies est & nox est. E. aut dies non est aut nox non est & contra.⁴³

$$3.1 \quad P \rightarrow 'P \vee Q'$$

$$3.2 \quad Q \rightarrow 'P \vee Q'^{44}$$

$$4.1 \quad '(P \cdot Q)'\rightarrow -Q$$

$$4.2 \quad '(P \cdot Q)'\rightarrow -P$$

A positione unius partis [copulativi] ad negationem alterius v.c. ex majori negativâ⁴⁵

He justified this rule on the grounds that a negated conjunction could be reduced to two conditionals⁴⁶:

$$4.3 \quad '(P \cdot Q)'\rightarrow 'P \supset -Q'$$

$$4.4 \quad '(P \cdot Q)'\rightarrow 'Q \supset -P'$$

Doubtless he owed this insight to Fonseca.⁴⁷

$$5.1 \quad 'P \vee Q', -P \rightarrow Q$$

$$5.2 \quad 'P \vee Q', -Q \rightarrow P^{48}$$

$$6 \quad 'P \supset Q', P \rightarrow Q^{49}$$

$$7 \quad 'P \supset Q', -Q \rightarrow -P^{50}$$

The only other authors to discuss hypothetical propositions in the context of the theory of consequence were Regius and Campanella; although Mercado did the reverse. Mercado gave one additional rule, which Campanella explicitly denied:

$$8 \quad 'P \cdot Q' \rightarrow 'P \vee Q'^{51}$$

Sixteenth and seventeenth century logicians were, of course, aware of many more propositional inference forms than the above, but the place to look for them is in the chapters on hypothetical syllogisms rather than the chapters on consequence.⁵²

3 After this brief survey one can only conclude that the theory of consequence suffered an abrupt decline after the first part of the sixteenth century. The one outstanding writer on the subject was Andreas Kesler, but he stands out for a single insight, rather than for any awareness of the ramifications of the theory. Unlike his sources, he saw that all of formal logic could be subsumed under the basic notion of consequence, and he was able to exclude extraneous material, but that was as far as he went. About the definition and division of consequence, and about consequential rules, he had nothing to say but what had been said before him by Fonseca and Regius. Nor did he betray any knowledge of earlier writers, although some at least must have been available to him in Wittenberg. For once those who deplore the loss of mediaeval insights during the sixteenth century seem to be justified.

NOTES

1. See my paper "The Theory of Consequence in the Late Fifteenth and Early Sixteenth Centuries," to appear in *Notre Dame Journal of Formal Logic*, vol. XIV (1973), No. 3, pp. 289-315.
2. Andreas Kesler, *De Consequentia Tractatus Logicus*, Wittenberg (1623), p. 2. "Consequentia in genere est oratio, in qua ex aliquo aliquid colligitur."
3. Petrus Fonseca, *Institutionum Dialecticarum libri octo*, Coimbra (1590), Vol. II, p. 1. Cf. L. Carbo, *Introductiones in Logicam*, Venice (1597), p. 122.
4. J. Caesarius, *Dialectica*, Coloniae (1559) [no pagination]. Cf. Oddus Illuminatus, *Logica peripatetica ad mentem Scoti*, Panormi (1664), p. 55, for a brief reference to this as an alternative to his definition: "Consequentia formaliter est habitudo necessaria illationis antecedentis ad consequens."
5. J. Regius, *Commentariorum ac disputationum logicarum libri IV*, Wittenberg (1608), p. 647; C. Timplerus, *Logicae Systema Methodicus*, Hanover (1612), p. 612.
6. Oddus, *loc. cit.*; John of St. Thomas, *Outlines of Formal Logic*, translated by F. C. Wade, Milwaukee (1955), p. 103.
7. John of St. Thomas, *loc. cit.*
8. Kesler, pp. 5-9. "Bona consequentia est, concretivè definiendo, oratio in qua ex una vel pluribus enunciationibus alia colligitur legitimâ deductione, ita ut verè haec tanquam consequens ab illa vel pluribus tanquam antecedente dependeat. . . . Bona consequentia, abstractè & formaliter considerando, est nexus habitudo, & dependentia vera enunciationum, quâ ex unâ vel pluribus alia deducitur legitimè, & ad rem ipsam accommodatè. . . . ponunt autores generalem hunc canonem, pro discernendâ consequentiâ bonâ à vitiosa. In bonâ consequentiâ antecedens non potest esse verum sine consequente." Cf., Fonseca, p. 4. "Bona dicitur ea, in qua aliquid ex aliquo vere colligitur." He gave the last two criteria on p. 5 and p. 6.
9. Kesler, p. 27.
10. The following authors stated that in a valid consequence it is impossible for the antecedent to be true and the consequent false, or that the truth of the consequent necessarily follows from that of the antecedent: Caesarius; S. Carvisius, *Catena aurea in totam logicam*, Venetiis (1561), p. 117; Thomas de Mercado, *Commentarii lucidissimi in textum Petri Hispani*, Hispali (1571), p. 67^{VO}; M. Santolaria, *In Dialecticam Integram perfecta quaedam institutio*, Oscae (1583), p. 156; Regius, p. 649; J. Scharfius, *Institutiones Logicae*, Wittenberg (1632), p. 490; J. Seton, *Dialectica*, London (1574) [no pagination]; and Timplerus, p. 585 and p. 613. The criterion of repugnance was given by T. Campanella, *Philosophiae Rationalis Partes quinque*, Paris (1638), p. 372; Carbo, p. 122; Carvisius, *loc. cit.*; Gabriel of St. Vincent, *Logica*, Romae (1669), p. 35; Regius, *loc. cit.*; Scharfius, p. 491; and Timplerus, p. 613.
11. P. Melancthon, *Erotemata Dialectices in Opera quae supersunt omnia*, edited by C. G. Breitschneider, Halle and Braunschwig (1834-1860), Vol. XIII, p. 595. "Quid est bona consequentia? Est recta connexio omnium partium in Argumento, in qua nullum praeceptum Dialecticae violatur."
12. P. Vincentius, *Compendium Dialectices*, Wratislaviae (1597), p. 49.

13. Kesler, pp. 12-13. Of the first view he said: "Fonseca & alii consequentia formalis est, quae vi formae concludit, hoc est, cujus formam modumque colligendi si retinueris, in quacumque materiâ etiam impossibili aptè concludes. Materialis, quae vi solius materiae concludit, hoc est, quae vi in quidem formae nihil colligit, sed tamen talis est ut assumptâ simili materiâ semper in eadem forma aptè concludatur." Cf. Fonseca, p. 7; and Regius, pp. 650-651.
14. Regius said, p. 650: "Formalis à Javello definitur, quod sit, in qua oppositum consequentis formaliter & manifestè repugnat antecedente ut nullus ambigere possit . . . materialem consequentiam definit quod sit in qua oppositum consequentis repugnat antecedenti, sed non ita manifestè, quia posset quis opinari, quod oppositum consequentis consistat cum antecedente, licet falso." Cf., C. Javellus, *Compendium Logicae*, Paris (1573), pp. 225-225^{vo}.
15. Javellus, p. 225^{vo}.
16. Kesler, p. 14. ". . . non verò dicitur materialiter & formaliter bonus, propter minus & magis manifestum concludendi rationem."
17. See J. H. Alsted, *Logicae Systema Harmonicum*, Herbonae Nassoviorum (1614), p. 384; Caesarius; Carbo, p. 123; Carvisius, p. 117^{vo}; Gabriel of St. Vincent, p. 35; Oddus, p. 55; R. Sanderson, *Logicae Artis Compendium*, Oxford (1741), p. 128; Santolaria, p. 157; Timplerus, p. 585; F. Toletus, *Introductio in dialecticam Aristotelis*, Rome (1601), pp. 174-175. Seton mentioned the distinction without offering a definition.
18. Carbo, *loc. cit.*; Gabriel of St. Vincent, *loc. cit.* They are following Fonseca here, as was Kesler. See Fonseca, pp. 8-10 and Kesler, pp. 14-16. These elements were quality, quantity, number and supposition of terms, and the copula.
19. For references, see below, note 30.
20. Fonseca, p. 17.
21. Regius, p. 654. Cf. Javellus, pp. 225^{vo}-226. See also [Cologne] *Textus omnium Tractatum Petri hispani . . . juxta processum magistrorum Colonie in bursa Montis regentium*, Cologne (1493), fo. ciiii. Most logicians of the earlier period maintained that an impossible proposition like "Man is an ass" was repugnant to any proposition, including itself.
22. [Cologne], *loc. cit.*
23. Kesler, pp. 18-20; Fonseca, pp. 13-14; Cf., Carbo, p. 124; Gabriel of St. Vincent, pp. 35-36.
24. See J. Eckius, *In Summulas Petri Hispani extemporaria et succincta*, Augustae Vindelicorum (1516), fo. c^{vo}. [Mainz] *Modernorum Summule logicales* (1489?) [no pagination]. J. Trutvetter, *Breviarum dialecticum*, Erphordie (1500) [no pagination].
25. Eckius, fo. ci.
26. Kesler, p. 22. These rules were commonplace among all logicians, whether they discussed the theory of consequence or not.
27. Kesler, p. 24. For comment, see previous note.

28. Kesler, p. 25. For 3.1, *cf.*, Carvisius, p. 118; John of St. Thomas, p. 126; Mercado, p. 67^{VO}; Oddus, p. 57; Santolaria, p. 157; Timplerus, p. 590; Toletus, p. 177. For 3.2, *cf.*, Carvisius, *loc. cit.*; Mercado, *loc. cit.*; Timplerus, *loc. cit.*; Toletus, p. 179.
29. Kesler, pp. 25-26. *Cf.*, Santolaria, p. 157; Timplerus, p. 590; Toletus, p. 177.
30. Kesler, pp. 26-27. For 5.1, *cf.*, Carvisius, p. 118; Mercado, p. 67^{VO}; Oddus, p. 58; Santolaria, p. 157; Toletus, p. 159. For 5.2, *cf.*, John of St. Thomas, p. 26; Mercado, *loc. cit.*
31. Kesler, p. 27. *Cf.* Alsted, p. 385; Timplerus, p. 590.
32. Kesler, p. 28. *Cf.*, Alsted, p. 385; John of St. Thomas, p. 126; Mercado, p. 68; Oddus, p. 58; Santolaria, p. 157; Timplerus, p. 590.
33. Kesler, p. 29. For 8.1, *cf.*, Alsted, p. 385; Carvisius, p. 118; Timplerus, p. 590; Toletus, pp. 179-180. For 8.2, *cf.*, T. Blundeville, *The Art of Logike*, London (1599); Caesarius; Campanella, p. 374; Carvisius, p. 118; John of St. Thomas, p. 126; A. Libavius, *Dialectica Philippo-Ramaea*, Francofurti ad Moenum (1608), p. 546; Melancthon, p. 627; Mercado, p. 68; Oddus, p. 58; Toletus, pp. 179-180.
34. See Fonseca, pp. 15-24. The complete set of Fonseca's rules was also given by Carbo, pp. 124-126; Gabriel of St. Vincent, pp. 36-37; and Regius, pp. 652-656.
35. Kesler, p. 22. *Cf.*, Oddus, p. 58; Carvisius, p. 118.
36. Kesler, p. 31. Fonseca, p. 2, said "Primum est cum qualibet enunciatio ex se ipsa, aut ex sua aequipollente colligitur." *Cf.*, Carbo, p. 122; Gabriel of St. Vincent, p. 35. The other types were (2) where antecedent and consequent have the same words but a different value, as in the inference from universal to particular; (3) where they have the same words but in a different order, as in conversion; and (4) where they have different words, as in the syllogism.
37. Carvisius, p. 119^{VO}. He gives as an example: "Necesse est omnem cygnum esse album, ergo omnis cygnus est albus." *Cf.*, Mercado who said, p. 67: "Et tenet consequentia ab propositione de necessario ad deinesse, tametsi non sit immediate reducibilis."
38. Carvisius, p. 119^{VO}.
39. Carvisius, p. 118. *Cf.*, John of St. Thomas, p. 126; Mercado, p. 67; Oddus, p. 57; Toletus, p. 177.
40. Carvisius, p. 118; *cf.*, Blundeville, p. 138; Caesarius; Campanella, p. 374; Mercado, p. 68; Oddus, p. 58; Santolaria, p. 157.
41. Santolaria, p. 157.
42. Kesler, p. 86. *Cf.*, Campanella, p. 379; Carvisius, p. 120^{VO}; Mercado, p. 69; Regius, p. 658.
43. Kesler, *loc. cit.* *Cf.*, Mercado, *loc. cit.*
44. Kesler, *loc. cit.* *Cf.*, Campanella, p. 380; Mercado, p. 71.
45. Kesler, p. 141.

46. Kesler, p. 142. "Commodè potest copulativus referri ad conditionalem, Nam major copulativi cum majore conditionalis convenit, ut: Non & frustra Christus mortuus est, & fides nostra firma est, convenit cum hac: si frustra Christus mortuus est fides nostra non est firma, & si fides nostra est firma Christus non est frustra mortuus."
47. Fonseca, p. 173, pp. 201-202. See my article, "Petrus Fonseca and Material Implication," *Notre Dame Journal of Formal Logic*, vol. IX (1968), pp. 227-228.
48. Kesler, p. 146. Cf., Campanella, *loc. cit.*; Mercado, *loc. cit.*; Regius, *loc. cit.*
49. Kesler, p. 143. Cf., Campanella, p. 381; Mercado, *loc. cit.*; Regius, *loc. cit.*
50. Kesler, *loc. cit.* Campanella, *loc. cit.*; Mercado, *loc. cit.*; Regius, pp. 658-659.
51. Mercado, p. 69. Campanella, p. 380.
52. See my article, "Propositional Logic in the sixteenth and early seventeenth centuries," *Notre Dame Journal of Formal Logic*, vol. IX (1968), pp. 179-192.

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