

STRICT AND MATERIAL IMPLICATION IN THE EARLY SIXTEENTH CENTURY

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One of the favorite games played by historians of logic is that of searching their sources for signs of the Lewis-Langford distinction between strict and material implication. There are three ways of going about this, but the first two are often reminiscent of the conjurer searching for his rabbit, and only the third has real merit, for it alone involves the study of what was said about the conditional as such. I shall look at each way in turn, in relation to writers of the early sixteenth century.

The first way, which I have at times pursued myself, involves spotting the equivalence.¹ If one discovers that an author admits the inference of ' $\neg P \vee Q$ ' from 'if P then Q ', then one has only to point to his acceptance of the rule ' $P \vee Q, \neg P$, therefore Q ' and to saddle him with both ' $P \equiv \neg\neg P$ ' and the principle of conditionalization in order to claim that he was implicitly aware of the equivalence ' $(P \supset Q) \equiv (\neg P \vee Q)$ ' and hence, of material implication. There are three drawbacks to this procedure. In the first place, if an acute logician like Caubraith or Enzinas, who both admitted the inference in question, was implicitly aware of material implication, why did the awareness never become explicit in this context? In the second place, had they become explicitly aware of the possibility of such an interpretation of the conditional they could well have rejected it. In the third place, those who discussed the matter made it quite clear that the disjunction derived from a conditional had to be a necessary one. All true conditionals are necessary, and no contingent proposition can be implied by a necessary proposition.²

Whether one could go the other way and derive a conditional from a disjunction was discussed in detail by Robert Caubraith.³ He said that from a non-necessary disjunction like "Either Socrates does not run or Plato disputes" one could not derive the conditional "If Socrates runs, then Plato disputes", but that from "Either Socrates runs or Socrates does not run" one could derive the conditional "If Socrates does not run, then Socrates does not run." He also said that "Either Socrates does not run or an animal does" materially implies "If Socrates runs, then an animal runs",

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but the force of 'materially implies' here is that the inference in question is valid on semantical rather than syntactical grounds. Both the disjunction and the conditional are necessarily true, but the necessity rests on the meaning of the terms involved rather than on the syntactical or formal properties of the propositions themselves.

The second way is to take what authors said about consequences which were valid *ut nunc* and to interpret this as being akin to a recognition of material implication. The chief snag here is that the searcher may be forced to blur the distinction between a true conditional and a valid inference, and although early logicians were often imprecise in their language, this distinction was obviously clear in their minds. Usually those who discuss consequences valid *ut nunc* either have something quite different from material implication in mind, or they are being so imprecise about the nature of inference that it hardly seems worthwhile to credit them with the discovery of the Lewis-Langford distinction. One doesn't stumble across a distinction as one might a hitherto unknown animal. In some cases, the authors obviously intend there to be a connection of terms, and Hundt said explicitly that this connection is contingent in an *ut nunc* consequence, but necessary in a simply valid consequence.⁴ Eckius gave as an example of an *ut nunc* consequence "Every man is a thinker, therefore you are", but said that it is not really valid unless the premiss "You are a man" is added.⁵ Niphus said that an *ut nunc* consequence like "Socrates runs, therefore something white runs" becomes valid *per se* with the addition of the true premiss "Socrates is white."⁶ In neither of these cases is there evidence that the relation between premisses and conclusion was seen as being merely truth-functional. Pardo explicitly rejected "John sits, therefore William runs", saying that this was not a kind of consequence used by logicians;⁷ and Almain said that "Socrates runs, therefore Plato runs" was not valid *ut nunc*, even if both were running.⁸ On the other hand, John of Glogavia accepted "A man is sitting, therefore a stick is in the corner" as valid *ut nunc*, provided that things were as described; and John Major accepted "John is a priest, therefore John is an ass," which presumably had a false antecedent.⁹ However, in the absence of any explanation of how these examples could count as inferences, and in the absence of any attempt to relate them to conditional propositions, I have no grounds for claiming that the authors were displaying any unusual insight here.

The third way, and the only way which I find satisfactory, is to read what the authors of the period had to say about the conditional proposition itself. Most of them stick to the standard view that the truth-conditions for a conditional are the same as the conditions for the validity of an inference, and for the purpose of this paper I will assume that this is equivalent to an acceptance of strict implication.¹⁰ However, they were not unaware that 'si' could have various uses. It might have illative force, but it might also be used in questions, vows, promises, or statements about the conditions necessary for performing some activity as in "If I had books, I would study."¹¹ The promissory use was that which was singled out for special

attention, the most common example being "If you come to me, I will give you a horse." Not all the discussion was relevant to logic, some later authors insisting that the truth of a promissory conditional depended on the presence of an intention and that even if I gave you a horse, the above conditional would be false if I had not so intended at the time of uttering the sentence.¹² Of the authors whose discussion is related to our purposes, I will examine Caubraith, since his account is the fullest and clearest, but it must be recognized that the substance of his argument was by no means peculiar to him.¹³

Caubraith begins by stating that for the truth of "If Socrates comes to me, I will give him a horse" it is not required either that the antecedent be true or that the consequent be true; nor is it required that it be impossible for the antecedent to be true without the consequent also being true. The necessary and sufficient condition for truth is simply that if the antecedent is true, then the consequent is also true. For the truth of a negated promissory conditional, the falsity of the corresponding affirmative is necessary and sufficient; that is, the antecedent should be true and the consequent false. Promissory conditionals can be either impossible, necessary, or contingent. If the antecedent is contingent and the consequent impossible, the conditional is impossible; but it must be noted that here Caubraith is for once in error. Enzinas pointed out that "If you come to me, I will turn you into an ass" is not impossible but true, provided that you never come to me.¹⁴ Celaya's example "If God exists, then I will give you a horse and I will not give you a horse" is correct, given that the antecedent was regarded as necessary.¹⁵ Caubraith went on to say that if the antecedent is impossible or the consequent necessary, then the conditional is necessary. Otherwise, it is contingent; and because contingency is admitted we know that a true promissory conditional is not equivalent to a valid inference, as is the true illative conditional. When a promissory conditional turns out to be equivalent to a valid inference given that the 'si' is taken illatively, it is necessarily true; but not all those which are equivalent to an invalid inference can be labelled as false or impossible. All illative conditionals imply a promissive conditional with the same terms, but the reverse does not hold. No rules were given.¹⁶

I think it is fair to conclude by saying that some early sixteenth century logicians were beyond doubt aware of the distinction between strict and material implication; and that no special pleading is necessary to establish this.

NOTES

1. See p. 183 of my paper "Propositional Logic in the Sixteenth and Early Seventeenth Centuries," *Notre Dame Journal of Formal Logic*, vol. IX (1968), pp. 179-192; and my paper "Petrus Fonseca and Material Implication," *Ibid.*, vol. IX (1968), pp. 227-228.

2. See R. Caubraith, *Quadrupertim in Oppositiones, Conversiones, Hypotheticas et Modales*, Paris (1510), fo. lxx. See also F. Enzinas, *Oppositionum Liber*, Paris (1528), fo. xx^{vo}. Enzinas says that the disjunction must be necessary "quia quum conditionalis illa necessaria fit per regulam dictam non potest esse talis disiunctiva contingens. quia ex necessario non sequitur contingens." References to the rule permitting the inference of a disjunction from a conditional are also to be found in J. Dolz, *Disceptationes super primum tractatum summularum*, Paris (1512) [no pagination]; Dominicus de Soto, *Introductiones Dialectice*, Burgos (1529), fo. lxxiii; Johannes de Celaya, *Expositio in primum tractatum summularum*, Paris (1515?) [no pagination]; and J. Major, *Consequentie inchoate perfecte ab Anthonio Coronel*, Paris (c. 1503) [no pagination].
3. *Loc. cit.*
4. M. Hundt, *Compendium totius logices*, Leipzig (1507) [no pagination]. Cf., [Cologne] *Textus omnium tractatum Petri Hispani . . . juxta processum magistrorum Colonie . . .*, Cologne (1493), fo. c^{vo}.
5. J. Eckius, *In summulas Petri Hispani Extemporia et succincta*, Augustae Vindelicorum (1516), fo. c^{vo}.
6. A. Niphus, *Super libros priorum Aristotelis*, Venetiis (1554), p. 11^{vo}.
7. H. Pardo, *Medulla dialectices*, Paris (1505), fo. x.
8. J. Almain, *Consequentiae*, Paris (1508) [no pagination].
9. Johannes de Glogavia, *Exercitium super omnes tractatus parvorum logicalium Petri Hispani*, Argentine (1517), fo. lxxix^{vo}; Major, *op. cit.*
10. For a fuller discussion, see my paper "The Theory of Consequence in the late Fifteenth and Early Sixteenth Centuries," *Notre Dame Journal of Formal Logic*, forthcoming.
11. See Caubraith, *op. cit.*, fo. lxxviii; Celaya, *op. cit.*; Dolz, *op. cit.*; Enzinas, *op. cit.*, fo. xx; Eckius, *op. cit.*, fo. xvii; Soto, *op. cit.*, fo. lxxvi; Major, *op. cit.*; Pardo, *op. cit.*, fo. xl^{vo}; Hieronymus of St. Mark, *Compendium preclarum quod parva logica seu summe dicitur*, Coloniae (1507) [no pagination]. For an earlier source, see J. Dorp's commentary in J. Buridan, *Summula de dialectica*, Paris (1487) [no pagination].
12. See [Alcalá] *Collegii Complutenses Sancti Cyrilli . . . Disputationes in Aristotelis Dialecticam et Philosophiam naturalem*, Lugduni (1668), p. 17; and Oddus Illuminatus, *Logica Peripatetica*, Panormi (1664), p. 52. Other authors outside my period of interest who mention promissory conditionals are Thomas de Mercado, *Commentarii Lucidissimi in textum Petri Hispani*, Hispali (1571), p. 64^{vo}; and Petrus de Oña, *Dialecticae Introductio*, Panormi (1621), p. 111.
13. Cf., Enzinas, *op. cit.*, fo. xxi; Celaya, *op. cit.*; Soto, *loc. cit.*; and Major, *op. cit.*
14. Enzinas, *loc. cit.*
15. Celaya, *loc. cit.*
16. Caubraith, *op. cit.*, fo. lxxi^{vo}-fo. lxxii. Part of the text is as follows: "ad veritatem huius si Sortes veniet ad me dabo tibi equum non requiritur quod Sortes veniat ad me nec requiritur quod dem illi equum: etiam non requiritur quod non sit possibile quod Sortes veniat ad me quin ego dem illi equum: sed sufficit et requiritur quod si ponatur antecedens ponatur consequens: hoc est:

quod si hec propositio est vera. Sortes venit ad me. hec est vera dabo illi equum.

1. ad veritatem conditionalis promissive non requiritur quod ponatur antecedens nec quod ponatur consequens: et in hoc imitatur naturam conditionalis illative.

2. secundo infertur quod ad veritatem conditionalis promissive non requiritur quod non sit possibile dare ita esse sicut significatur per antecedens: quin ita sit sicut significatur per consequens: et in hoc promissiva vera differt a conditionalis illativa vera. Et ex illis duobus correlariis satis claret quod conditionalis promissiva et conditionalis illativa in aliquo conveniunt et in aliquo differunt.

3. . . . omnis promissiva que esset bona consequentia si ly si caperetur illative est vera et necessaria

6. sexto infertur a conditionalis illativa ad conditionalem promissivam ex eisdem terminis compositam valet argumentum et non e contra: quia in tali modo arguendi a propositione habente pauciores causas veritatis ad propositionem habentem plures."

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