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SOMMERS ON EMPTY DOMAINS AND EXISTENCE

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Many philosophers have held the question of what there is to be a logical question. They have argued that a logical examination of our language will reveal our existential commitment. A source of difficulty for such a view has been the empty domain.

1. Can there be an empty domain of discourse? Quine's well known answer was that such a domain, though logically possible, is best ignored.[1] Quine wanted to retain all normal quantified theorems, e.g.

(1) $(\exists x)\ (Fx \vee -Fx)$
(2) $(x)\ (Fx) \supset (\exists x)\ (Fx)$

as true in any domain. Thus he was forced to assume (so he thought) that there is always at least one object in any domain which could be the value of these bound variables; otherwise such theorems would be false. Quine simply would not allow an empty domain of discourse.

Karl Potter has charged that this position into which Quine has forced himself is the result of his failure to recognize the denial-negation distinction.[2] Negation (-) may be construed as a unary propositional connective which has the effect of reversing the truth value of any proposition upon which it operates. Denial (′ or *non*) is an operation upon a predicate alone. It will be seen that denial has the effect of reversing the ontic commitment made by the proposition in which the denied predicate occurs. Keeping this distinction in mind Potter gives a restricted version of Quine's criterion for ontic commitment:

OC: Given an asserted formula S, to be is to be a value of a bound variable in S if either (1) S is categorical and tilde-free, or (2) S logically implies a formula which is categorical and tilde-free.

1. "Meaning and Existential Inference," *From a Logical Point of View*, New York (1953), p. 160.

2. "Negation, Names, and Nothing," *Philosophical Studies*, vol. 15 (1954).

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A formula is categorical just in case its content is an atomic formula or the conjunction of atomic formulae.

Look now at (1) above. (1) is equivalent to

(1.1) $(\exists x)\ (Fx \supset Fx)$

but neither formula is categorical and tilde-free. Thus, by OC, (1) makes no ontic commitment. However, Quine has taken (1) to be the same as

(1.2) $(\exists x)\ (Fx \vee F'x)$.

According to Potter, (1.2) is false in the empty domain. Since Quine thinks (1.2) equals (1), which is a theorem, he eliminates empty domains in order to preserve (1). But (1) does not equal (1.2). (1) says that for some x, Fx is either true or false, while (1.2) says that some x is either F or non$F$. (1) is clearly true even in any empty domain. This has best been shown by Bas C. van Fraassen in "Singular Terms, Truth-value Gaps, and Free Logic."[3] Briefly, van Fraassen's account would allow that when the variable in a formula is not assigned any member of the domain (this would be the case for any variable in the empty domain) two value assignments (classical valuations) are possible for each atomic formula: one T the other F. A formula is a theorem just in case its supervaluation is T (a supervaluation assigns T to any formulae which gets a T by every classical valuation and an F to any formula which gets F by every classical valuation, otherwise it leaves a truth-value gap). Thus in the empty domain one classical valuation assigns T to Fx and F to $-Fx$ and the other classical valuation assigns F to Fx and T to $-Fx$. However, by definition of "$\vee$" both classical valuations assign T to "$(Fx \vee -Fx)$". Since the existential quantifier is interpreted as a disjunctive assignment of each member of the domain, and the empty domain has no members, both classical valuations will assign T to "$(\exists x)\ (Fx \vee -Fx)$." The supervaluation will therefore assign T to (1) and thus (1) is valid even in the empty domain.

Now look at (2) above. (2) is not categorical and tilde-free, and thus makes no ontic commitment. Yet (2) still remains problematic for Quine. For him, universally quantified formulae are true in the empty domain while existentially quantified formulae are false in the empty domain. Thus (2) becomes false just in the empty domain. Since Quine wants (2) as a theorem, he rejects empty domains. Potter's way out is to make tilde-containing formulae true and tilde-free formulae false in the empty domain.[4] Thus while Quine would make A and E statements true and I and O statements false, Potter makes A and I statements false and E and O

3. *Journal of Philosophy*, vol. 63 (1966).

4. Karl Lambert uses a similar gambit in "Singular Terms and Truth," *Philosophical Studies*, vol. 10 (1959).

statements true. (2), then, becomes true even in the empty domain, where $(x)Fx$ would be false.

2. We have seen that the main problem with the empty domain is the preservation of truth for certain theorems. Potter's solution was simply to assign truth values in the empty domain in such a way that truth would be retained for such theorems. The main reason that Quine assigned the truth values he did in the empty domain was his reading of the existential quantifier, $(\exists x)$. Quine read it as "something x exists which is such that . . ." Clearly on such a reading *I* and *O* statements would be false in the empty domain. If the problems inherent in the empty domain are to be avoided without simply ignoring the empty domain as Quine did, one of the following ways must be taken: (a) Arbitrarily assign truth values in the empty domain in such a way that any required truth is preserved. This was Potter's method. (b) Read the existential quantifier in such a way that existential commitment is not necessarily made by every existential statement. (c) Reject empty domains on the grounds that they are logically impossible. (d) Reject quantification altogether and construe domains of discourse in some way other than sets of variable values.

Suggestion (b) has been made explicitly or implicitly by several philosophers including H. S. Leonard,[5] N. Rescher,[6] and R. M. Martin.[7] But by far the best presentation of (b) has been made by C. Lejewski in "Logic and Existence."[8] Briefly, what Lejewski proposes is a new, "unrestricted," reading of existential quantifiers to replace the traditional Quinian, "restricted," interpretation. Under the restricted interpretation, for any model, quantifiers quantify only members of the domain. The universal quantifier is interpreted as a conjunctive assignment of each member of the domain while the existential quantifier is interpreted as a disjunctive assignment of each member of the domain. Under the unrestricted interpretation these assignments need not be of a single member of the domain; they may be to none, one, or many domain members. Since the restricted interpretation assumes that variable values must exist (i.e. that variables must be assigned members of the domain) formulae such as (1) and (2) must be false when the domain is empty. On the other hand, under the unrestricted interpretation, variables need not be assigned members of the domain. So even when the domain is empty (1) and (2) will be valid.

5. "The Logic of Existence," *Philosophical Studies*, Vol. 7 (1956).

6. "On the Logic of Existence and Denotation," *Philosophical Review*, vol. 68 (1959).

7. See "Of Time and the Null Individual," *Journal of Philosophy*, vol. 62 (1965) and the reply by J. W. Swanson, "The Singular Case of the Null Individual in the Empty Domain," *Journal of Philosophy*, vol. 63 (1966).

8. In *Logic and Philosophy*, edited by G. Iseminger, New York (1968).

3. In effect Lejewski's solution to the empty domain problem is to eliminate the existential import carried by quantifiers under the traditional restricted interpretation. According to Lejewski, "$(Fy \vee -Fy)$" is true for any interpretation in any domain. Such a formula says of Fy and $-Fy$ that one is true and the other false. But according to L. J. Cohen this is an unwarranted assumption.[9] Cohen argues that when the domain of discourse is empty there is no way available for assigning truth values to formulae. His answer is to eliminate empty domains. But, unlike Quine, who accepted the logical possibility of empty domains while ignoring them, Cohen rejects empty domains on the grounds that they are logically impossible. Briefly, Cohen argues that given any domain D, to say that k-things do not exist is to say that the class k does not intersect with the class D. Since D intersects with D, D cannot possibly be empty. Of course the argument will not work. The premise that D intersects with D holds only on the assumption that D is not empty, which is the conclusion.

We will see that what is most interesting in Cohen's discussion is not his negative thesis that the empty domain is logically impossible but the positive thesis concerning existential statements. According to this thesis: to say that k-things exist is to say that the class k intersects with some domain of discourse. Obviously there are as many senses of "exist" as there are domains of discourse. In fact, if any class can be a domain of discourse, then anything can be said to exist (in some sense). What Cohen's theory requires is a restriction on the kinds of classes that can be domains of discourse. We will see later that Sommers has supplied that restriction in his theory.

4. One obvious difficulty in Cohen's paper is his failure to make clear the nature of a domain of discourse. Intuitively a domain of discourse seems to be something like "what-is-being-talked-about." But for any assertion, what is being talked about? I tell you that Simon Legree was a Northerner. Am I talking about Simon Legree, or Northerners, or perhaps Southerners, or slave foremen, or men, or fictitious men, or . . . what? Let us compare three views. Nelson Goodman has argued that in one sense of "about" what a statement is about is the union of the extensions of all of its terms.[10] "Crows are black" is about the two classes of crows and black things. It says that the class of crows is included in the class of black things. One danger, at least, with this view is the problem involved when the class mentioned by the subject term is null. Thus, "Carnivorous cows ϕ" is true for any ϕ since the null set is included in any class.

Cohen's notion of what a statement can be about is not as clear as Goodman's. According to Cohen, "Cows moo" is in some sense about terrestrial animals; "Winged horses fly" is about celestial animals. It

9. "Logic and the Empty Universe," Iseminger, *op. cit.*

10. "About," *Mind*, vol. 70 (1961).

seems as if what a statement is about is the domain of discourse with reference to which the statement is made, and the domain of discourse is simply any class sufficiently large enough to include the individual or class mentioned by the subject term. On one interpretation of Goodman's view, statements with empty subject terms are always true. For Cohen, on the other hand, there can be no such thing as an empty subject term. "Winged horses," for example, could not be empty since the obvious domain of discourse in such a case would be celestial animals. A class is only empty if it is not included in the domain of discourse.

Against both of these views Sommers has proposed that a domain of discourse for a given statement is actually the intersection of all the categories mentioned in the statement, while what the statement is about is the intersection of those categories.[11] A category is a set of things all of which are spanned by a given term. A term spans a thing if it is truly affirmed or denied of that thing. Thus "P" spans g just in case either g is P or g isn't P (written "$Pg \vee P'g$"). A category is always a category with respect to a given term. The category with respect to "red" consists of all those things that are spanned by "red," i.e. are either red or somehow fail to be red; this would include fire engines, applies, lemons, and the moon, but not numbers, thoughts, or the equator.

Recall that for Goodman "Crows are black," being about those two classes, must predicate "black" of each crow individually or it must predicate "included in the class of black things" of the individual class of crows. Under the first interpretation "Crows are black" says of each thing which is a crow that it is black (i.e. $(x)\ (\text{Crow}_x \supset \text{Black}_x)$). Under the second interpretation "Crows are black" says that the class of crows is included in the class of black things (i.e. $(\text{Crow} \subset \text{Black})$). According to Sommers, this is the result of what he has referred to as "the Fregean dogma," the belief that any predication must be the predicating of a term to an individual. Once this dogma has been abandoned we can see that "Crows are black" simply predicates "black" of crows (not each individual crow, nor the class of crows). "Crows are black" says of crows that they are black (i.e. $(\text{Black}_{\text{crow}})$).

Some additional notation might be helpful before continuing to show how Sommers would treat the empty domain. The logical contrary of P (written "$\overline{P}$" or "non$P$") will be the disjunction of all the contraries of P. Thus "$\overline{\text{red}}$" equals "green or blue or white or pink or . . ." To deny P of an *individual* is to affirm $\overline{P}$ of it. So: $P'g \equiv \overline{Pg}$ (i.e. "g isn't P if and only if g is non$P$"). To deny P of a *plural* subject (e.g. k-things) is to negate the affirmation of P to it. Thus: $P'k \equiv -Pk$ (i.e. "k-things aren't P if and only if it is not the case that k-things are P"). The expression "$/P/$" (read

11. See especially "Types and Ontology," *Philosophical Review*, vol. 72 (1963), and "On a Fregean Dogma," *Problems in the Philosophy of Mathematics*, Amsterdam (1967).

"absolute P") can be interpreted either as *the category with respect to* P *or the term* "P or non P," since whenever P is truly affirmed or denied of an individual "P or non$P$" can be affirmed to it. Two terms are said to be "N-related" just in case the categories with respect to those terms do not intersect with one another.

As we have seen, a domain of discourse for Sommers is the intersection of all the categories with respect to each term in the statement. Given "Crows are black" the domain is /crow/$\cap$/black/ (i.e. the intersection of the class of things that are crows or noncrows and the class of things that are black or nonblack). If the category with respect to "crow" is the class of all animals and the category with respect to "black" is all coloured things, then "Crows are black" is in one sense about coloured animals.

For Cohen the discerning of a domain of discourse was more or less independent of any given statement except that the domain should include the individual or class mentioned by the subject of the statement. Thus, there might be several domains with respect to which a given statement was made. We saw that Cohen was unable to disallow any class from serving as a domain of discourse so that existence statements became ridiculously ambiguous. For Sommers, on the other hand, the domain of discourse is not arbitrary but determined uniquely by the statement.

Statements whose grammatical subjects are *something* or *everything* say in effect that all or some of the members of the domain are such-and-such. Thus, "$(x)Fx$" says that all the members of the domain are F; in other words, whatever is /F/ is F (i.e. $F_{/F/}$). For plural subject k, "No k-things are F" says "All k-things are non$F$" (i.e. $\overline{F}k$). So, "Nothing is F" says "Everything is non$F$" (i.e. $\overline{F}_{/F/}$). "Something is F" says "Not everything is non$F$" (i.e. $-\overline{F}_{/F/}$ or $\overline{F}'_{/F/}$). Finally, "Something is not F" says "Not everything is F" (i.e. $-F_{/F/}$ or $F'_{/F/}$). "Everything is ϕ" then is not about everything that exists, but merely about all things which are either ϕ or nonϕ.

What about the empty domain? According to Sommers, a domain of discourse is empty if any two terms in the statement are N-related. Recall that if two terms are N-related then their categories do not intersect. So, if the domain is just the intersection of all the appropriate categories, when two terms are N-related there will be no such intersection; the domain will be empty. "Prime numbers are red": here the domain is the intersection of /prime/ and /red/, which is empty since "prime" and "red" are N-related.

As I pointed out earlier, the crux of the problem about the empty domain can be seen by viewing formulae (1) and (2). For the "quantificationalists," if an empty domain of discourse is allowed, both of these become false. Let us look at how Sommers could handle (1) and (2). First of all, notice that when a term spans everything in a domain, the distinction between affirming the logical contrary of that term and negating the affirmation of that term is lost. If we are talking only of things in the category /F/, then whatever we refer to is either F or non$F$. So if it is not the case that g is F then g is non$F$. In other words, when the domain of discourse is $/F/$, $-Fg \equiv \overline{F}g$.

Formula (1) says that something is such that it is F or it is not the case that it is F. According to Sommers, the domain must be $/F/$. To say of something that it is not the case that it is F is to say that it is, in the domain of $/F/$, non F. So what (1) says is that something is F or $\overline{F}$, which amounts to "Something is $/F/$." But now, using Sommers' way of eliminating quantification (where $(\exists x)Px = \overline{P'}_{/P/}$), this can be translated as $/F/'_{/F/}$ ((all) the members of the domain aren't non/F/). Since $/F/$ is plural, we have:

(1s) $-\overline{/F/}_{/F/}$

A simple proof for (1s) is in Appendix I.

Let us turn to (2). What (2) says is that if everything in the domain is F then something in the domain is F (i.e. $F_{/F/} \supset \overline{F'}_{/F/}$). In other words, since $/F/$ is a plural subject:

(2s) $F_{/F/} \supset -\overline{F}_{/F/}$

This formula is a theorem by virtue of the "law of incompatibility": for any subject s and any predicate P, $-(Ps \cdot \overline{P}s)$.

Now, what about (1s) and (2s) when the domain of discourse is empty? Again, given any statement, to say that the domain of discourse with respect to that statement is empty is to say that the statement contains an N-related pair of terms. Therefore (1s) and (2s) will have as their domains the null set whenever F is equivalent to an N-related pair of terms. Let "F" equal "is a red number," then, since $\overline{/F/}g \equiv Fg \cdot \overline{F}g$, (1s) says that it is not the case that whatever is or fails to be a red number is a red number and fails to be a red number. This is clearly the case since *no* term spans whatever is or fails to be a red number. So when the domain is empty (i.e. when F equals an N-related pair of terms) (1s) is valid. (For a proof see Appendix II.) Likewise, (2s) is valid when the domain is empty since by the same reasoning as above its antecedent is false and its consequence is true. (In fact, for the empty domain, A and E statements will be false, and I and O statements will be true.)

5. It might help to understand Sommers' nonquantificationalism more clearly if we looked at how he could handle two very important rules for quantification: universal instantiation and existential generalization. The first may be construed as:

(UI) $(x)Fx \supset Fg$

the second as:

(EG) $Fg \supset (\exists x)Fx$

Quantificationalists have been greatly exercised over these rules since they seem to work only upon the assumption that g exists. But they have been relatively unsuccessful in explicating the notion of existence. I suggest that we understand by "g exists" (in the sense required by the quantificationalists) that g is a member of the domain of discourse. This was Cohen's notion. Coupling this suggestion with Sommers' formulations of the rules we would get:

(UIs) $(g \epsilon /F/ \cdot F_{/F/}) \supset Fg$
(EGs) $(g \epsilon /F/ \cdot Fg) \supset -F_{/F/}$

Both of these are quite obviously valid.

The quantificationalists' problem was one of existence. They had to assume that their domains were not empty and that every individual existed. As we have seen, Sommers, in rejecting quantificationalism, has eliminated both problems. Empty domains are possible and have no effect upon the validity of theorems. One need not assume that an individual exists, he needs only to assume that it belongs to the domain of discourse with respect to which statements about that individual are made. Existence for Sommers is a notion independent of symbolization. The quantificationalist is in the position of having to say that everything exists. Sommers is not. Indeed, to say that everything exists is to say that what /exists/ exists (letting E mean "exists", we have $E_{/E/}$, which entails $E_{\bar{E}}$).

Not long ago Sommers attempted to give a proof that necessarily something exists.[12] Later his proof was shown to be wrong in principle.[13] I think that we are now ready to do what he could not do and more. I shall conclude this paper by offering the following two proofs; the first to the effect that necesssrily something does not exist, the second to the effect that necessarily something does exist. (Again let E be "exists"):

I	1. $/E/_{/E/}$	Theorem
	2. $/E/_{E} \cdot /E/_{\bar{E}}$	1.
	3. $/E/_{\bar{E}}$	2.
	4. $-(/E/_{\bar{E}} \cdot \overline{\underline{/E/_{\bar{E}}}})$	Incompatibility
	5. $-\underline{/E/}_{\bar{E}} \vee -\overline{/E/}_{\bar{E}}$	4.
	6. $-\overline{/E/}_{\bar{E}}$	3, 5.
	7. $-E_{/E/}$	6. Inversion (Sommers' form of contraposition)

QED

In other words, something does not exist.

II	1. $/E/_{/E/}$	Theorem
	2. $/E/_{E} \cdot /E/_{\bar{E}}$	1.
	3. $/E/_{E}$	2.
	4. $-(E/_{E} \cdot \overline{/E/_{E}})$	Incompatibility

12. "Why is There Something and not Nothing?" *Analysis*, vol. 26 (1966).

13. H. Guerry, "Sommers' Ontological Proof," *Analysis*, vol. 27 (1967).

5. $-/E/_E \vee -\overline{/E/}_E$	4.
6. $-\overline{/E/}_E$	3, 5.
7. $-\overline{E}_{/E/}$	6. Inversion

QED

In other words, something does exist.

Appendix I. Proof of (1s) $-\overline{/F/}_{/F/}$

1. $-(/F/_{/F/} \cdot \overline{/F/}_{/F/})$	Incompatibility
2. $-/F/_{/F/} \vee -\overline{/F/}_{/F/}$	1.
3. $/F/_{/F/}$	Theorem
4. $-\overline{/F/}_{/F/}$	2, 3.

QED

Appendix II. Proof that (1s) is valid in the empty domain:
The domain with respect to (1s) will be empty whenever F is equivalent to a pair of N-related terms. Let F equal RN, where R and N are N-related (e.g. "red" and "number"). (1s) will be valid in the empty domain if we can show that $(-\overline{/RN})_{/RN/})$ is valid.

1. $-(R_{/RN/} \cdot \overline{R}_{/RN/})$	Incompatibility
2. $-R\overline{R}_{/RN/}$	1.
3. $-RN\overline{R}_{/RN/}$	2.
4. $-(N_{/RN/} \cdot \overline{N}_{/RN/}$	Incompatibility
5. $-N\overline{N}_{/RN/}$	4.
6. $-RN\overline{N}_{/RN/}$	5.
7. $-RN\overline{R}_{/RN/} \cdot -RN\overline{N}_{/RN/}$	3, 6.
8. $-(RN\overline{R} \text{ or } RN\overline{N})_{/RN/}$	7.
9. $-(RN(\overline{R} \text{ or } \overline{N}))_{/RN/}$	8.
10. $-(RN(\overline{RN}))_{/RN/}$	9.
11. $-(\overline{\overline{RN} \text{ or } RN})_{/RN/}$	10.
12. $-\overline{/RN/}_{/RN/}$	11.

QED

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