

UNCERTAINTIES OVER DISTRIBUTION DISPELLED

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The classical doctrine of distribution has received a number of crushing blows administered by Peter Geach but has not been discarded as an inference device in modern texts on syllogistic logic. Geach has attacked not only the utility of the concept of distribution (for he has substituted a variant interpretation ([3], pp. 61-64) of categorical sentences which makes the doctrine dispensable), but he has impugned the doctrine's intelligibility as well. And no wonder, for even the most sophisticated and otherwise excellent texts leave much to be desired in explicating the doctrine. For instance, Michalos explains that "a term is distributed in a categorical sentence if and only if the sentence makes an assertion about every object denoted by the term" ([6], p. 84). He then considers a sample 0-proposition, "Some roses are not flowers." In order to apply his definition to it, he paraphrases the proposition as "Given any flower at all, only some are roses [sic]." Now it is notoriously difficult to show that the predicate ('flowers' in this case) is distributed in an 0-proposition, thus text authors will often be found to forsake or forget their own sound principles just to establish distribution in this sort of proposition. Note that the original proposition is negative, whereas the paraphrase is thoroughly and unabashedly affirmative. Michalos' revised version also clearly depends on a reading of 'some' as 'some are and some are not,' yet this flies in the face of his earlier point, ([6], p. 54) that 'some' means 'at least one.' Note too that the paraphrase is ungrammatical in a way that affects the thought—can it be that given any flower, only some [of *them*?] are roses?

Perhaps we can preserve the intent of Michalos to provide a general pattern of paraphrase for 0-propositions as (1) 'In the entire *class* of flowers, some of its elements are not roses,' or (2) 'Given any flower, *it* may not be a rose.' Let us ignore the fact that these, like Michalos' own paraphrase appear to translate 'some flowers are not roses.'

(1), however, makes an assertion about the class of flowers and, therefore, not about "every object denoted by the term" 'flower'—a violation of the definition. In any event, to resort to classes at this elementary level may needlessly incur nominalistic objections, and, further, the paraphrase (1) is

a mixed bag. It is an assertion about a class and about its elements, though (contrary to the existential force of 'some') it may have no elements. When we construct an unmixed version, (1a) 'The class of flowers has a subclass devoid of roses,' nothing much is gained; for any class has such a subclass, namely, the empty class. If we try again as in (1b) 'The class of flowers has a non-empty subclass devoid of roses,' we are thereby also bound to maintain that the superclass corresponding to the predicate be non-empty and, hence, bar ourselves from saying things like, 'some animals are not unicorns.' That may or may not be too high a price to pay for a sound doctrine of distribution, unfortunately it is of no avail since other parts of the doctrine get lost in the shuffle. For in paraphrases like (1b) it is by no means clear that the subject, 'roses,' retains its initial status of being undistributed. By logical parity with the usual sort of "proof" of the distribution of the terms in an E-proposition, one can claim that (1b) is an assertion about every member of the class of roses. Since the non-empty subclass excludes *all* roses, the subject term, 'roses,' must be conceded as distributed not only in the (supposedly) equivalent paraphrase but in the original as well. In order to safeguard the distribution of the predicate of an O-proposition, we end up compelled to accept the distribution of its subject.

What about (2)? Though it avoids those troubles afflicting talk of classes, a form like (2) is inadequate for an equally serious reason: failure to capture the meaning of an 'S o P' sentence. This type of modal paraphrase, unlike its prototype categorical, will be false only when 'PaS' is necessary. *In re* Michalos' example, on the one hand 'Some roses are not flowers' is false; on the other hand 'Given any flower at all, it may not be a rose' ought reasonably to be considered true. As if this were not sufficient, the quite parallel modal paraphrase of 'S i P' as 'Given any P at all, it may be an S' has the untoward effect of establishing the distribution of 'P' in I-propositions.

Rescher ([7], pp. 116-117) explains that a term is distributed, when a categorical proposition containing it, discourses about its entire extension rather than only some part of it. The true test of this criterion is once again O-propositions. Rescher remarks on his example, 'Some motor vehicles are not taxis' that

the predicate term actually is distributed because the subject term is excluded altogether from its extension, that is, the X's that are at issue in the statement 'Some X's are not Y's' are put wholly and entirely outside the Y's.

Now the fact that the denotata of the subject are put "wholly and entirely" outside the denotation class of the predicate would seem to be a reason for counting the *subject* as distributed—and, of course, it's not—rather than a reason for regarding the predicate as distributed. Furthermore, Rescher seems to be operating on the notion that the subject term is 'some motor vehicles' rather than simply 'motor vehicles.' This is not a negligible slip, for patently the extension of 'motor vehicles' is not "altogether excluded"

from the extension of 'taxis.' Then, too, what the extension of 'some motor vehicles' would be, is something of a mystery. It certainly could not be constant from one proposition to another.

By the definite referring phrase, "the *X*'s that are at issue," Rescher is committed to the existence of such *X*'s when in fact there may be none—as in a false 0-proposition like 'Some frogs are not animals.' One could contend here that "some *X*'s" were at least intended to be at issue, and that is what counts. Once again, however, in rescuing the explanation for the 0-proposition, trouble for the general doctrine pops up elsewhere. One could then argue that both terms in an I-proposition, like 'Some *X*'s are *Y*'s,' are distributed, because *all* the *X*'s (intended to be) at issue are identical with *all* the *Y*'s (intended to be) at issue.

Copi ([2], p. 153) has yet another explanation: a proposition distributes a term if it *refers* to all members of the class designated by the term. Barker ([1], p. 42) rightly rejects explanation in terms of referring as too vague, and, in addition provides a counter-example, which would jolt any such effort (even if the mode of reference in a categorical were precisely stated): 'All equilateral triangles are equiangular triangles.' Certainly if this proposition distributes 'equilateral triangles' because it refers to all members of the class designated by that term, it ought equally to distribute the predicate, 'equiangular triangles' for the same reason. If one objected that the speaker of the proposition (under examination) may not know that the two terms are coextensive and, hence, need not mentally refer to all the equiangular triangles, then one has abandoned a criterion in *logic*, for the dubious safety of a psychological criterion. Even with the psychological criterion, would someone who knew of the coextensiveness then be using an A-proposition that distributed its predicate? Suppose the counterexample were 'All triangles are triangles'; here even Goodman's secondary extensions do not help, should one have regarded the proposition as referring to tokens within it. This sort of example is not the only problem, for consider this I-proposition, 'Some current American (national government) president is named 'Nixon'.' Here the extension of the subject is a unit class, and if the proposition refers at all, it certainly refers to each element of this extension. Consequently, the subject is, by Copi's criterion, distributed.

Barker has by far the most elaborate and ingenious explication of distribution that I have ever seen:

Suppose that *T* is a term which occurs as subject or predicate in a categorical sentence *s*. Where *T'* is any other term, let *s'* be the sentence that is exactly like *s* except for containing the compound term *T'* and *T*, where *s* contains *T*. Now *T* is said to be distributed in *s* if and only if, for every term *T'*, *s* logically implies *s'*. ([1], pp. 41-42)

For ease in remembering, we can regard Barker as saying that if you can always replace a term by one that denotes a subset of the extension of the original term, and if the resulting proposition is logically implied by the original, the term so replaceable is distributed. Despite the technical merits of this explanation, there are drawbacks here too, and it is worth noting some details. First, suppose that the subject of an A-proposition is

necessarily empty as in *s*: 'All unicorns are animals.' We can now argue that the predicate of *s* is distributed, since *s* logically implies any *s'* containing a compound of 'animals' (i.e., which denotes a subset of animals). Thus *s* logically implies *s'*: 'All unicorns are tall animals,' as it is logically impossible for *s* to be true and *s'* false. If that example is troublesome, suppose for a moment, what is true in some restrictive syllogistic systems, that empty terms are out of bounds. Suppose further that the subject and predicate of an I-proposition necessarily have the same extension as in 'Some equilateral triangles are equiangular triangles.' Barker's explanation would clearly make both terms distributed. In fact if the extension of one term of an I-proposition were such as to be included in that of the other, the former term would also according to Barker's criterion be reckoned as distributed.

Naturally it would be remarkable, if there were no problems with O-propositions. So let us suppose, finally, the extension of the subject term of an O-proposition to be necessarily empty, as in *s*: 'Some round square is not a triangle.' Clearly it would be logically impossible for *s* to be true and any appropriate *s'* to be false, because *s* could not be true in the first place on Barker's existential reading of 'some'. Since *s* logically implies *s'*, the subject has to be considered distributed.

In all these cases we have not shown that Barker's test ever requires a term to be *undistributed*, when its location in a proposition (e.g., as subject of a universal affirmative—there are eight locations in all) would, according to tradition, lead us to label it 'distributed.' Thus we can imagine someone insisting that a term is distributed, if, and only if, it occupies a location in a proposition such that no other term in the same location is ever required by Barker's test to be undistributed. If this is accepted, then even when an occasional fluke might temporarily make us think that the predicate of an A-proposition, say, is distributed, we have only to remember that, when Barker's test is not constant with respect to such a location, this very inconstancy reveals the term's real lack of distribution. Nevertheless some formidable obstacles to accepting this resolution of the problem remain. Although it can be demonstrated that Barker's criterion provides a necessary condition for deeming a term distributed, the resolution in question would not provide any theoretical consideration to guide us in evaluating syllogisms, but only a record of various experiments involving the locations of terms. Even more important, to my mind, however, is that such a view abandons all hope for a nonsyntactic test of distribution. One may just as well arbitrarily define 'distribution' directly in terms of location, i.e., as applying to subjects of universals and predicates of negatives. Such a move, however, would give no hint of the semantic requirements of syllogisms, where validity seems to depend on distribution at crucial places in the argument.

Toms, ([8]) in an endeavor to fend off Geach's attacks, ([4]: Ch. 1 and [3]: Ch. 2.1) offers two defences of the doctrine of distribution. First, he tries to show that "from the proposition 'some S is not P' we can infer a proposition of the form 'every member of P is Q,' where Q is to be such

that this last proposition has a standard (viz. universal) form." This will, he believes, serve to vindicate the doctrine at the point where most of its troubles begin. Geach ([3], p. 65n.) praises Toms' effort: "nobody that I know of has given us an argument any better than Toms gives," and after restating it, forthwith rejects the argument by a circuitous logical analogy.

A more direct rebuttal is in order. Taking X to be "a member of S "—the needless shift into class language can be disregarded—that is not a P , Toms produces this type of proposition as being of the desired form: (F) 'Every member of P is something other than X .' An immediate problem is that, when an 0-proposition is false, there is no such X ; are we then to deem ' P ' undistributed by default? But Toms asks us to suppose the 0-proposition to be true, then he tells us "we could actually find at least one member of S which is not any member of P ." Of course, many existence proofs in mathematics assure us that a certain condition can be fulfilled, but we are unable to determine by what. This can happen in "real life" as well; suppose the truth of the proposition, 'Some soldier was not on guard last night.' If all means of discovery have vanished, then there is no way to specify Toms' X . A proper name is obviously unavailable. A singular definite description might suggest that exactly one soldier was derelict when there might have been more. The most promising indefinite description—already a departure from Toms' X , that "we could find"—leads us into tautology: Everyone on guard last night is something other than a soldier not on guard last night. There is, therefore, no way we could always depend on in order to satisfy Geach's requirement by a proposition of form (F); thus, contra Geach, Toms has not even proved that "for any given readings of ' S ' and ' P ', the *truth* of 'Some S is not P ' entails that there is a reading of ' Q ' such that 'Every P is Q ' is true" (*italics added*).

Matters are even worse than the above suggests. One might think that if Geach's desideratum, a proposition of form, (G): 'Every P is Q ', were somehow discoverable then all would be well. More realistically, we could hold high hopes for a "location" argument, saying that if Geach's desideratum ever proves a term in a certain location to be distributed, all terms in that location are distributed. But the distressing fact, as Geach ([3], p. 66) informs us, is that we could always produce a proposition of form (G) to "demonstrate" the distribution of the undistributed predicate of ' S i P .'

Following Toms' ground rule that in applying (G) we are to avoid necessary truths, Geach himself conveniently offers us: Every P is a thing that either is an S or is different from some P . But of course, if Geach's proposition can "show" the distribution of undistributed terms, then his original test involving (G), must be considered irrelevant to the general question of distribution. Thus, in the case of ' S o P ' we could "establish" distribution of P with 'Every P is a thing that is different from some S '.

If it were not for the opportunity to shed light on an important subject, we could probably safely ignore Toms' other argument, intended to parry Geach's thrust to the effect that the doctrine of distribution is discredited because it invalidates the derivation of propositional inverses—an inverse

has the contradictory of the original subject. The fairly well known crux of inversion is that by our using only standard rules of inference, ' $S'oP$ ' (which distributes ' P ') can be derived from ' $S a P$,' where ' P ' is undistributed. Since at the heart of the problem is a rule which always seemed to me to apply only to syllogisms—namely, a term cannot be distributed in the conclusion, if it is undistributed in the premisses—no great difficulty would have been anticipated here. But as one distributionist, a Professor Ray, has argued, "if a term is taken in the premiss to mean *at least* one thing denoted by it, it cannot in the conclusion be taken to mean all things denoted by it." (quoted in [5], pp. 106-107) Now, as Toms points out, one could claim that all the trouble originates in going from a universal to a particular, which is, on the Boolean view at least, an existential fallacy. Nevertheless, Toms bypasses this easy solution, preferring instead to revamp obversion. Toms reconstructs the rule for obverting ' $S a P$ ' as

$$\text{not everything is } P \longrightarrow (S a P \longrightarrow S e P')$$

and comments that the condition 'not everything is P ' distributes ' P '. If so, it will be no shock to find ' P ' distributed in the final step of

$$\text{not everything is } P \longrightarrow (S a P \longrightarrow \dots \longrightarrow S' o P).$$

But 'distribution,' so far has been "defined" only in connection with the four categoricals (A, E, I, and O), so justification for regarding ' P ' as distributed in 'not everything is P ' is still due from Toms (and Keynes before him). Note that we cannot freely rewrite the phrase as 'Some existent is not P ', on account of the (widely disputed) logical status of 'existent', as a term. We could however, for appropriate ' S ' use ' $S i P$ ' to indicate that ' P ' is not universally applicable. Let us invoke, as an aid, Boolean equations and Venn diagrams to show that Toms' condition ($P' \neq 0$) does safeguard the derivation of the inverse. I use the x -bar- x notation recommended by Quine to represent particular propositions (by indicating the possibly occupied regions) and, as it happens, Toms' condition as well; I also use the convention that the left circle always represents ' S ' and the right ' P '.

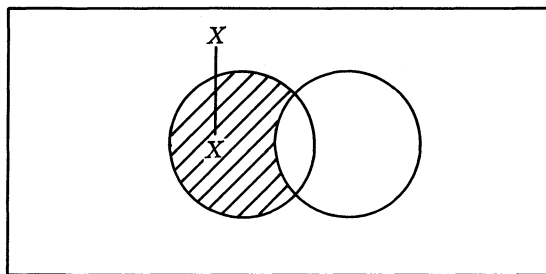


Figure 1

The fact that, of the two possible x 's, only the upper is actual, guarantees that $S'P' \neq 0$ or, in other notation, $S'oP$.

As for the general revamping of obversion, Toms will need to stipulate non-empty extensions for every term and its contradictory—something Toms actually endorses, though apparently for other reasons—because any one of these might show up in the predicate of a proposition to be obverted. Keynes, of course, assumed the same thing, and it does preserve the usual inferences of traditional formal logic. It has, however, the undesirable consequence of precluding this logic from treating terms “denoting” hypothetical entities, and the like. Furthermore, this *stipulation* makes every term (and its contradictory) distributed—since, to speak loosely, there will, according to the stipulation, always be something excluded from the entire extension of any term—whereas we had been used to having the distribution of a term determined solely by the proposition containing it. Toms' solution simply makes every predicate (of a proposition which might be obverted) distributed, but in so doing necessitates a change in the syllogistic rules, to the effect that the rules pertain only to the sort of distribution determined by the premisses themselves. Now if we allow that these rules pertain only to propositionally determined distribution, the original objection brought by Ray against inversion either still holds or (if you please) is irrelevant, so there was nothing gained by introducing the type of distribution wrought by Toms' stipulation in the first place.

We are still faced with the task of presenting a coherent semantic explanation of distribution. I think the only way a semantic sense can be established for the doctrine is by recourse to the stratagem of dividing and conquering. Distribution is one thing for universals and another for particulars; though, of course, the divergent features can be subsumed under an artificial unifying description. Let ' T ' and ' U ' represent either simple or compound terms—for instance, either the simple term 'soldiers' or the compound term 'soldier and nonperfectionist.' Whenever a universal proposition decrees (whether truly or falsely) with respect to terms ' T ' and ' U ', that all the T 's are excluded from being U 's (or non- U 's), the proposition distributes ' T '. Whenever a particular proposition decrees (whether truly or falsely), with respect to terms ' T ' and ' U ' that all T 's are different from some U , the proposition distributes ' T '. As applied to Venn diagrams whenever a term's area (often a circle, but not so for a term which is the contradictory of a term already represented by a circle) is shaded or else ignored by an ' x ' (or chain of x 's linked by bars), that term is distributed. Incidentally, this point may further clarify why Toms' condition (depicted in figure 1) distributed ' P '. The defining conditions were purposely stated to allow a proposition to distribute (incidentally, always antithetically) the contradictory of a term, when only the uncontradicted term occurs in the proposition. Thus ' S a P ' distributes not only ' S ' but ' P '; in the Venn diagram (see figure 2) both all S 's and all P 's are excluded from being S -and- P 's.

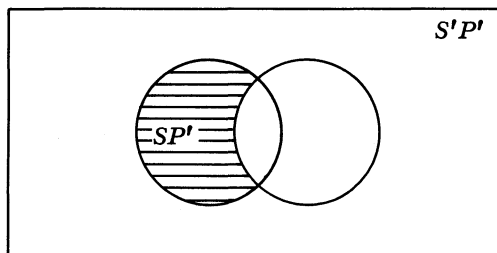


Figure 2

Figure 3 shows that the doctrine of distribution now applies to propositions like 'Some *S* is not a thing that is *S*, *M*, and *P*.' Here the subject is undistributed and the predicate distributed.

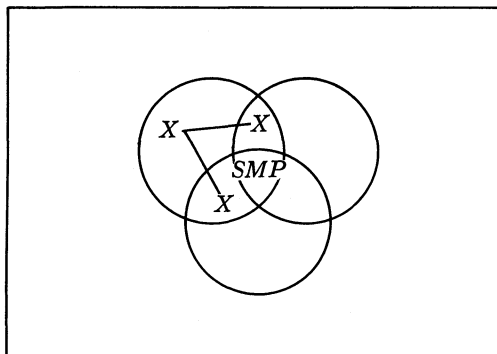


Figure 3

I remarked before that the unifying feature in common to the two modes of distributing terms seemed artificial. Even though the phrase 'all *T*'s' occurs in both defining conditions possibly gladdening the heart of the traditional distributionist, I nevertheless do not think that we can hastily conclude that distribution is a monolithic notion, for the operations affecting all the *T*'s in the two cases are quite different. The deceptive commonality can be further mitigated in a new—and, in some respects, simpler—statement of the test for distribution in terms of Boolean (in)equations. In any equation where the product of the term letters is zero, each of these terms is distributed; but if the product is not zero, the contradictory of each term is distributed. For example, for '*S* a *P*' (i.e., $SP' = 0$), '*S*' and '*P*' are distributed; while for '*S* i *P*' (i.e., $SP \neq 0$), '*S*' and '*P*' are distributed.

The doctrine as expounded so far seems well suited for resolving most quandries concerning distribution, yet there is one more rather esoteric problem I should like to treat. Geach ([4], pp. 16-18), reviving an issue from the fifteenth century, asks about the distribution of 'villager' in the following syllogism:

- I. Every donkey that belongs to a villager is running in the race;
- II. Brownie is not running in the race;
- III. Ergo, Brownie is not a donkey that belongs to a villager.

This is not the usual sort of problem about the distribution of the whole subject or the whole predicate of a premiss, nor is it even one about that sort of constituent of a compound subject or predicate (for instance, like 'donkey' or 'belongs to a villager') which applies to entities denoted by each subject and predicate in the entire syllogism. One consequence of not being such a constituent is that the Venn diagram required for the syllogism need have no area(s) for 'villager'. The question Geach poses could be handled in several ways. First we could rule it out of order because traditional syllogistic is not expected to treat complications due to the polyadic nature of relations. Syllogistic theory can, of course, accommodate a dyadic relation when it is used as a monadic term, like ' x is owned by a villager'; but it cannot also simultaneously treat a term applying to the other relata (i.e., the y 's in ' x is owned by y '). One could also argue that as a practical matter the puzzle could be put aside, since being able to use the standard test for validity depends only on understanding what it is for a whole subject or predicate to be distributed. Still, it is important for the respectability of the doctrine to extend it as far as Geach demands; in fact, I try to extend it in such a manner as to cover even polyadic terms themselves. The problem about 'villager' is that one can contend that it is distributed in the conclusion but not in the first premiss. In I, replacing 'a villager' by 'every villager' fails to preserve the same sense; but if 'some villager' is the replacement, all is well. This is supposed to prove 'villager' undistributed in the premiss. In III, a replacement that does preserve the same sense is 'any villager', a circumstance which seems to indicate distribution in the conclusion. Geach claims that 'any' and 'every' ought equally and without distinction to be considered signs of distribution, and the fact that 'any villager' is a suitable replacement in the premiss—thus indicating distribution there too—is immaterial. My explanation for all this is to point out that words like 'some' can have different meanings in logically different places in a proposition; just as an existential quantifier, applying only to a term in the antecedent of a conditional, must become universal, when removed to the front so as to have as its scope the whole of the unquantified part of the formula.

Extension of the doctrine of distribution (by a proposition) is best carried out through technical statement in more modern logic. To apply this new test, translate the propositions into symbolic logic, rendering all universals as universally quantified, unnegated conditional formulae and all particulars as existentially quantified unnegated conjunctions. Next put each formula into prenex normal form and have negations apply only to predicate letters. When the main truth function is a conditional—this occurs only when the original proposition was universal—we can say a term is distributed, if each of its variables is governed by a universal quantifier and the term is either in the antecedent unnegated or in the consequent negated. If

the main truth function is a conjunction (this will be the case when the original proposition is a particular), a term is distributed if each of its variables is governed by a universal quantifier, or if each of its variables is governed by an existential quantifier and the term is negated. In Ia and IIIa below I render I and III into symbolic form; in Ib and IIIb, I carry out enough transformations to show that 'villager' is distributed according to the above test in both I and III. The proper name 'Brownie' is eliminated in favor of a Quinean predicate represented as 'Bx'. 'Bxy' represents 'x belongs to y.' Notice that all terms distributed in Ib (even 'Bxy') are distributed in IIIb (the syllogistic conclusion).

- Ia. $(x) \{ [Dx \cdot (\exists y) (Vy \cdot Bxy)] \supset Rx \}$
 Ib. $(x) (y) \{ [Dx \cdot (Vy \cdot Bxy)] \supset Rx \}$
 IIIa. $(x) \{ Bx \supset \sim [Dx \cdot (\exists y) (Vy \cdot Bxy)] \}$
 IIIb. $(x) (y) \{ Bx \supset [\sim Dx \vee \sim Vy \vee \sim Bxy] \}$

This test also is a guide to distributing Geach's last "unhackneyed" example, 'Every donkey that belongs to every villager is running in the race.' The formula IV below easily shows that 'villager' is not distributed; Geach had, no doubt, correctly asserted that the *traditional* doctrine was of no service in deciding this matter.

- IV. $(x) (\exists y) \{ [Dx \cdot (Vy \supset Bxy)] \supset Rx \}$

Less technical rules, corresponding to the symbolic versions could be given; but they would be so numerous—for instance, covering all possible occurrences of 'every' in subject and predicate, negated and unnegated, etc.—that there is not much point in doing so. I think, however, that since there is now a way to treat Geach's puzzles by a fairly natural extension and tightening of the traditional doctrine, the most serious reservations about the *concept* of distribution have been dispelled.

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