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BURIDAN: 'EVERY PROPOSITION IS FALSE' IS FALSE

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In his *Sophismata*,[1] Buridan proposes as a sophism:

(1) Every proposition is false.

A sophism for Buridan is a proposition[2] which appears to be both demonstrably true and demonstrably false. We are to hypothesize a background situation in which "all true propositions should be destroyed and false ones remain."[3] This sophism is a version of the classical paradox, the one which van Frassen calls "the weakened liar paradox." On the one hand, the problem proposition, (1), seems to describe aptly the hypothetical situation in which all true propositions have been destroyed, and thus to be true. On the other hand, if *it* is an existent proposition and is true, then it itself is no longer true. If it is false, however, it seems to make itself true once more. This is certainly sufficient reason for Buridan to pronounce (1) a sophism. Buridan's solution is that 'Every proposition is false' is in fact false. I should like to argue that Buridan's solution to the weakened liar paradox is acceptable. He has both an intuitive and a demonstrative argument for his assignment of truth value, and those arguments are correct. But Buridan's system is precisely the kind which should, according to Tarski, eventually produce liar-type paradoxes which it cannot solve. I shall argue that Buridan's representation of natural language is indeed just strong enough to produce a sentence whose truth value is in one sense obvious, but is in addition such that the assignment of that truth value introduces inconsistency into the system. That sentence is the so-called "ordinary liar paradox," 'I am speaking falsely.' Buridan does not block the formation of the sentence expressing the ordinary liar paradox.[4] Nor, I shall argue, does he depart from his basic commitment to semantic bivalence. The result is that he can have either his solution to the ordinary liar paradox, or systematic consistency, but not both.

Buridan has two arguments for the falsity of (1). In the first argument, which I shall call "intuitive," he uses the notion of "virtual implication" to move from:

(2) No proposition is true.

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uttered by Socrates, to (2)'s virtual implicate:

(3) 'No proposition is true' is true.

(3) is false, presumeably because it is self-falsifying. (2), then, which leads to (3), is false.[5] The point of the argument is clear enough. Even disregarding the curious notion of "virtual implication," it would be plausible to suppose that a proposition which, when assumed true, implies its own falsity, is false, if it has a truth value. We can afford to overlook the fact that Buridan started with (2), and not with (1).[6] Since there are assumed to be propositions in existence, he could just as easily have constructed an analogous argument for the falsity of (1). He seems simply to have been led off target by an extra example which he himself interposed.

Buridan offers the intuitive argument to illustrate the working of the principle of virtual implication: From a proposition *C* which exists, there follows by virtual implication the proposition '*C* is true'. He has at this point already given his deductive argument for the falsity of (1):

> Now we must look into the truth and falsity of these propositions. And, briefly, I believe that the proposition would be false. For it would be either false or not false. If it is false, I have my conclusion. And if it is not false, it follows that it is true, from which also it is. And if it is true, it follows that it is false, as was previously argued. Hence I have my conclusion, namely that it is false.[7]

Filling in the "previous argument" to which Buridan alludes, we have a constructive dilemma:

i.	Either 'Every proposition is false' is false or 'Every proposition is false' is not false.	
ii.	Assume: 'Every proposition is false' is false.	
iii.	'Every proposition is false' is false.	(ii)
iv.	Assume: 'Every proposition is false' is not false.	
v.	'Every proposition is false' is true.	(iv)
vi.	'Every propositions is false' is (exists).	(v)
vii.	Not every proposition is false.	(v, vi)
viii.	'Every proposition is false' is false.	(vii)
ix.	'Every proposition is false' is false.	(i, iii, v, viii)

Thus, assuming either that the problem proposition is not false or that it is false, we can conclude that it is false. There is no parallel argument which shows it to be true on both assumptions. It would, of course, follow from the assumption that (1) is true that (1) is true. But from the assumption that (1) is not true, it would follow only that some proposition or other is true, not that (1) itself is the one which is true.

The argument is a little strange. It is not so very unusual to be able to derive *p* from *not-p* in a proof. But this usually means that the derived *p* is either a logical truth or something contained in the assumptions for the proof and produced at the critical point by the rule of repetition. "'Every proposition is false' is false" is not for Buridan a logical truth. He says explicitly of (1) that it "could be as it signifies," that it might be true of, or describe, a possible world.[8] Its falsity is therefore not for Buridan a logical

necessity. And (viii) comes straightforwardly from (v), not from some previous assumption or some other line of the proof. Formally there is nothing wrong with the proof. It just so happens that in this system it is sometimes possible to derive the contingent *p* from the contingent *not-p*. This happens, Buridan says, "When a proposition has or can have reflection on itself."[9] So far, the only conceivable objection to what Buridan has done is that he may have failed to see that, given his argument, (1) is logically false. It seems fair to say that Buridan's system has survived its first encounter with a liar-type paradox.

Buridan's success is somewhat unexpected. The emergence of liar-type paradoxes in his system is what we ought to have expected—but not their solution. Buridan's language offers us a paradigm case of what Tarski calls "a semantically closed language."[10] The language includes propositions; and it contains ways of referring to these propositions. An expression may be represented materially by another token of roughly the same type.[11] In addition, an expression may be referred to by such expressions as 'this sentence' or included in the reference of 'every sentence'. Obviously, the language also contains the predicates 'true' and 'false'. Nothing blocks formation of any expression of the form '*p* is true (false)', so long as *p* is replaced by an expression which represents or refers to a syntactically well-formed proposition. The ordinary laws of logic as Buridan knows them hold throughout the system.[12] But Buridan does *not* follow Tarski's suggestion for solving semantic paradoxes by presenting us with a metalanguage which is what Tarski calls "essentially richer" than the object language.[13] If it were, the move from (v) to (viii) would be blocked by the fact that 'Every proposition is false' is of the wrong type to be taken as comprehended in the reference of 'Every proposition' in 'Every proposition is false'. (Alternatively, we could speak of blocking the move from (v) to (vii) or from (vii) to (viii).) Buridan, then, does not accept Tarski's proposed solution. And he has survived at least one encounter with a liar-type paradox. The question is whether Buridan will be able to deal with *all* the liar-type paradoxes so easily. I shall argue that he cannot.

It is significant that Buridan does not use the principle of virtual implication explicitly to assign a truth value to (1). He never does use the principle when he can avoid doing so. But in his solution to the ordinary liar paradox:

(4) I am speaking falsely.

where we are to hypothesize that the speaker utters (4) and only (4), he is forced to use the principle directly.

> I answer that the sophism is false because from it and the proposition expressing the case, a false proposition follows. Yet, since this proposition expressing the case is said to be true, and that false, what follows is that the sophism is both true and false at once. But a proposition is false from which, together with its truth, a false proposition follows.[14]

Buridan's statement of his argument is not very clear, but by reference to a previous argument[15], it can be reconstructed:

(i)	I am speaking falsely.	
(ii)	'I am speaking falsely' is (exists).	(i)
(iii)	'I am speaking falsely' is true.	(i, ii)
(iv)	'I am speaking falsely' is false.	(iii)
(v)	'I am speaking falsely' is false.	(i, iii, iv)

The "false proposition" that follows from (4) is that (4) is both true and false. The "proposition expressing the case" is, presumeably, that (4) exists, i.e., is spoken. (iii) and (iv) taken together constitute a contradiction; and so (4), that is (i), is false. In this case, Buridan has to use the principle of virtual implication to derive (iii), from which, in turn, he derives (iv). But now he has a serious problem. From "'I speak falsely' is false" it seems to follow that 'I speak falsely' is true. If so, then the assignment of a truth value to (4) produces a contradiction. Buridan attempts to answer the charge:

> I answer that it does not follow that if it is false, then it is true. But I do indeed agree that it follows that if it is false, then it is. I concede also that from it and from the fact that it is, it follows that it is true. But I do not concede the antecedent. Rather I deny the antecedent, because the antecedent was composed of it and its being. And I deny it, so I also deny the consequent, namely that it is true.[16]

Let us assume that Buridan has not made an elementary logical error, that he meant to say that the consequent had to be accepted if *and only if* the antecedent was acceptable. This still leaves plenty to discuss. Buridan says that he denies the antecedent, that the antecedent is composed of "it and its being." There is a good enough reason, intuitively speaking, for rejecting (4) and (4)'s being. (4) construed self-referentially—and that is the only way it can be construed—implies a contradiction. Any "world" described by (4) thus fails to be a possible world. From (4) and the claim which Buridan apparently accepts, that (4) is either true or false, a contradiction follows. Nonetheless, (4)'s "being" follows from (4)'s falsity. And there thus seems to be no reason at all for claiming that

(5) 'I am speaking falsely' is true.

does not follow from (4) and its being by virtual implication. There is, so far as I can see, no way to make Buridan's argument work without repudiating one or more of his principles in an *ad hoc* way.

The problem could be avoided only *if* Buridan meant by "denying" (4) to reject (4) as a genuine proposition or to refuse to assign it a truth value at all. But there is no textual evidence that he meant to do either. We must conclude that the price of Buridan's ideal of completeness, i.e., semantic assessment of every well-formed proposition of natural language, is inconsistency. The inconsistency is not in what Tarski would term "the sentences of the object language," but in the sentences of the meta-language which express the assignment of truth values.

Buridan has, it seems, landed himself in serious trouble. It seems significant that the difficulties arise only in those cases in which he has to use the principle of virtual implication explicitly to make an initial assignment of truth value. Some inspection of that principle and how it works

seems in order. For propositions which do not involve self-reference, Buridan does have something like a satisfaction theory of truth. An affirmative categorical proposition is true if and only if the subject and predicate terms supposit for or refer to the same entities.[17] Two terms in a proposition have the same supposition if their extensions, determined by what they signify mediately, and suitably modified by the semantically relevant features of the surrounding propositional context, are the same.[18] When Buridan states this account, he indicates quite explicitly that it will not suffice for the *insolubilia* of the liar-paradox type. The simple criterion will fail, he says, because "such a proposition either simply or with accompanying circumstances, implies a contradiction."[19]

We can see from Buridan's treatment of the weakened liar paradox why the normal criterion will not suffice in such cases:

> Again, if it is false, the howsoever it signifies, so it is. For it signifies only that every proposition is false, and so it is. Therefore it is true.[20]

Buridan is here experimenting with applying the normal criterion. If we assume that the proposition is false, then its subject term, standing for all the false propositions left after the destruction of the true ones *and* for itself, stands for precisely the same entities for which the predicate term 'false' stands. The normal criterion thus yields the value 'true'—from which it follows that the proposition is false. And so Buridan adds the principle of virtual implication for the evaluation of propositions involving self-reference:

> Hence, because of this, it will be said that when a proposition does or can have reflection on itself it does not suffice for the truth of an affirmative that the terms stand for the same, as is said elsewhere. But it is required that *in such a consequent*, the terms stand for the same.[21]

"Such a consequent" Buridan has specified as "another proposition so that of the subject standing for the (original) proposition, there is affirmed the predicate 'true'."[22] This is what we have called "the principle of virtual implication." In a case involving self-reference, then, if '*p* is true' is false, then so, by *modus tollens*, is *p*.

The new truth criterion based on virtual implication allows Buridan to capture a semantic feature of propositions which are or can be self-referential which would otherwise elude him. In effect, it allows Buridan to ask whether such a proposition can truly describe a possible world *in* which it has existence and the value 'true'. The notion of "virtual implication" thus provides a way of stating the connection between a self-referential proposition containing an alethic predicate and its truth conditions *within* the language in which the proposition occurs. This drops a vital part of what would normally be considered metalanguage, the expression of truth value assignments to sentences containing 'true' or 'false' as predicates, into a now extended object language. For propositions which *may* be self-referential and which are self-falsifying when so construed, the principle merely generates queer arguments. For propositions which must have self-reflection and which are self-falsifying, it generates inconsistency in the extended

object language. There is no sense in quibbling about the sense of "implication" involved; this is what the principle *does*.[23] The operation of the principle may thus be seen as of a piece with the kind of type theory commonly found in Ockhamist systems of logic.

But it is simply not clear that Buridan thinks he has a "different kind of truth," "truth on a different level," or any of the other alternatives suggested by some current scholars. One factor is surely that propositions *not* involving reflection on themselves satisfy the new criterion trivially. A proposition which does not involve self-reference is true just in case its virtual implicate is true. Buridan may see himself as simply making explicit the fact that, unlike non-self-referential propositions, self-referential propositions include themselves explicitly in the world they purport to describe. Although Buridan does not appeal explicitly to the principle of virtual implication in his deductive argument for the falsity of 'Every proposition is false', its ramifications legitimate the move from (v) to (viii). That, if fact, is why the proof looks odd.

The interesting question is why Buridan can deal successfully with the weakened liar paradox, but not with the ordinary liar paradox. Proper understanding of the principle of virtual implication provides us with an answer. 'Every proposition is false' is not, as Buridan points out, inescapably self-contradictory. It might even be true of a world in which it was not itself included. In a world in which it is included, it is self-falsifying. But the ordinary liar paradox as Buridan presents it is inescapably productive of contradictions, given semantic bivalence. Unlike the weakened liar paradox, it mandates its inclusion in any world to which it applies. Its self-reference is the only reference it can have, and it has it as soon as it exists. The principle of virtual implication will make this obvious by producing inconsistency in these cases.

The introduction of the principle of virtual implication is, then, not *ad hoc*; and it is not a stupid mistake. It is, in fact, more of a piece with Buridan's semantics than he is inclined to see. It insures a uniform treatment of self-referential propositions which is an extension of the treatment of propositions which do not involve self-reference.

Unfortunately, it will not preserve systematic consistency if Buridan insists on a bivalent semantics which completely expresses natural language. But it does bring out an interesting parallel between systems of semantics for natural language and problems in the representation of mathematics demonstrated by Gödel. It is, one might say, inconsistent in an interesting way.[24]

Notes

1. John Buridan, *Sophisms on Meaning and Truth*. Transl. T. K. Scott (New York, 1966), hereafter cited as *Sophisms*. Ch. VIII, p. 191. All page references to Buridan's text will be to this translation. When the Latin is given, the reference is to my own transcription of an *incunabulum*, ca. 1500.

2. Buridan takes the *proposition* whose truth value he is debating to be the sophisms, though many of the propositions with which he concerns himself are expressions of enthymatic inferences, e.g., 'Every proposition is affirmative, so none is negative,' p. 180.

3. *Sophisms,* p. 191.

4. *Sophisms,* p. 196. Buridan says of (1) that it is "possible, though it could not be true" in the hypothesized situation.

5. *Sophisms*, p. 196.

6. The text contains no argument that (1) and (2) are equivalent, even given the existence of some propositions.

7. *Sophisms*, p. 194. The "previous argument" for the falsity of the sophism is presumably the one used in setting up the problem, p. 191.

8. *Sophisms*, p. 196.

9. *Sophisms*, p. 195.

10. Alfred Tarski, "The Semantic Conception of Truth," in *Semantics and Necessary Truth*, ed. Linsky (Urbana, 1952), p. 348.

11. The criterion for this kind of representation has to be stated loosely. Buridan, like Ockham, concedes that the exigencies of surface grammar may produce a difference between the expression represented and the expression which materially represents it. 'Homo est asinus' may, for example, be represented by 'hominem esse asinum' in 'Hominem esse asinum est vera'. *Sophisms*, p. 101.

12. Buridan's method of assigning truth conditions is recursive, but it differs from Tarski's in several important respects. Quantified categorical propositions are taken as simple propositions. The language contains more quantifiers than Tarski's, and there are clauses for dealing with propositions containing such propositional operators as 'begins' and 'ceases'.

13. Tarski, *op. cit.*, says that the language under discussion, if based on a logical theory of types, is "essentially richer" than the relevant object language if it contains variables of a higher type than those of the object language, p. 351.

14. *Sophisms*, p. 204.

15. In the previous case, which has to do with cross-reference, we are to suppose that Plato says, "Socrates speaks falsely," and that Socrates says, "Plato speaks falsely." Buridan's solution is that both Socrates' and Plato's propositions are false because each implies that it itself is both true and false.

16. *Sophisms*, p. 204. "Respondeo quod non sequitur si est falsa quod est vera: sed bene concedo quod sequitur si est false quod ipsa est. et concedo etiam quod ex ipsa et quod ipsa est sequitur bene quod ipsa est vera. licet ego non concedo antecedens; immo ego nego antecedens; quia antecedens erat compositum ex ipsa et quod ipsa est et ipsam nego ideo etiam nego consequens secundum quod ipsa est vera."

17. *Sophisms*, p. 90.

18. Terms signify concepts of the mind immediately, *Sophisms*, p. 70. "Mediately" or "ultimately" they stand for the entities conceived under the concept, *Sophisms*, p. 75. The Total mediate signification of a term is the *totality* of individuals signified by its component parts. The total mediate signification of 'white men', then, includes all white things and all men. The *supposition* of a term in a proposition is the *overlap* of the mediate significations of the

component terms, modified by the logically relevant features of the surrounding propositional context, *Sophisms*, pp. 89-91. The supposition of 'All white men' in 'All white men now run' is all presently existing men who are also white things.

19. *Sophisms*, p. 92.

20. *Sophisms*, p. 192.

21. *Sophisms*, p. 195. (underlining mine).

22. *Sophisms*, p. 195 (insertion mine).

23. Buridan seems to have what Herzberger, "Truth and Modality in Semantically Closed Languages," in *The Paradox of the Liar*, ed., Martin,(New Haven, 1970) calls "a language which has general closure," i.e., "L contains the means for expressing the whole of its semantic theory," p. 26. It would, of course, require further argument to show that Buridan's language expresses not only all truth value assignments, but also Buridan's metalogical assumptions. What Buridan's language does *not* have are the truth value gaps Herzberger attributes to it, and which would allow Buridan to avoid inconsistency.

24. This paper was first presented at the eleventh annual medieval conference, Kalamazoo, Michigan, May 3-6, 1976.

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