Notre Dame Journal of Formal Logic
Volume XIX, Number 3, July 1978
NDJFAM

PRIMITIVITY IN MEREOLOGY. II

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CHAPTER IV: CARDINAL-DEPENDENT PRIMITIVE TERMS

The results in this chapter arose from consideration of the term **w-dscr** . We notice that **w-dscr**, where restricted to names of cardinality less than three, is not primitive, i.e., the primitivity of **w-dscr** demands there be more than two objects. In this chapter we define a sequence of terms which are primitive provided a certain number of objects exist.* We begin with the definition of **sbstm**. Intuitively, **sbstm** $\{b\}$ means that b is a model for mereology. The definition states that b is closed under the terms **Kl** and $\backslash$, that is, relative complement.

D25 $[b] \therefore \mathbf{sbstm}\,\{b\} \,.\!\equiv\!:\, [a] : a \subset b \;.\!\supset\!.\; \mathbf{Kl}(a) \subset b \,.\, \mathbf{Kl}(b) \backslash \mathbf{Kl}(a) \subset b$

Notice that if **pr**, **Kl**, and = are restricted to b and **sbstm** $\{b\}$, then b satisfies the axioms of mereology. Henceforth, if we state **sbstm** $\{b\}$ in a hypothesis, we will assume we are working within that subsystem unless otherwise indicated. For this reason, in the proof lines we will also merely note results as they are stated in previous theorems, without explicitly showing they are restricted. Occasionally, for clarity we will indicate exactly which subsystem we are working in with subscript notation, e.g., $\mathbf{pr}_a$, $\mathbf{Kl}_a$, etc.

T197 $[A] : A \,\varepsilon\, A \;.\!\supset\!.\; A \,\varepsilon\, \mathbf{Kl}(\mathbf{el}(A))$ [*D2*]

T198 $[A] : A \,\varepsilon\, A \;.\!\supset\!.\; \mathbf{sbstm}\,\{\mathbf{el}(A)\}$

PR $[A] \therefore \mathrm{Hp}(1) \;.\!\supset\!:$

2. $A \,\varepsilon\, \mathbf{Kl}(\mathbf{el}(A)) :$ [1; *T197*]
3. $[b] : b \subset \mathbf{el}(A) \;.\!\supset\!.\; \mathbf{Kl}(b) \subset \mathbf{el}(\mathbf{Kl}(\mathbf{el}(A))) :$ [*T11*]
4. $[b] : b \subset \mathbf{el}(A) \;.\!\supset\!.\; \mathbf{Kl}(b) \subset \mathbf{el}(A) :$ [2; 3]
5. $[b] : b \subset \mathbf{el}(A) \;.\!\supset\!.\; \mathbf{Kl}(\mathbf{el}(A)) \backslash \mathbf{Kl}(b) \subset \mathbf{el}(A) :$ [4; *D11*]

$\mathbf{sbstm}\,\{\mathbf{el}(A)\}$ [4; 5; *D25*]

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*The first part of this paper appeared in *Notre Dame Journal of Formal Logic*, vol. XIX (1978), pp. 25-62.

Received August 30, 1971

T199 $\mathbf{sbstm}\ \{\mathbf{el}(\wedge)\}$ [*D25*; *D12*; *T8*]
T200 $[A]:\rightarrow\{A\}\ .\supset.\ \mathbf{sbstm}\ \{\mathbf{el}(A)\}$ [*T198*; *T199*]
T201 $[a]\,.\ \mathbf{sbstm}\ \{\mathbf{el}(\mathbf{Kl}(a))\}$ [*A4*; *T200*]
T202 $[ABa]:\mathbf{sbstm}\ \{a\}\,.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,a\,.\,A\,\varepsilon\,\mathbf{lnk}(B)\ .\supset.\ \mathsf{Cd}\,\{a\} > 3$
PR $[ABa]:\mathrm{Hp}(4)\ .\supset.$
5\. $A\wedge B\,\varepsilon\,A\wedge B\,.$ [1; 4; *D5*; *D8*; *D25*]
6\. $A\wedge B \neq A\,.$ [2; 4; 5; *T33*; *T17*]
7\. $A\wedge B \neq B\,.$ [3; 4; 5; *T33*; *T18*]
8\. $A\wedge B\,\varepsilon\,a\,.$ [1; 2; 3; 5; *D5*; *D25*]
9\. $A\vee B\,\varepsilon\,A\vee B\,.$ [1; 2; 3; *D4*; *D25*]
10\. $A\vee B \neq A\,.$ [2; 3; 4; 9; *T19*; *T33*]
11\. $A\vee B \neq B\,.$ [2; 3; 4; 9; *T20*; *T33*]
12\. $A\vee B \neq A\wedge B\,.$ [4; *D4*; *D5*; *D8*]
13\. $A\vee B\,\varepsilon\,a\,.$ [1; *D25*; 9]
14\. $A \neq B\,.$ [4; *T33*]
$\mathsf{Cd}\,\{a\} > 3$ [2; 3; 6; 7; 8; 10-14; **ON**]

We will depart, for a theorem, from our usual notation concerning subsystems to make the following theorem clearer.

T203 $[ABa]:\mathbf{sbstm}\ \{a\}\,.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,\mathbf{atm}_{\mathbf{el}_a(A)}\ .\supset.\ B\,\varepsilon\,\mathbf{atm}_a$
PR $[ABa]\therefore\mathrm{Hp}(3)\ .\supset:$
4\. $\mathbf{sbstm}\ \{\mathbf{el}_a(A)\}\,.$ [1; *D25*; 2; *T200*]
5\. $B\,\varepsilon\,\mathbf{el}_a(A):$ [3; *D25*]
6\. $[C]:C\,\varepsilon\,\mathbf{el}_{\mathbf{el}_a(A)}(B)\ .\supset.\ C = B:$ [3; *T50*]
7\. $[C]:C\,\varepsilon\,\mathbf{el}_a(A)\,.\,C\,\varepsilon\,\mathbf{el}_a(B)\ .\supset.\ C = B:$ [6; *D25*]
8\. $[C]:C\,\varepsilon\,\mathbf{el}_a(B)\ .\supset.\ C = B:$ [1; *D25*; 7; 5; *T4*]
$B\,\varepsilon\,\mathbf{atm}_a$ [8; *T50*]

The previous theorem makes clear why we will avoid such notation if at all possible. We will also adopt another convention in our induction proofs. By *050*, to show $\theta(n)$ we must first show $\theta(0)$ and $[m]:m < n\,.\,\theta(m)\ .\supset.\ \theta(n)$. In our induction proofs, it will generally be clear that we will begin our induction at 1 instead of 0, so we will assume that we can just as well begin our induction from 1 or for that matter from any finite number. We will set off the induction step with horizontal lines and denote the induction hypothesis by **IH**.

T204 $[A]:A\,\varepsilon\,A\,.\ \mathbf{sbstm}\ \{A\}\ .\supset.\ A\,\varepsilon\,\mathbf{at}(A)$
PR $[A]\therefore\mathrm{Hp}(2)\ .\supset:$
2\. $[B]:B\,\varepsilon\,\mathbf{el}(A)\ .\supset.\ B = A\,.$ [1; 2; *D25*]
3\. $A\,\varepsilon\,\mathbf{atm}\,.$ [2; *D9*]
$A\,\varepsilon\,\mathbf{at}(A)$ [3; *T36*]
T205 $[Aa]:\mathbf{sbstm}\ \{a\}\,.\ \mathsf{Fin}\,\{a\}\,.\sim(\rightarrow\{a\})\,.\,A\,\varepsilon\,a\ .\supset.\ [\exists B]\,.\,B\,\varepsilon\,\mathbf{at}(A)$
PR $[Aa]\therefore\mathrm{Hp}(4)\ .\supset:$
$[\exists CD]:$

$$\left.\begin{array}{ll} 5. & C\,\varepsilon\,a\,. \\ 6. & C\,\varepsilon\,\mathbf{Kl}(a)\,. \end{array}\right\}\qquad [1;\ 3;\ D25;\ A5]$$

$$\left.\begin{array}{ll} 7. & D\,\varepsilon\,a\,. \\ 8. & D \neq C\,. \end{array}\right\}\qquad [3]$$

9.	$D \varepsilon \mathbf{pr}(C)$.	[1; *D25*; 7; 6; *D2*; *D1*]
10.	$D \varepsilon D$.	[7; **ON**]
11.	$\mathbf{sbstm} \{\mathbf{el}(D)\}$.	[10; *T200*]
12.	$\mathbf{el}(D) \subsetdot a$.	[5; 9]
13.	$\mathsf{Cd} \{\mathbf{el}(D)\} < \mathsf{Cd} \{a\}$:	[2; 12; **ON**]
14.	$[E] : E \varepsilon \mathbf{el}(D) \;.\supset.\; [\exists F] . F \varepsilon \mathbf{at}_{\mathbf{el}(D)}(D)$:	[11; 13; **IH**; *T204*]
15.	$[E] : E \varepsilon \mathbf{el}(D) \;.\supset.\; [\exists F] . F \varepsilon \mathbf{at}(D)$:	[14; *T203*]
16.	$[E] : E \varepsilon a \;.\supset.\; [\exists F] . F \varepsilon \mathbf{at}(E)$:	[1; 12; 15; 9; *T203*]
	$[\exists B] . B \varepsilon \mathbf{at}(A)$	[4; 16]
T206	$[aA] : \mathbf{sbstm} \{a\} . \mathsf{Fin} \{a\} . A \varepsilon a \;.\supset.\; [\exists B] . B \varepsilon \mathbf{at}(A)$	[*T204*; *T205*]
T207	$[aA] : \mathbf{sbstm} \{a\} . A \varepsilon a . \sim (A \varepsilon \mathbf{Kl}(a)) \;.\supset.\; \mathbf{el}(A) \subsetdot a$	
PR	$[aA] : \mathrm{Hp}(3) \;.\supset.$	
	$[\exists B]$.	
4.	$B \varepsilon a$.	[1; *D25*]
5.	$B \varepsilon \mathbf{Kl}(a)$.	
6.	$A \varepsilon \mathbf{el}(B)$.	[2; 5; *D2*]
7.	$A \neq B$.	[3; 5]
8.	$A \varepsilon \mathbf{pr}(B)$.	[6; 7; *D1*]
9.	$\sim (B \varepsilon \mathbf{pr}(A))$.	[8; *A2*]
10.	$\sim (B \varepsilon \mathbf{el}(A))$.	[7; 9; *D1*]
11.	$\mathbf{el}(A) \subset a$.	[1; *D25*; 2]
	$\mathbf{el}(A) \subsetdot a$	[4; 10; 11; **ON**]
T208	$[a] : a \varepsilon a \;.\supset.\; \mathsf{Cd} \{a\} = 1$	[*D032*]

Henceforth we make the assumption that the variable *n*, *m*, etc., denote natural numbers, i.e., $\mathsf{Nn}(n)$, $\mathsf{Nn}(m)$, etc. This will shorten the statements of many theorems.

T209	$[a] : a \varepsilon a \;.\supset.\; [\exists n] . \mathsf{Cd} \{a\} = n$	[*T208*]
T210	$[Aa] : \mathbf{sbstm} \{a\} . \mathsf{Fin} \{a\} . A \varepsilon a . \mathsf{Cd} \{\mathbf{at}(A)\} = 1 \;.\supset.\; A \varepsilon \mathbf{at}(A)$	
PR	$[Aa] \therefore \mathrm{Hp}(4) \;.\supset$:	
	$[\exists B]$:	
5.	$B \varepsilon \mathbf{at}(A)$.	[4; **ON**]
6.	$B \varepsilon \mathbf{el}(A)$:	[5; *D10*]
7.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; A \backslash B \varepsilon a$:	[3; 1; *D25*]
8.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; A \backslash B \varepsilon \mathbf{el}(A)$:	[7; *D11*]
9.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; [\exists C] . C \varepsilon \mathbf{at}(A \backslash B)$:	[7; 1; 2; 3; *T206*]
10.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; [\exists C] . C \varepsilon \mathbf{atm} . C \varepsilon \mathbf{ex}(B)$:	[9; *D11*; *D10*]
11.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; [\exists C] . C \varepsilon \mathbf{atm} . C \neq B$:	[10; *T33*]
12.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; [\exists C] . C \varepsilon \mathbf{at}(A) . C \neq B$:	[9; *T203*; 11]
13.	$B \varepsilon \mathbf{pr}(A) \;.\supset.\; \mathsf{Cd} \{\mathbf{at}(A)\} \geqslant 2$:	[5; 12]
14.	$B = A$:	[*D1*; 6; 4; 13]
	$A \varepsilon \mathbf{at}(A)$	[5; 14]
T211	$[AB] : A \varepsilon \mathbf{ex}(B) \;.\supset.\; (A \vee B) \backslash A = B$	
PR	$[AB] : \mathrm{Hp}(1) \;.\supset.$	
2.	$A \vee B \varepsilon A \vee B$.	[1; 2; 3; *D4*]
3.	$(A \vee B) \backslash A \varepsilon (A \vee B) \backslash A$.	[1; 2; *T19*; *D11*]
4.	$(A \vee B) \backslash A \circ (A \vee B) \wedge \mathbf{Cm}(A)$.	[1; 2; *T19*; *T47*]

5. $(A \vee B)\backslash A \circ (A \wedge \mathbf{Cm}(A)) \vee (B \wedge \mathbf{Cm}(A))$. [4; **BA**]
6. $(A \vee B)\backslash A \circ B \wedge \mathbf{Cm}(A)$. [5; **BA**]
7. $B \varepsilon \mathbf{el}(\mathbf{Cm}(A))$. [1; *T27*]
8. $(A \vee B)\backslash A \circ B$. [6; 7; *T60*]

$(A \vee B)\backslash A = B$. [3; 8; **ON**]

T212 $[ABC] : A \varepsilon \mathbf{ex}(B \vee C) . A \vee B = A \vee C .\supset. B = C$

PR $[ABC] : \mathrm{Hp}(2) .\supset.$

3. $A \varepsilon \mathbf{ex}(B)$.
4. $A \varepsilon \mathbf{ex}(C)$. [1; *T30*]
5. $(A \vee B)\backslash A = B$. [3; *T211*]
6. $(A \vee C)\backslash A = C$. [4; *T211*]
7. $(A \vee B)\backslash A = (A \vee C)\backslash A$. [2; **ON**]

$B = C$ [5; 6; 7]

T213 $[ABC] \therefore A \varepsilon \mathbf{ex}(B \vee C) .\supset: A \vee B = A \vee C .\equiv. B = C$ [*T212*; **ON**]

D26 $[abC] : C \varepsilon a * b .\equiv. [\exists AB] . A \varepsilon a . B \varepsilon b . C = A \vee B$

This mereological term defines a new name from two given names. It forms the Boolean sum of two names, one from each of the given names.

T214 $[ABC] : C \varepsilon A \vee B .\equiv. A \varepsilon A . B \varepsilon B . C \varepsilon A * B$ [*D4*; *D26*]

AD1 $[BC\boldsymbol{\sigma}] : B \varepsilon \boldsymbol{\sigma}(C) .\equiv. B \varepsilon B . [\exists Aa] . A \varepsilon A . B = A \vee C . C \varepsilon a$

T215 $[Aa] : \mathsf{Fin}\{a\} . A \varepsilon \mathbf{ex}(\mathbf{Kl}(a)) .\supset. \mathsf{Cd}\{A * a\} = \mathsf{Cd}\{a\}$

PR $[Aa] \therefore \mathrm{Hp}(2) .\supset:$

3. $\sim(A \varepsilon a) :$ [2; *T11*]
4. $[B] : B \varepsilon a .\supset. A \varepsilon \mathbf{ex}(B) :$ [2; *T11*; *T4*; *D7*]
5. $[B] : B \varepsilon a .\supset. A \vee B \varepsilon \boldsymbol{\sigma}(B) :$ [*AD1*; 2]
6. $[C] : C \varepsilon A * a .\supset. [\exists D] . D \varepsilon a . C = A \vee D :$ [*D26*]
7. $[C] : C \varepsilon A * a .\supset. [\exists D] . C \varepsilon \boldsymbol{\sigma}(D) :$ [6; *AD1*]
8. $[EF] : E \varepsilon a . F \varepsilon a . E = F .\supset. E \vee A = F \vee A :$ [**ON**]
9. $[CD] : C \varepsilon a . D \varepsilon a . A \vee C = A \vee D .\supset. C = D :$ [2; *T213*]
10. $A * a \overset{\sigma}{\infty} a$. [1; 5; 7; 8; 9; **ON**]

$\mathsf{Cd}\{A * a\} = \mathsf{Cd}\{a\}$ [1; 10; **ON**]

T216 $[ab] : \sim(a \cap b \circ \Lambda) .\supset. \sim(\mathbf{Kl}(a) \wedge \mathbf{Kl}(b) \circ \Lambda)$

PR $[ab] : \mathrm{Hp}(1) .\supset.$

$[\exists A]$.

2. $A \varepsilon a \cap b$. [1; **ON**]
3. $A \varepsilon a$.
4. $A \varepsilon b$. [2; **ON**]
5. $A \varepsilon \mathbf{el}(\mathbf{Kl}(a))$. [3; *T11*; *T5*]
6. $A \varepsilon \mathbf{el}(\mathbf{Kl}(b))$. [4; *T11*; *T5*]
7. $A \varepsilon \mathbf{el}(\mathbf{Kl}(a)) \cap \mathbf{el}(\mathbf{Kl}(b))$. [5; 6; **ON**]
8. $A \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(\mathbf{Kl}(a)) \cap \mathbf{el}(\mathbf{Kl}(b))))$. [7; *T11*; *T5*]
9. $A \varepsilon \mathbf{el}(\mathbf{Kl}(a) \wedge \mathbf{Kl}(b))$. [8; *D5*]

$\sim(\mathbf{Kl}(a) \wedge \mathbf{Kl}(b) \circ \Lambda)$ [9; **ON**]

T217 $[ab] : \mathbf{Kl}(a) \wedge \mathbf{Kl}(b) \circ \Lambda .\supset. a \cap b \circ \Lambda$ [*T216*]

T218 $[ab] : \mathbf{Kl}(a) \wedge \mathbf{Kl}(b) \circ \Lambda . \mathsf{Fin}\{a\} . \mathsf{Fin}\{b\} . \sim(\rightarrow \{a\}) .\supset.$
$\mathsf{Cd}\{a * b\} = \mathsf{Cd}\{a\} \cdot \mathsf{Cd}\{b\}$

PR $[ab]:\mathrm{Hp}(4)\ .\supset.$

$[\exists A].$

5. $A\,\varepsilon\,a.$ [4; **ON**]

6. $\sim(A = a).$ [4; **ON**]

7. $\mathrm{Cd}\,\{a - A\} + 1 = \mathrm{Cd}\,\{a\}.$ [5; **ON**]

8. $\mathrm{Cd}\,\{a - A\} < \mathrm{Cd}\,\{a\}.$ [7; 2; **ON**]

9. $\mathrm{Cd}\,\{(a - A) * b\} = \mathrm{Cd}\,\{a - A\}\cdot\mathrm{Cd}\,\{b\}.$ [3; 8; **IH**]

10. $(a - A) * b \cup A * b \circ a * b.$ [*D26*]

11. $(a - A) * b \cap A * b \circ \Lambda.$ [1; *T217*; *D26*; *T217*]

12. $\mathrm{Cd}\,\{(a - A) * b\} + \mathrm{Cd}\,\{A * b\} = \mathrm{Cd}\,\{a * b\}.$ [10; 11; **ON**]

13. $\mathrm{Cd}\,\{a - A\}\cdot\mathrm{Cd}\,\{b\} + \mathrm{Cd}\,\{b\} = \mathrm{Cd}\,\{a * b\}.$ [9; 12; *T215*; 3; 5; 1]

14. $(\mathrm{Cd}\,\{a - A\} + 1)\cdot\mathrm{Cd}\,\{b\} = \mathrm{Cd}\,\{a * b\}.$ [13; **ON**]

$\mathrm{Cd}\,\{a * b\} = \mathrm{Cd}\,\{a\}\cdot\mathrm{Cd}\,\{b\}$ [7; 14; **ON**]

T219 $[ab]:\mathbf{Kl}(a)\wedge\mathbf{Kl}(b)\circ\Lambda.\ \mathrm{Fin}\,\{a\}.\ \mathrm{Fin}\,\{b\}\ .\supset.\ \mathrm{Cd}\,\{a * b\} = \mathrm{Cd}\,\{a\}\cdot\mathrm{Cd}\,\{b\}$
[*T215*; *T218*; *D26*]

T220 $[ABa]:\mathbf{sbstm}\ \{a\}.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A))\ .\supset.$
$\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) \cup \mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) * B \cup B \subset a$ [1; *D25*; *D26*; **ON**]

T221 $[ABCa]:\mathbf{sbstm}\ \{a\}.\,A\,\varepsilon\,a\,.\,C\,\varepsilon\,a\,.\,B\,\varepsilon\,\mathbf{at}(\mathbf{Cm}\,(A))\,.$
$B\,\varepsilon\,\mathbf{pr}\,(C)\ .\supset.\ C\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) * B$

PR $[ABCa]:\mathrm{Hp}(5)\ .\supset.$

6. $C\backslash B\,\varepsilon\,C\backslash B.$ [1; 3; 4; 5; *T45*; *D25*]

7. $B\,\varepsilon\,\mathbf{el}\,(\mathbf{Cm}(A)).$ [4; *D10*]

8. $\mathbf{Cm}\,(A)\backslash B\circ\mathbf{Kl}\,(\mathbf{el}\,(\mathbf{Cm}\,(A))\cap\mathbf{ex}\,(B)).$ [7; *T46*]

9. $A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)\circ A\,\mathbf{v}\,\mathbf{Kl}\,(\mathbf{el}\,(\mathbf{Cm}\,(A))\cap\mathbf{ex}\,(B)).$ [8; **ON**]

10. $A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)\circ\mathbf{Kl}\,(\mathbf{el}\,(A)\cup(\mathbf{el}\,(\mathbf{Cm}\,(A))\cap\mathbf{ex}\,(B))).$ [9; *D4*; *T197*; *T13*]

11. $C\backslash B\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)).$ [6; 10; **BA**]

12. $C\backslash B\,\mathbf{v}\,B\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) * B.$ [11; *D26*; **BA**]

$C\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) * B$ [12; *T44*]

T222 $[ABCa]:\mathbf{sbstm}\ \{a\}.\,A\,\varepsilon\,a\,.\,C\,\varepsilon\,a\,.\,B\,\varepsilon\,\mathbf{at}(\mathbf{Cm}\,(A))\,.\,B\,\varepsilon\,\mathbf{ex}\,(C)\ .\supset.$
$C\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B))$

PR $[ABCa]:\mathrm{Hp}(5)\ .\supset.$

6. $B\,\varepsilon\,\mathbf{el}\,(\mathbf{Cm}(A)).$ [4; *D10*]

7. $A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)\circ A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\wedge\mathbf{Cm}\,(B)).$ [6; *T47*]

8. $A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)\circ(A\,\mathbf{v}\,\mathbf{Cm}\,(A)\wedge(A\,\mathbf{v}\,\mathbf{Cm}\,(B)).$ [7; **BA**]

9. $A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)\circ\mathbf{Un}\wedge(A\,\mathbf{v}\,\mathbf{Cm}\,(B)).$ [8; *D6*]

10. $A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)\circ A\,\mathbf{v}\,\mathbf{Cm}\,(B).$ [*T10*; 9; *D3*; *T63*]

11. $C\,\varepsilon\,\mathbf{el}\,(\mathbf{Cm}\,(B)).$ [5; *T25*; *T27*]

$C\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B))$ [10; 11; 1; 2; 3; *D25*]

T223 $[ABCa]:\mathbf{sbstm}\ \{a\}.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,\mathbf{at}(\mathbf{Cm}\,(A))\,.\,C\,\varepsilon\,a\ .\supset.$
$C\,\varepsilon\,\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B) * B \cup \mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) \cup B$ [*T39*; *T221*; *T222*]

T224 $[ABa]:\mathbf{sbstm}\ \{a\}.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,\mathbf{at}(\mathbf{Cm}\,(A))\ .\supset.$
$a\circ\mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) * B \cup \mathbf{el}\,(A\,\mathbf{v}\,(\mathbf{Cm}\,(A)\backslash B)) \cup B$ [*T220*; *T223*]

T225 $[a]:\mathbf{sbstm}\ \{a\}.\ \mathrm{Fin}\,\{a\}.\sim(\rightarrow\{a\})\ .\supset.\ [\exists n].\ \mathrm{Cd}\,\{a\} = 2^n - 1$

PR $[a]\therefore\mathrm{Hp}(3)\ .\supset::$

$[\exists AB]::$

4. $A\,\varepsilon\,a.$

5. $\sim(A\,\varepsilon\,\mathbf{Un}).$

$\}$ [3; 1; *D25*]

6. $B \varepsilon a.$
7. $B \varepsilon \mathbf{Cm}(A) \therefore$ $\Big\}$ [3; 4; 5; *D6*]

$[\exists C] \therefore$

8. $C \varepsilon \mathbf{at}(\mathbf{Cm}(A)).$
9. $C \varepsilon a:$ $\Big\}$ [1; 2; 7; *T206*]

$[\exists D]:$

10. $D \varepsilon a.$
11. $D \varepsilon A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash C).$ [1; 4; 8; *D4*]
12. $C \varepsilon \mathbf{ex}(D).$ [8; *D11*; *T27*; *T30*]
13. $\mathbf{el}(D) \Subset a.$ [12; 1; 9; *T33*]
14. $\mathrm{Cd}\,\{\mathbf{el}(D)\} < \mathrm{Cd}\,\{a\}.$ [2; 13; **ON**]

$[\exists m].$

15. $\mathrm{Cd}\,\{\mathbf{el}(D)\} = 2^m - 1.$ [14; **IH**]
16. $\mathrm{Cd}\,\{\mathbf{el}(D) * C\} = \mathrm{Cd}\,\{\mathbf{el}(D)\}.$ [15; 12; *T215*]
17. $\mathbf{el}(D) * C \cap \mathbf{el}(D) \circ \wedge.$
18. $\mathbf{el}(D) * C \cap C \circ \wedge.$
19. $\mathbf{el}(D) \cap C \circ \wedge.$ $\Big\}$ [12; *D26*]
20. $\mathrm{Cd}\,\{\mathbf{el}(D) \cup \mathbf{el}(D) * C \cup C\} =$
$\mathrm{Cd}\,\{\mathbf{el}(D)\} + \mathrm{Cd}\,\{\mathbf{el}(D) * C\} + \mathrm{Cd}\,\{C\}.$
[17; 18; 19; **ON**]
21. $\mathbf{el}(D) \cup \mathbf{el}(D) * C \cup C \circ a.$ [1; 4; 8; *T224*]
22. $\mathrm{Cd}\,\{a\} = \mathrm{Cd}\,\{\mathbf{el}(D)\} + \mathrm{Cd}\,\{\mathbf{el}(D) * D\} + \mathrm{Cd}\,\{C\}.$
[20; 21]
23. $\mathrm{Cd}\,\{a\} = 2^m - 1 + 2^m - 1 + 1.$
[22; 15; *T215*; **ON**]
24. $\mathrm{Cd}\,\{a\} = 2^{m+1} - 1 ::$ [23; **ON**]

$[\exists n].\, \mathrm{Cd}\,\{a\} = 2^n - 1$ [24; *050*]

T226 $[AB]: B \varepsilon \mathbf{el}(\mathbf{Cm}(A)) \;.\!\supset\!.\; A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash B) \circ \mathbf{Cm}(B)$

PR $[AB]: \mathrm{Hp}(1) \;.\!\supset\!.$

2. $\mathbf{Cm}(A) \backslash B \circ \mathbf{Cm}(A) \wedge \mathbf{Cm}(B).$ [1; *T47*]
3. $A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash B) \circ A \,\mathbf{v}\, (\mathbf{Cm}(A) \wedge \mathbf{Cm}(B)).$ [2; **ON**]
4. $A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash B) \circ (A \,\mathbf{v}\, \mathbf{Cm}(A)) \wedge (A \,\mathbf{v}\, \mathbf{Cm}(B)).$ [3; **BA**]
5. $A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash B) \circ \mathbf{Un} \wedge (A \,\mathbf{v}\, \mathbf{Cm}(B)).$ [4; *D6*]
6. $A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash B) \circ A \,\mathbf{v}\, \mathbf{Cm}(B).$ [5; *T10*; *T60*]
7. $A \varepsilon \mathbf{el}(\mathbf{Cm}(B)).$ [1; **BA**]

$A \,\mathbf{v}\, (\mathbf{Cm}(A) \backslash B) \circ \mathbf{Cm}(B)$ [6; 7; *T63*]

T227 $[ABa]: \mathbf{sbstm}\ \{a\}.\, A \varepsilon a.\, B \varepsilon \mathbf{at}(\mathbf{Cm}(A)) \;.\!\supset\!.\; a \circ \mathbf{el}(\mathbf{Cm}(B))$
$\cup\ \mathbf{el}(\mathbf{Cm}(B)) * B \cup B$ [*T224*; *T226*; *D10*]

T228 $[an]: n > 1.\, \mathrm{Cd}\,\{a\} = 2^n - 1.\, \mathbf{sbstm}\ \{a\} \;.\!\supset\!.\; \mathrm{Cd}\,\{\mathbf{atm}_a\} = n$

PR $[an]: \mathrm{Hp}(3) \;.\!\supset\!.$

$[\exists A].$

4. $A \varepsilon a.$
5. $A \varepsilon \mathbf{at}(\mathbf{Kl}(a)).$ $\Big\}$ [1; 2; 3; *T206*]
6. $A \neq \mathbf{Kl}(a).$ [1; 2; 5]
7. $\mathbf{Cm}(A) \varepsilon a.$ [6; 3; *D6*; *D25*]
8. $\mathbf{el}(\mathbf{Cm}(A)) \Subset a.$ [6; 7; *T207*]
9. $\mathbf{sbstm}\ \{\mathbf{el}(\mathbf{Cm}(A))\}.$ [8; *T200*]

10. $\mathbf{el}(\mathbf{Cm}(A)) \cup \mathbf{el}(\mathbf{Cm}(A)) * A \cup A \circ a$. [3; 7; 5; *T36*; *T227*]

11. $\mathrm{Cd}\,\{\mathbf{el}(\mathbf{Cm}(A)) \cup \mathbf{el}(\mathbf{Cm}(A)) * A \cup A\} = \mathrm{Cd}\,\{a\}$. [9; **ON**]

12. $\mathbf{el}(\mathbf{Cm}(A)) \cap \mathbf{el}(\mathbf{Cm}(A)) * A \circ \Lambda$. [*D6*; *D26*]

13. $\mathbf{el}(\mathbf{Cm}(A)) \cap A \circ \Lambda$. [*D6*]

14. $\mathbf{el}(\mathbf{Cm}(A)) * A \cap A \circ \Lambda$. [*D6*; *D26*]

15. $\mathrm{Cd}\,\{\mathbf{el}(\mathbf{Cm}(A))\} + \mathrm{Cd}\,\{\mathbf{el}(\mathbf{Cm}(A)) * A\} + \mathrm{Cd}\,\{A\} = \mathrm{Cd}\,\{a\}$.
[11-14; **ON**]

16. $2 \cdot \mathrm{Cd}\,\{\mathbf{el}(\mathbf{Cm}(A))\} + 1 = 2^n - 1$. [5; *T215*; 2]

17. $\mathrm{Cd}\,\{\mathbf{el}(\mathbf{Cm}(A))\} = 2^{n-1} - 1$. [16; **ON**]

18. $\mathrm{Cd}\,\{\mathbf{atm}_{\mathbf{el}(\mathbf{Cm}(A))}\} = n - 1$. [8; 17; **IH**]

19. $\mathbf{atm}_a \circ \mathbf{atm}_{\mathbf{el}(\mathbf{Cm}(A))} \cup A$. [*T39*; *T203*; 5; 10]

20. $\mathrm{Cd}\,\{\mathbf{atm}_a\} = n - 1 + 1 = n$. [18; 19; **ON**]

$\mathrm{Cd}\,\{\mathbf{atm}_a\} = n$ [20]

T229 $[an] : n \geqslant 0 \,.\, \mathrm{Cd}\,\{a\} = 2^n - 1 \,.\, \mathbf{sbstm}\,\{a\} \,.\!\supset\!.\, \mathrm{Cd}\,\{\mathbf{atm}_a\} = n$
[*T204*; *T228*; *T199*]

We now define the terms which will be primitive provided a certain number of objects exist. We begin this with some definitions which generalize the idea of **pr** and **lnk**. We notice that, ultimately, our primitive terms defined on a name say that no individuals of that name are outside one another.

The definitions of **cl**$(2,a)$ and **cl**(n,a) do not follow the rule of definition given by Leśniewski. It should be noted that the definition scheme may be converted to a proper mereological definitions by using the method of Frege for reducing inductive definitions to proper definitions.

AD2 $[ABan] : \mathbf{sfc}\,\{ABan\} \,.\!\equiv\!.\, A \,\varepsilon\, a \,.\, B \,\varepsilon\, a \,.\, \mathbf{sbstm}\,\{a\} \,.\, \mathrm{Cd}\,\{a\} = 2^n - 1$.

D27 $[a] :: \mathbf{ch}(a) \,.\!\equiv\therefore [AB] \therefore A \,\varepsilon\, a \,.\, B \,\varepsilon\, a \,.\, A \ddagger B \,.\!\supset: A \,\varepsilon\, \mathbf{pr}(B) \,.\!\vee\!.\, B \,\varepsilon\, \mathbf{pr}(A)$

D28 $[a] \therefore \mathbf{fl}(a) \,.\!\equiv: [AB] : A \,\varepsilon\, a \,.\, B \,\varepsilon\, a \,.\, A \ddagger B \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{lnk}(B)$

D29 $[a] \therefore \mathbf{cl}(a) \,.\!\equiv: \mathbf{ch}(a) \,.\!\vee\!.\, \mathbf{fl}(a) : \sim(\rightarrow \{a\})$

D30 $[a] : \mathbf{cl}(2a) \,.\!\equiv\!.\, [\exists AB] \,.\, A \,\varepsilon\, a \,.\, B \,\varepsilon\, a \,.\, A \ddagger B \,.\, \mathbf{cl}(A \cup B)$

T230 $[AB] : A \,\varepsilon\, \mathbf{ex}(B) \,.\!\supset\!.\, \sim(\mathbf{cl}(A \cup B)) \,.\, A \ddagger B$

PR $[AB] : \mathrm{Hp}(1) \,.\!\supset\!.$

2. $A \ddagger B$. [1; *T33*]

$$\left.\begin{array}{ll} 3. & A \,\varepsilon\, A \cup B \,. \\ 4. & B \,\varepsilon\, A \cup B \,. \end{array}\right\} \quad [\mathbf{ON}]$$

$$\left.\begin{array}{ll} 5. & \sim(A \,\varepsilon\, \mathbf{pr}(B)) \,. \\ 6. & \sim(B \,\varepsilon\, \mathbf{pr}(A)) \,. \end{array}\right\} \quad [1;\ T33]$$

7. $\sim(\mathbf{ch}(A \cup B))$. [2; 3; 4; 5; 6; *D27*]

8. $\sim(A \,\varepsilon\, \mathbf{lnk}(B))$. [1; *T33*]

9. $\sim(\mathbf{fl}(A \cup B))$. [2; 3; 4; 8; *D28*]

10. $\sim(\mathbf{cl}(A \cup B))$. [7; 9; *D29*]

$\sim(\mathbf{cl}(A \cup B)) \,.\, A \ddagger B$ [2; 10]

T231 $[AB] : \sim(\mathbf{cl}(A \cup B)) \,.\, A \ddagger B \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{ex}(B)$

PR $[AB] : \mathrm{Hp}(2) \,.\!\supset\!.$

$$\left.\begin{array}{ll} 3. & \sim(\mathbf{ch}(A \cup B)) \,. \\ 4. & \sim(\mathbf{fl}(A \cup B)) \,. \end{array}\right\} \quad [1;\ 2;\ D29]$$

$$\left.\begin{array}{ll}5. & \sim(A\,\varepsilon\,\mathbf{pr}(B))\,.\\ 6. & \sim(B\,\varepsilon\,\mathbf{pr}(A))\,.\end{array}\right\}$$ [3; *D27*]

7. $\sim(A\,\varepsilon\,\mathbf{lnk}(B))\,.$ [4; *D28*]

$A\,\varepsilon\,\mathbf{ex}(B)$ [2; 5; 6; 7; *T33*]

T232 $[AB]:A\,\varepsilon\,\mathbf{ex}(B)\ .\equiv.\ A\neq B\,.\sim(\mathbf{cl}(A\cup B))$ [*T230*; *T231*]

Hence, **cl** is a primitive term.

T233 $[ab]:\mathbf{ch}(a)\,.\,b\subset a\ .\supset.\,\mathbf{ch}(b)$ [*D27*]

T234 $[ab]:\mathbf{fl}(a)\,.\,b\subset a\ .\supset.\ \mathbf{fl}(b)$ [*D28*]

T235 $[ab]:\mathbf{cl}(a)\,.\,b\subset a\,.\sim(\rightarrow\{b\})\ .\supset.\ \mathbf{cl}(b)$ [*D29*; *T233*; *T234*]

T236 $[ABC]\therefore A\,\varepsilon\,\mathbf{pr}(B)\,.\,A\,\varepsilon\,\mathbf{pr}(C)\ .\supset:\ B=C\ .\mathrm{v}.\ B\,\varepsilon\,\mathbf{pr}(C)\ .\mathrm{v}.$
$C\,\varepsilon\,\mathbf{pr}(B)\ .\mathrm{v}.\ B\,\varepsilon\,\mathbf{lnk}(C)$ [*D7*; *T33*]

T237 $[ab]:\mathbf{ch}(a)\,.\,\mathbf{ch}(b)\ .\supset.\ \mathbf{ch}(a\cap b)$

PR $[ab]:\mathrm{Hp}(2)\ .\supset.$

3. $a\cap b\subset a\,.$ [1; 2; **ON**]

$\mathbf{ch}(a\cap b)$ [1; 3; *T233*]

T238 $[AB]:A\,\varepsilon\,\mathbf{ex}(B)\ .\equiv.\ A\neq B\,.\sim(\mathbf{cl}(2A\cup B))$ [*T232*; *D30*]

Hence $\mathbf{cl}(2a)$ is primitive.

T239 $[a]\therefore \mathbf{cl}(a)\ .\equiv:\ \sim(\rightarrow\{a\}):[AB]:A\,\varepsilon\,a\,.\,B\,\varepsilon\,a\,.\,A\neq B\ .\supset.\ \sim(A\,\varepsilon\,\mathbf{ex}(B))$
[*D27-D29*; *T33*]

T240 $[a]:\mathbf{cl}(a)\ .\supset.\ \mathbf{cl}(2a)\,.$ [*D29*; *D30*]

T241 $[AB]\therefore A\neq B\ .\supset:\ \mathbf{cl}(A\cup B)\ .\equiv.\ \mathbf{cl}(2A\cup B)$ [*D30*; *T240*]

DS1 $[an]\therefore \mathbf{cl}(na)\ .\equiv:\ n>2\,.\,\mathbf{cl}(n-1a):[A]:A\,\varepsilon\,a\ .\supset.\ \mathbf{cl}(n-1a-A)$

T242 $[an]:\mathbf{cl}(na)\ .\supset.\ \mathbf{cl}(n-1a)$ [*DS1*]

T243 $[an]\therefore \mathbf{cl}(na)\ .\supset:\ [m]:2\leqslant m\leqslant n\ .\supset.\ \mathbf{cl}(ma)$ [*T242*]

T244 $[a]:A\,\varepsilon\,A\,.\sim(A\,\varepsilon\,a)\ .\supset.\ a-A\circ a$ [**ON**]

T245 $[an]:\mathbf{cl}(na)\,.\,A\,\varepsilon\,A\,.\sim(A\,\varepsilon\,a)\ .\supset.\ \mathbf{cl}(na-A)$ [*T244*]

T246 $[an]\therefore \mathbf{cl}(na)\ .\equiv:\ n>2\,.\,\mathbf{cl}(n-1a):[A]:A\,\varepsilon\,A\ .\supset.\ \mathbf{cl}(n-1a-A)$
[*DS1*; *T245*]

T247 $[Aa]:A\circ\Lambda.\supset.\ a-A\circ a$ [**ON**]

T248 $[an]\therefore \mathbf{cl}(na)\ .\equiv:\ n>2\,.\,\mathbf{cl}(n-1a):[A]:\rightarrow\{A\}\ .\supset.\ \mathbf{cl}(n-1a-A)$
[*T246*; *T247*]

T249 $[ab]:\mathbf{cl}(a)\,.\sim(\rightarrow\{a\cap b\})\ .\supset.\ \mathbf{cl}(a\cap b)$

PR $[ab]::\mathrm{Hp}(2)\ .\supset\therefore$

3. $[AB]\therefore A\,\varepsilon\,a\,.\,B\,\varepsilon\,a\,.\,A\neq B\ .\supset:\ A\,\varepsilon\,\mathbf{pr}(B)\ .\mathrm{v}.\ B\,\varepsilon\,\mathbf{pr}(A)\ .\mathrm{v}.\ A\,\varepsilon\,\mathbf{lnk}(B)\therefore$
[1; *T239*; *T33*]

4. $[AB]\therefore A\,\varepsilon\,a\cap b\,.\,B\,\varepsilon\,a\cap b\,.\,A\neq B\ .\supset:\ A\,\varepsilon\,\mathbf{pr}(B)\ .\mathrm{v}.\ B\,\varepsilon\,\mathbf{pr}(A)\ .\mathrm{v}.$
$A\,\varepsilon\,\mathbf{lnk}(B)\therefore$ [3; **ON**]

$\mathbf{cl}(a\cap b)$ [3; 4; *T239*; *D29*]

T250 $[an]\therefore \mathbf{cl}(na)\ .\equiv:\ n>2:[A]\rightarrow\{A\}\ .\supset.\ \mathbf{cl}(n-1a-A)$ [*T247*; *T248*]

T251 $[AB]:A\,\varepsilon\,\mathbf{ex}(B)\,.\,\mathrm{Cd}\,\{a\}=2\,.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,a\ .\supset.\ \sim(\mathbf{cl}(2a))$

PR $[AB]:\mathrm{Hp}(4)\ .\supset.$

5. $A\neq B\,.$ [1; *T33*]

6. $a\circ A\cup B\,.$ [2; 3; 4; 5; **ON**]

7. $\sim(\mathbf{cl}(2A\cup B))\,.$ [1; *T238*]

$\sim(\mathbf{cl}(2\,a))$ [6; 7]

T252 $[ABan] : n \geqslant 3 . \mathrm{Cd}\{a\} = n . A \varepsilon a . B \varepsilon a . A \neq B .\supset.$
$[\exists bC] . b \subset a . A \varepsilon b . B \varepsilon b . \mathrm{Cd}\{b\} = n - 1 . C \varepsilon a .$
$\sim (C \varepsilon b) . a \circ b \cup C .$
PR $[ABan] : \mathrm{Hp}(5) .\supset.$
$[\exists C] .$
6. $C \varepsilon a .$
7. $C \neq A .$ [1; 2; 5]
8. $C \neq B .$
9. $a - C \subset a .$ [**ON**]
10. $A \varepsilon a - C .$ [3; 7]
11. $B \varepsilon a - C .$ [4; 8]
12. $\mathrm{Cd}\{a - C\} = n - 1 .$ [2; 6; **ON**]
13. $a \circ a - C \cup C .$ [**ON**]
14. $\sim (C \varepsilon a - C) .$ [**ON**]
$[\exists bC] . b \subset a . A \varepsilon b . B \varepsilon b . \mathrm{Cd}\{b\} = n - 1 . C \varepsilon a . \sim (C \varepsilon b) . a \circ b \cup C$
[9; 10; 11; 12; 13; 14]

T253 $[ABan] : A \varepsilon \mathbf{ex}(B) . \mathrm{Cd}\{a\} = n . n \geqslant 3 . A \varepsilon a . B \varepsilon a .\supset. \sim (\mathbf{cl}(na))$
PR $[ABan] : \mathrm{Hp}(5) .\supset.$
6. $A \neq B .$ [1; *T33*]
$[\exists bC] .$
7. $A \varepsilon b . B \varepsilon b . b \subset a . \mathrm{Cd}\{b\} = n - 1 .$ [2; 3; 4; 5; 6; *T252*]
8. $C \varepsilon a . \sim (C \varepsilon b) . a \circ b \cup C .$
9. $\sim (\mathbf{cl}(n - 1 b)) .$ [1; 7; **IH**]
10. $b \cup C - C \circ b .$ [8; **ON**]
11. $a - C \circ b .$ [8; 10; **ON**]
12. $\sim (\mathbf{cl}(n - 1 a - C)) .$ [9; 11]
13. $\sim (\mathbf{cl}(na)) .$ [8; 12; *DS1*]
$\sim (\mathbf{cl}(na))$ [13; *T251*]

T254 $[ABan] : n \geqslant 2 . \mathrm{Cd}\{a\} = n . A \varepsilon \mathbf{ex}(B) . A \varepsilon a . B \varepsilon a .\supset. \sim (\mathbf{cl}(na))$
[*T251*; *T253*]

T255 $[AB] \therefore A \varepsilon \mathbf{ex}(B) .\supset: [an] : n \geqslant 2 . \mathrm{Cd}\{a\} = n . A \varepsilon a . B \varepsilon a .$
$A \neq B .\supset. \sim (\mathbf{cl}(na))$ [*T254*; *T33*]

T256 $[a] : \mathbf{sbstm}\,\{a\} .\supset. \mathbf{Un} \circ \mathbf{Kl}(a)$ [*D3*]

T257 $[Aan] :: \mathbf{sfc}\,(AAan) . A \varepsilon \mathbf{pr}(\mathbf{Kl}(a)) .\supset\therefore [\exists B] \therefore A \varepsilon \mathbf{el}(B) .$
$B \varepsilon \mathbf{pr}(\mathbf{Kl}(a)) : [C] : B \varepsilon \mathbf{pr}(C) .\supset. C \varepsilon \mathbf{Kl}(a) .$
PR $[Aan] \because \mathrm{Hp}(2) .\supset::$
3. $A \varepsilon a .$ [1; *AD2*]
4. $\mathbf{Cm}(A) \varepsilon a ::$ [1; *AD2*; 2; 3; *D6*]
$[\exists BD] ::$
5. $D \varepsilon \mathbf{at}(\mathbf{Cm}(A)) .$ [1; *AD2*; 4; *T206*]
6. $B \varepsilon \mathbf{Cm}(D) .$ [1; *AD2*; 4; 5]
7. $B \varepsilon \mathbf{pr}(\mathbf{Kl}(a)) .$ [1; *AD2*; 5; 6]
8. $A \varepsilon \mathbf{el}(\mathbf{Cm}(D)) .$ [5; *D10*; **BA**]
9. $A \varepsilon \mathbf{el}(B) :$ [6; 8]
10. $[C] : B \varepsilon \mathbf{pr}(C) .\supset. C \backslash B \varepsilon a \therefore$ [1; *AD2*; *T45*]
$[\exists E] \therefore$
11. $[C] : B \varepsilon \mathbf{pr}(C) .\supset. E \varepsilon \mathbf{at}(C \backslash B) :$ [10; 1; *AD2*; *T206*]

12. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;E\,\varepsilon\,\mathbf{at}(C\wedge\mathbf{Cm}(B)):$ [11; *D10*; *T47*]

13. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;E\,\varepsilon\,\mathbf{at}(C\wedge D):$ [12; 6; *T23*]

14. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;E\,\varepsilon\,\mathbf{at}(D):$ [11; 5; *T18*; *T36*]

15. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;E=D\;\therefore$ [5; 14]

16. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;B\vee D\,\varepsilon\,\mathbf{el}(C):$ [15; 11; *D10*; *D11*; *D4*]

17. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;\mathbf{Un}\,\varepsilon\,\mathbf{el}(C):$ [6; *D6*; 16]

18. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;\mathbf{Un}=C:$ [17; *T6*; *T10*]

19. $[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;C\,\varepsilon\,\mathbf{Kl}(a)::$ [18; 1; *AD2*; *T256*]

$[\exists B]\therefore A\,\varepsilon\,\mathbf{el}(B)\,.\,B\,\varepsilon\,\mathbf{pr}(\mathbf{Kl}(a)):[C]:B\,\varepsilon\,\mathbf{pr}(C)\;.\supset.\;C\,\varepsilon\,\mathbf{Kl}(a)\,.$ [9; 7; 19]

T258 $[ABa]:\mathbf{sfc}\,\{ABa2\}\,.\,A\,\varepsilon\,\mathbf{pr}(B)\;.\supset.\;[\exists b]\,.\,b\subset a\,.\,A\,\varepsilon\,b\,.$
$B\,\varepsilon\,b\,.\,\mathbf{ch}(b)\,.\,\mathrm{Cd}\,\{b\}=2$

PR $[ABa]:\mathrm{Hp}(2)\;.\supset.$

$$\left.\begin{array}{ll}3. & A\,\varepsilon\,A\cup B\,.\\ 4. & B\,\varepsilon\,A\cup B\,.\end{array}\right\}\qquad[1;\ AD2;\ \mathbf{ON}]$$

5. $A\neq B\,.$ [2; *T33*]

6. $\mathrm{Cd}\,\{A\cup B\}=2\,.$ [3; 4; 5; **ON**]

7. $A\cup B\subset a\,.$ [1; **ON**]

8. $\mathbf{ch}(A\cup B)\,.$ [2; 3; 4; 5; *D27*]

$[\exists b]\,.\,b\subset a\,.\,A\,\varepsilon\,b\,.\,B\,\varepsilon\,b\,.\,\mathbf{ch}(b)\,.\,\mathrm{Cd}\,\{b\}=2$ [3; 4; 6; 7; 8]

T259 $[ABan]:\mathbf{sfc}\,\{ABan\}\,.\,A\,\varepsilon\,\mathbf{pr}(B)\,.\,B\,\varepsilon\,\mathbf{Un}\;.\supset.\;[\exists b]\,.\;b\subset a\,.\,A\,\varepsilon\,b\,.$
$B\,\varepsilon\,b\,.\,\mathrm{Cd}\,\{b\}=n\,.\,\mathbf{ch}(b)$

PR $[ABan]::\mathrm{Hp}(3)\;.\supset\therefore$

$[\exists C]\therefore$

$$\left.\begin{array}{ll}4. & A\,\varepsilon\,\mathbf{el}(C)\,.\\ 5. & C\,\varepsilon\,\mathbf{pr}(B):\\ 6. & [D]:C\,\varepsilon\,\mathbf{pr}(D)\;.\supset.\;D=B:\end{array}\right\}\qquad[1;\ 2;\ 3;\ T256;\ T257]$$

7. $\mathbf{sbstm}\;\{\mathbf{el}(C)\}\,.$ [5; *T200*]

8. $\mathbf{el}(C)\subsetdot a\,.$ [5; 1; *AD2*]

9. $\mathrm{Cd}\,\{\mathbf{el}(C)\}<\mathrm{Cd}\,\{a\}\,.$ [8; **ON**; *AD2*]

10. $B\backslash C\,\varepsilon\,B\backslash C$ [5; *T45*]

11. $\mathbf{Un}\backslash C\,\varepsilon\,\mathbf{Cm}(C):$ [*D3*; *D11*; *D6*; **BA**]

12. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;E\,\varepsilon\,\mathbf{ex}(C):$ [10; *D6*; *T24*]

13. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;E\vee C\,\varepsilon\,E\vee C:$ [12; *D7*; *D4*]

14. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;C\,\varepsilon\,\mathbf{pr}(E\vee C):$ [12; 13; *T19*; *D7*]

15. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;E\vee C\,\varepsilon\,\mathbf{Un}:$ [3; 6; 14]

16. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;E\wedge C\circ\Lambda:$ [12; *T24*]

17. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;E\,\varepsilon\,\mathbf{Cm}(C):$ [12; 5; 15; 16; *D6*]

18. $[E]:E\,\varepsilon\,\mathbf{el}(B\backslash C)\;.\supset.\;E=B\backslash C:$ [17; 11; 3]

19. $B\backslash C\,\varepsilon\,\mathbf{atm}\,.$ [18; *D9*]

20. $\mathbf{Cm}(C)\,\varepsilon\,\mathbf{atm}\,.$ [3; 11]

21. $a\circ\mathbf{el}(C)\cup\mathbf{el}(C)*\mathbf{Cm}(C)\cup\mathbf{Cm}(C)\,.$ [1; *AD2*; 20; *T224*]

22. $\mathrm{Cd}\,\{a\}=2^n-1\,.$ [1; *AD2*]

23. $\mathbf{el}(C)\cap\mathbf{el}(C)*\mathbf{Cm}(C)\circ\Lambda.$ [*D26*; *D6*]

24. $\mathbf{el}(C)\cap\mathbf{Cm}(C)\circ\Lambda.$ [*D6*]

25. $\mathbf{el}(C)*\mathbf{Cm}(C)\cap\mathbf{Cm}(C)\circ\Lambda.$ [*D26*; *D6*]

26. $\mathrm{Cd}\,\{a\}=\mathrm{Cd}\,\{\mathbf{el}(C)*\mathbf{Cm}(C)\}+\mathrm{Cd}\,\{\mathbf{el}(C)\}+\mathrm{Cd}\,\{\mathbf{Cm}(C)\}\,.$
[21; 23; 24; 25]

27. $2^n - 1 = 2 \cdot \mathrm{Cd}\,\{\mathbf{el}(C)\} + 1\,.$ [26; 22; *T275*]

28. $\mathrm{Cd}\,\{\mathbf{el}(C)\} = 2^{n-1} - 1 \therefore$ [27; **ON**]

$[\exists d] \therefore$

29. $d \subset \mathbf{el}(C)\,.\,C \varepsilon d\,.\,A \varepsilon d\,.\,\mathbf{ch}(d)\,.\,\mathrm{Cd}\,\{d\} = n - 1\,.$ [28; **IH**; *T258*]

30. $d \cup B \subset a\,.$ [1; *AD2*; 8]

31. $d \cap B \circ \Lambda.$ [1; 5; 29]

32. $\mathrm{Cd}\,\{d \cup B\} = n.$ [29; 31; 3; **ON**]

33. $A \varepsilon d \cup B\,.$ [29; **ON**]

34. $B \varepsilon d \cup B :$ [3; **ON**]

35. $[D] : D \varepsilon d\; .\supset.\; D \varepsilon \mathbf{pr}(B) :$ [5; 29; *A1*]

36. $\mathbf{ch}(d \cup B) \therefore$ [*D27*; 29; 35]

$[\exists b]\,.\,b \subset a\,.\,A \varepsilon b\,.\,B \varepsilon b\,.\,\mathrm{Cd}\,\{b\} = n\,.\,\mathbf{ch}(b)\,.$ [30; 32; 33; 34; 36]

T260 $[ABan] : \mathbf{sfc}\,\{ABan\}\,.\,A \varepsilon \mathbf{pr}(B)\,.\sim(B \varepsilon \mathbf{Un})\; .\supset.\; [\exists b]\,.\,b \subset a\,.\,A \varepsilon b\,.$
$B \varepsilon b\,.\,\mathbf{ch}(b)\,.\,\mathrm{Cd}\,\{b\} = n$

PR $[ABan] :\therefore \mathrm{Hp}(3)\; .\supset ::$

4. $B \varepsilon \mathbf{pr}(\mathbf{Un}) ::$ [1; *AD2*; 3; *T10*; *D1*]

$[\exists D] ::$

5. $B \varepsilon \mathbf{el}(D)\,.$

6. $D \varepsilon \mathbf{pr}(\mathbf{Un}) :$ [1; 4; *T257*]

7. $[C] : D \varepsilon \mathbf{pr}(C)\; .\supset.\; C \varepsilon \mathbf{Un} :$

8. $\mathbf{sbstm}\,\{\mathbf{el}(D)\}.$ [6; *T200*]

9. $\mathrm{Cd}\,\{\mathbf{el}(D)\} = 2^{n-1} - 1\,.$ [8; proofs of *T257*; *T259*]

10. $\mathbf{el}(D) \subsetdot a \therefore$ [1; *AD2*; 6]

$[\exists d] \therefore$

11. $d \subset \mathbf{el}(D)\,.\,A \varepsilon d\,.\,B \varepsilon d\,.\,\mathbf{ch}(d)\,.\,\mathrm{Cd}\,\{d\} = n - 1 :$
[**IH**; 2; 5; 6; 7; 8; 9; *T258*]

12. $[E] : E \varepsilon d\; .\supset.\; E \varepsilon \mathbf{pr}(\mathbf{Un}) :$ [1; *AD2*; 6; 11; *A1*]

13. $\mathbf{ch}(d \cup \mathbf{Un})\,.$ [11; 12; *D27*]

14. $d \cap \mathbf{Un} \circ \Lambda.$ [6; 11]

15. $A \varepsilon d \cup \mathbf{Un}\,.$

16. $B \varepsilon d \cup \mathbf{Un}\,.$ [11; **ON**]

17. $d \cup \mathbf{Un} \subset a ::$ [1; *AD2*; 6; 11]

$[\exists b]\,.\,b \subset a\,.\,A \varepsilon b\,.\,B \varepsilon b\,.\,\mathbf{ch}(b)\,.\,\mathrm{Cd}\,\{b\} = n$ [13; 14; 15; 16; 17; 11; **ON**]

T261 $[ABan] : n \geqslant 2\,.\,\mathbf{sfc}\,\{ABan\}\,.\,A \varepsilon \mathbf{pr}(B)\; .\supset.\; [\exists b]\,.\,b \subset a\,.\,A \varepsilon b\,.\,B \varepsilon b\,.$
$\mathbf{ch}(b)\,.\,\mathrm{Cd}\,\{b\} = n$ [*T250*; *T259*; *T260*]

T262 $[AB]\,.\,\mathbf{at}(A \wedge B) \cap \mathbf{at}(A \backslash (A \wedge B)) \circ \Lambda$ [*T42*; *D11*]

T263 $[AB]\,.\,\mathbf{at}(A \wedge B) \cap \mathbf{at}(B \backslash (A \wedge B)) \circ \Lambda$ [*T262*]

T264 $[AB]\,.\,\mathbf{at}(A \backslash (A \wedge B)) \cap \mathbf{at}(B \backslash (A \wedge B)) \circ \Lambda$ [*T42*; *D11*; *D11*]

T265 $[AB]\,.\,\mathbf{at}(A \wedge B) \cap \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda$ [*T42*; **BA**]

T266 $[AB]\,.\,\mathbf{at}(A \backslash (A \wedge B)) \cap \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda$ [*T42*; **BA**]

T267 $[AB]\,.\,\mathbf{at}(B \backslash (A \wedge B)) \cap \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda$ [*T266*]

T268 $[ABan] : \mathbf{sfc}\,\{ABan\}\; .\supset.\; \mathrm{Cd}\,\{\mathbf{at}(A \wedge B)\} + \mathrm{Cd}\,\{\mathbf{at}(A \backslash (A \wedge B)) +$
$\mathrm{Cd}\,\{\mathbf{at}(B \backslash (A \wedge B)) + \mathrm{Cd}\,\{\mathbf{at}(\mathbf{Cm}(A \vee B))\} = n$
[*T262*; *T263*; *T264*; *T265*; *T266*; *T267*; *T48*; *T229*]

AD3 $[AB]\,.\,\boldsymbol{\delta}(AB) = \mathrm{Cd}\,\{\mathbf{at}(A \wedge B)\}$

AD4 $[AB]\,.\,\boldsymbol{\alpha}(AB) = \mathrm{Cd}\,\{\mathbf{at}(A \backslash (A \wedge B))\}$

AD5 $[AB].\boldsymbol{\beta}(AB) = \mathrm{Cd}\,\{\mathbf{at}(B\backslash(A \wedge B))\}$

AD6 $[AB].\boldsymbol{\gamma}(AB) = \mathrm{Cd}\,\{\mathbf{at}(\mathbf{Cm}(A \mathbf{v} B))\}$

These definitions are merely auxiliary definitions used to simplify notation.

T269 $[Aan]: \mathbf{sfc}\,\{AAan\}.\,\mathrm{Cd}\,\{\mathbf{at}(A)\} = 1 .\supset.\ A\,\varepsilon\,\mathbf{Kl}(\mathbf{at}(A))$

PR $[Aan]: \mathrm{Hp}(2) .\supset.$

3. $A\,\varepsilon\,\mathbf{at}(A)$. [1; 2; *DA2*; *T210*]

4. $A\,\varepsilon\,\mathbf{Kl}(A)$. [3; *T5*]

5. $\mathbf{at}(A) \circ A$. [3; 2]

6. $\mathbf{Kl}(\mathbf{at}(A)) \circ \mathbf{Kl}(A)$. [5; **ON**]

$A\,\varepsilon\,\mathbf{Kl}(\mathbf{at}(A))$ [4; 6]

T270 $[AB]: B\,\varepsilon\,\mathbf{el}(A) .\supset.\ A = (A\backslash B)\,\mathbf{v}\,B$

PR $[AB]: \mathrm{Hp}(1) .\supset.$

2. $(A\backslash B)\,\mathbf{v}\,B = (A \wedge \mathbf{Cm}(B))\,\mathbf{v}\,B$. [1; *T47*; *D11*]

3. $(A\backslash B)\,\mathbf{v}\,B = (A\,\mathbf{v}\,B) \wedge (\mathbf{Cm}(B)\,\mathbf{v}\,B))$. [2; **BA**]

4. $(A\backslash B)\,\mathbf{v}\,B = (A\,\mathbf{v}\,B) \wedge \mathbf{Un}$. [3; *D6*]

5. $(A\backslash B)\,\mathbf{v}\,B = A\,\mathbf{v}\,B$. [4; *D5*; *D3*]

$(A\backslash B)\,\mathbf{v}\,B = A$ [5; 1; *T60*]

T271 $[AB]: B\,\varepsilon\,\mathbf{el}(A) .\supset.\ \mathbf{at}(A\backslash B) \circ \mathbf{at}(A) - \mathbf{at}(B)$

PR $[AB] \therefore \mathrm{Hp}(1) .\supset:$

2. $[C]: C\,\varepsilon\,\mathbf{at}(A\backslash B) .\equiv.\ C\,\varepsilon\,\mathbf{at}(A \wedge \mathbf{Cm}(B)):$ [1; *T47*]

3. $[C]: C\,\varepsilon\,\mathbf{at}(A\backslash B) .\equiv.\ C\,\varepsilon\,\mathbf{atm}\,.\,C\,\varepsilon\,\mathbf{el}(A \wedge \mathbf{Cm}(B)):$ [2; *D10*]

4. $[C]: C\,\varepsilon\,\mathbf{at}(A\backslash B) .\equiv.\ C\,\varepsilon\,\mathbf{atm}\,.\,C\,\varepsilon\,\mathbf{el}(A)\,.\,C\,\varepsilon\,\mathbf{el}(\mathbf{Cm}(B)):$
[3; *T17*; *T18*; *T4*]

5. $[C]: C\,\varepsilon\,\mathbf{at}(A\backslash B) .\equiv.\ C\,\varepsilon\,\mathbf{atm}\,.\,C\,\varepsilon\,\mathbf{el}(A)\,.\,C\,\varepsilon\,\mathbf{ex}(B):$ [4; *T27*]

6. $[C]: C\,\varepsilon\,\mathbf{at}(A\backslash B) .\equiv.\ C\,\varepsilon\,\mathbf{atm}\,.\,C\,\varepsilon\,\mathbf{el}(A)\,.\sim(C\,\varepsilon\,\mathbf{el}(B)):$ [5; *T39*]

7. $[C]: C\,\varepsilon\,\mathbf{at}(A\backslash B) .\equiv.\ C\,\varepsilon\,\mathbf{at}(A)\,.\sim(C\,\varepsilon\,\mathbf{at}(B)):$ [6; *D10*]

$\mathbf{at}(A\backslash B) \circ \mathbf{at}(A) - \mathbf{at}(B)$ [7; **ON**]

T272 $[Aanm]: \mathbf{sfc}\,\{AAan\}.\,\mathrm{Cd}\,\{\mathbf{at}(A)\} = m\,.\,m > 1 .\supset.\ A = \mathbf{Kl}(\mathbf{at}(A))$

PR $[Aanm]: \mathrm{Hp}(3) .\supset.$

$[\exists B].$

$$\left.\begin{array}{ll} 4. & B\,\varepsilon\,\mathbf{at}(A). \\ 5. & B \ddagger \mathbf{at}(A). \end{array}\right\} \quad [1;\ 2;\ 3;\ AD2]$$

6. $\mathbf{at}(A\backslash B) \circ \mathbf{at}(A) - \mathbf{at}(B)$. [4; *D10*; *T271*]

7. $\mathbf{at}(A\backslash B) \circ \mathbf{at}(A) - B$. [4; 5; 6; *T36*]

8. $\mathrm{Cd}\,\{\mathbf{at}(A\backslash B)\} = m - 1$. [7; 2; 3; 4]

9. $A\backslash B = \mathbf{Kl}(\mathbf{at}(A\backslash B))$. [8; **IH**]

10. $(A\backslash B)\,\mathbf{v}\,B = \mathbf{Kl}(\mathbf{at}(A\backslash B))\,\mathbf{v}\,B$. [9; **ON**]

11. $A = \mathbf{Kl}(\mathbf{at}(A\backslash B))\,\mathbf{v}\,B$. [4; 10; *T270*]

12. $A = \mathbf{Kl}(\mathbf{at}(A\backslash B) \cup B)$. [11; *D4*; *T13*]

13. $A = \mathbf{Kl}(\mathbf{at}(A) - B \cup B)$. [12; 7]

$A = \mathbf{Kl}(\mathbf{at}(A))$ [*T269*; 1; 8; 13; *050*]

T273 $[Aan]: \mathbf{sfc}\,\{AAan\} .\supset.\ A = \mathbf{Kl}(\mathbf{at}(A))$ [*T269*; *T272*]

T274 $[ABan]: \mathbf{sfc}\,\{ABan\}.\,A \wedge B \circ \Lambda .\supset.\ \mathrm{Cd}\,\{\mathbf{at}(A) * \mathbf{at}(B)\} =$
$\mathrm{Cd}\,\{\mathbf{at}(A)\} \cdot \mathrm{Cd}\,\{\mathbf{at}(B)\}$

PR $[ABan]: \mathrm{Hp}(2) .\supset.$

$$\left.\begin{array}{ll} 3. & A = \mathbf{Kl}(\mathbf{at}(A))\,. \\ 4. & B = \mathbf{Kl}(\mathbf{at}(B))\,. \end{array}\right\} \qquad [AD2;\ 1;\ T273]$$

5. $\mathbf{Kl}(\mathbf{at}(A)) \wedge \mathbf{Kl}(\mathbf{at}(B)) \circ \Lambda\,.$ [2; 3; 4]

$\mathsf{Cd}\,\{\mathbf{at}(A) * \mathbf{at}(B)\} = \mathsf{Cd}\,\{\mathbf{at}(A)\}\,.\,\mathsf{Cd}\,\{\mathbf{at}(B)\}$ [1; *AD2*; 5; *T219*]

T275 $[abc]\,.\,a*c \cup b*c \circ (a \cup b)*c$

PR $[abc]::$

1. $[D] \therefore D\,\varepsilon\,a*c \cup b*c\,.$

2. $\equiv:\ [\exists AC]\,.\,A\,\varepsilon\,a\,.\,C\,\varepsilon\,c\,.\,D = A \vee C :\!\vee\!: [\exists BC]\,.\,B\,\varepsilon\,b\,.\,C\,\varepsilon\,c\,.\,D = B \vee C :$ [*D26*]

3. $\equiv.\ [\exists EF]\,.\,E\,\varepsilon\,a \cup b\,.\,F\,\varepsilon\,c\,.\,D = E \vee F :$ [2; **ON**]

4. $\equiv.\ D\,\varepsilon\,(a \cup b)*c \therefore$ [2; *D26*]

$a*c \cup b*c \circ (a \cup b)*c$ [1; 2; 3; 4; **ON**]

T276 $[abc]\,.\,a*(b*c) \circ (a*b)*c$

PR $[abc] \therefore$

1. $[D] : D\,\varepsilon\,a*(b*c)\,.$

2. $\equiv.\ [\exists AE]\,.\,A\,\varepsilon\,a\,.\,E\,\varepsilon\,b*c\,.\,D = A \vee E\,.$ [*D26*]

3. $\equiv.\ [\exists ABC]\,.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,b\,.\,C\,\varepsilon\,c\,.\,D = A \vee (B \vee C)\,.$ [1; *D26*]

4. $\equiv.\ [\exists ABC]\,.\,A\,\varepsilon\,a\,.\,B\,\varepsilon\,b\,.\,C\,\varepsilon\,c\,.\,D = (A \vee B) \vee C\,.$ [2; *D4*]

5. $\equiv.\ [\exists FC]\,.\,F\,\varepsilon\,a*b\,.\,C\,\varepsilon\,c\,.\,D = F \vee C\,.$ [3; *D26*]

6. $\equiv.\ D\,\varepsilon\,(a*b)*c :$ [4; *D26*]

$a*(b*c) \circ (a*b)*c$ [1; 2; 3; 4; 5; 6; **ON**]

T277 $[ab] : a*b \circ b*a$ [*D26*; *T16*]

T278 $[abc] : \mathbf{Kl}(a) \wedge \mathbf{Kl}(b) \circ \Lambda\,.\,\mathbf{Kl}(b) \wedge \mathbf{Kl}(c) \circ \Lambda\,.\,\mathbf{Kl}(a) \wedge \mathbf{Kl}(c) \circ \Lambda\,.\ \mathsf{Fin}\,\{a\}\,.$
$\mathsf{Fin}\,\{b\}\,.\,\mathsf{Fin}\,\{c\}\,.\,\sim(a \circ \Lambda)\,.\,\sim(b \circ \Lambda)\,.\,\sim(c \circ \Lambda)\ .\supset.\ \mathsf{Cd}\,\{a*b*c\} =$
$\mathsf{Cd}\,\{a\}\,.\,\mathsf{Cd}\,\{b\}\,.\,\mathsf{Cd}\,\{c\}$ [*T219*; *T276*]

AD7 $[ABC] : C\,\varepsilon\,\boldsymbol{\Delta}(AB)\ .\equiv.\ A\,\varepsilon\,A\,.\,B\,\varepsilon\,B\,.\,[\exists D]\,.\,D\,\varepsilon\,\mathbf{at}(A \wedge B)\,.\,C = (A \wedge B)\backslash D\,.$

T279 $[ABamn] : \mathbf{sfc}(ABam)\,.\,\boldsymbol{\delta}(AB) = n\,.\,n > 1\ .\supset.$
$\mathsf{Cd}\,\{\boldsymbol{\Delta}(AB)\} = n$

PR $[ABamn] \therefore \mathrm{Hp}(3)\ .\supset:$

4. $\mathsf{Cd}\,\{\mathbf{at}(A \wedge B)\} = n :$ [2; *AD3*]

5. $[CD] : C\,\varepsilon\,\mathbf{at}(A \wedge B)\,.\,D\,\varepsilon\,\mathbf{at}(A \wedge B)\ .\supset.\ C = D\ .\equiv.\ (A \wedge B)\backslash C = (A \wedge B)\backslash D :$ [3; *T37*; *D11*; *T47*]

6. $\mathsf{Cd}\,\{\mathbf{at}(A \wedge B)\} = \mathsf{Cd}\,\{\boldsymbol{\Delta}(AB)\}\,.$ [5; **ON**]

$\mathsf{Cd}\,\{\boldsymbol{\Delta}(AB)\} = n\,.$ [4; 6]

T280 $[ABCD] : A\,\varepsilon\,\mathbf{lnk}(B)\,.\,C\,\varepsilon\,\mathbf{ex}(A \vee B)\,.\,D\,\varepsilon\,\mathbf{ex}(A \vee B)\ .\supset.\ A \vee C\,\varepsilon\,\mathbf{lnk}(B \vee D)$

PR $[ABCD] : \mathrm{Hp}(3)\ .\supset.$
$[\exists EFG]\,.$

4. $E\,\varepsilon\,\mathbf{el}(A)\,.$ [*D8*; 1]

5. $E\,\varepsilon\,\mathbf{el}(B)\,.$ [*D8*; 1]

6. $E\,\varepsilon\,\mathbf{el}(A \vee C)\,.$ [4; *T19*; *T4*]

7. $E\,\varepsilon\,\mathbf{el}(B \vee D)\,.$ [5; *T20*; *T4*]

$$\left.\begin{array}{ll} 8. & F\,\varepsilon\,\mathbf{el}(A)\,. \\ 9. & F\,\varepsilon\,\mathbf{ex}(B)\,. \end{array}\right\} \qquad [1;\ D8]$$

10. $F\,\varepsilon\,\mathbf{el}(A \vee C)\,.$ [8; *T19*; *T4*]

11. $F\,\varepsilon\,\mathbf{ex}(D)\,.$ [3; 8; *T30*; *D7*]

12. $F\,\varepsilon\,\mathbf{ex}(B \vee D)\,.$ [9; 11; *T30*]

13. $G\,\varepsilon\,\mathbf{el}(B)$.

14. $G\,\varepsilon\,\mathbf{ex}(A)$. [1; *D8*]

15. $G\,\varepsilon\,\mathbf{el}(B \vee D)$. [13; *T20*; *T4*]

16. $G\,\varepsilon\,\mathbf{ex}(C)$. [3; 14; *T30*; *D7*]

17. $G\,\varepsilon\,\mathbf{ex}(A \vee C)$. [14; 16; *T30*]

$A \vee C\,\varepsilon\,\mathbf{lnk}(B \vee D)$ [6; 7; 10; 12; 15; 17; *T31*]

T281 $[ABCD] : \boldsymbol{\delta}(AB) \geqslant 3 \,.\, C\,\varepsilon\,\boldsymbol{\Delta}(AB) \,.\, D\,\varepsilon\,\boldsymbol{\Delta}(AB) \,.\, C \neq D \;.\supset.\; C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABCD] : \mathrm{Hp}(4) \;.\supset.$

$[\exists EF]$.

5. $E\,\varepsilon\,\mathbf{at}(A \wedge B)$.

6. $C = (A \wedge B) \backslash E$. [2; *AD7*]

7. $F\,\varepsilon\,\mathbf{at}(A \wedge B)$.

8. $D = (A \wedge B) \backslash F$. [3; *AD7*]

9. $E \neq F$. [4; 6; 8]

10. $(A \wedge B) \backslash (E \vee F)\,\varepsilon\,\mathbf{el}(C)$. [6; *T4*; **BA**]

11. $(A \wedge B) \backslash (E \vee F)\,\varepsilon\,\mathbf{el}(D)$. [8; *T4*; **BA**]

12. $E\,\varepsilon\,\mathbf{el}(D)$. [5; 8; 9; *T39*]

13. $E\,\varepsilon\,\mathbf{ex}(C)$. [6; *D11*]

14. $F\,\varepsilon\,\mathbf{el}(C)$. [6; 7; 9; *T39*]

15. $F\,\varepsilon\,\mathbf{ex}(D)$. [8; *D11*]

$C\,\varepsilon\,\mathbf{lnk}(D)$ [10; 11; 12; 13; 14; 15; *T31*]

T282 $[ABCD] : \boldsymbol{\delta}(AB) \geqslant 3 \,.\, C\,\varepsilon\,\boldsymbol{\Delta}(AB) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \,.\, D\,\varepsilon\,\boldsymbol{\Delta}(AB) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \,.\, C \neq D \;.\supset.\; C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABCD] :: \mathrm{Hp}(4) \;.\supset\therefore$

$[\exists EFGH] \therefore$

5. $E\,\varepsilon\,\boldsymbol{\Delta}(AB)$.

6. $F\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A \vee B))$. [2; *D26*]

7. $C = E \vee F$.

8. $G\,\varepsilon\,\boldsymbol{\Delta}(AB)$.

9. $H\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A \vee B))$. [3; *D26*]

10. $D = G \vee H$:

11. $E \neq G \;.\vee.\; F \neq H$: [4; 5; 6; 7; 8; 9]

12. $E \neq G \;.\supset.\; E\,\varepsilon\,\mathbf{lnk}(G)$: [5; 8; 1; *T281*]

13. $F\,\varepsilon\,\mathbf{ex}(A \vee B)$. [6; *D10*; *T27*]

14. $H\,\varepsilon\,\mathbf{ex}(A \vee B)$: [9; *D10*; *T27*]

15. $E \neq G \;.\supset.\; C\,\varepsilon\,\mathbf{lnk}(D)$: [7; 10; 12; 13; 14; *T280*]

16. $F \neq H \;.\supset.\; F\,\varepsilon\,\mathbf{ex}(H)$: [6; 9; *T37*]

17. $F\,\varepsilon\,\mathbf{ex}(G)$. [6; 8; **BA**]

18. $F \neq H \;.\supset.\; F\,\varepsilon\,\mathbf{ex}(D)$. [16; 17; *T30*]

19. $H\,\varepsilon\,\mathbf{ex}(E)$. [5; 9; *AD7*; **BA**]

20. $F \neq H \;.\supset.\; H\,\varepsilon\,\mathbf{ex}(C)$: [18; 19; *T25*; *T30*]

21. $E \wedge G\,\varepsilon\,\mathbf{el}(C)$. [1; 5; 7; 8; *T4*; *T17*]

22. $E \wedge G\,\varepsilon\,\mathbf{el}(D) \therefore$ [1; 5; 8; 10; *T4*; *T18*]

$C\,\varepsilon\,\mathbf{lnk}(D)$ [*T31*; 11; 15; 7; 10; 18; 20; 21; 22]

T283 $[AB] : \boldsymbol{\alpha}(AB) > 0 \,.\, \boldsymbol{\beta}(AB) > 0 \,.\, \boldsymbol{\delta}(AB) > 0 \;.\supset.\; A\,\varepsilon\,\mathbf{lnk}(B)$

PR $[AB] : \mathrm{Hp}(3) \;.\supset.$

$[\exists CDE]$.

4. $C\,\varepsilon\,A\backslash(A \wedge B)$. [1; *AD4*]
5. $D\,\varepsilon\,B\backslash(A \wedge B)$. [2; *AD5*]
6. $E\,\varepsilon\,A \wedge B$. [3; *AD3*]
7. $E\,\varepsilon\,\mathbf{el}(A)$. [6; *T17*]
8. $E\,\varepsilon\,\mathbf{el}(B)$. [6; *T18*]
9. $C\,\varepsilon\,\mathbf{el}(A)$. [4; *D11*]
10. $C\,\varepsilon\,\mathbf{ex}(B)$. [4; *D5*; *D11*]
11. $D\,\varepsilon\,\mathbf{el}(B)$. [5; *D11*]
12. $D\,\varepsilon\,\mathbf{ex}(A)$. [5; *D5*; *D11*]

$A\,\varepsilon\,\mathbf{lnk}(B)$ [7; 8; 9; 10; 11; 12; *T31*]

T284 $[ABCD]:\boldsymbol{\delta}(AB) \geqslant 3\,.\,C\,\varepsilon\,\boldsymbol{\Delta}(AB) \ast \mathbf{at}(A\backslash(A \wedge B)) \ast \mathbf{at}(B\backslash(A \wedge B))\,.$
$D\,\varepsilon\,\boldsymbol{\Delta}(AB) \ast \mathbf{at}(A\backslash(A \wedge B)) \ast \mathbf{at}(B\backslash(A \wedge B))\,.\,C \neq D\;.\supset.\;C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABCD]::\mathrm{Hp}(4)\;.\supset\therefore$
$[\exists EFGHKM]\therefore$

$$\left.\begin{array}{ll} 5. & E\,\varepsilon\,\boldsymbol{\Delta}(AB)\,. \\ 6. & F\,\varepsilon\,\mathbf{at}(A\backslash(A \wedge B))\,. \\ 7. & G\,\varepsilon\,\mathbf{at}(B\backslash(A \wedge B))\,. \\ 8. & C = E \vee F \vee G\,. \end{array}\right\} \text{[2; } \mathit{D26}\text{]}$$

$$\left.\begin{array}{ll} 9. & H\,\varepsilon\,\boldsymbol{\Delta}(AB)\,. \\ 10. & K\,\varepsilon\,\mathbf{at}(A\backslash(A \wedge B))\,. \\ 11. & M\,\varepsilon\,\mathbf{at}(B\backslash(A \wedge B))\,. \end{array}\right\} \text{[3; } \mathit{D26}\text{]}$$

12. $D = H \vee K \vee M:$ [3; *D26*]
13. $E \neq H\,.\mathrm{v}.\,F \neq K\,.\mathrm{v}.\,G \neq M:$ [4; 8; 12]
14. $E \wedge H\,\varepsilon\,\mathbf{el}(C)$. [1; 5; 9; 8; *AD7*; **BA**]
15. $E \wedge H\,\varepsilon\,\mathbf{el}(D)$. [1; 5; 9; 12; *AD7*; **BA**]
16. $F\,\varepsilon\,\mathbf{el}(C)$. [8; *T20*]
17. $K\,\varepsilon\,\mathbf{el}(D):$ [12; *T20*]
18. $F \neq K\;.\supset.\;F\,\varepsilon\,\mathbf{ex}(D):$ [*T23*; *T31*; *AD7*; 6; 9; 10; 11; 12]
19. $F \neq K\;.\supset.\;K\,\varepsilon\,\mathbf{ex}(C):$ [*T23*; *T31*; *AD7*; 5; 6; 7; 8; 10]
20. $F \neq K\;.\supset.\;C\,\varepsilon\,\mathbf{lnk}(D):$ [14; 15; 16; 17; 18; 19; *T31*]
21. $G \neq M\;.\supset.\;C\,\varepsilon\,\mathbf{lnk}(D):$ [20]
22. $E \neq H\;.\supset.\;E\,\varepsilon\,\mathbf{lnk}(H):$ [*T281*]
23. $E \neq H\;.\supset.\;C\,\varepsilon\,\mathbf{lnk}(D)\therefore$ [22; *T280*; 8; 12; *T30*]

$C\,\varepsilon\,\mathbf{lnk}(D)$ [13; 20; 21; 23]

T285 $[ABCD]:\boldsymbol{\delta}(AB) \geqslant 3\,.\,C\,\varepsilon\,\boldsymbol{\Delta}(AB) \ast \mathbf{at}(A\backslash(A \wedge B)) \ast \mathbf{at}(B\backslash(A \wedge B))\,.$
$D\,\varepsilon\,\boldsymbol{\Delta}(AB) \ast \mathbf{at}(\mathbf{Cm}(A \vee B))\;.\supset.\;C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABCD]:\mathrm{Hp}(3)\;.\supset.$
$[\exists EFGHK]\,.$

$$\left.\begin{array}{ll} 4. & E\,\varepsilon\,\boldsymbol{\Delta}(AB)\,. \\ 5. & F\,\varepsilon\,\mathbf{at}(A\backslash(A \wedge B))\,. \\ 6. & G\,\varepsilon\,\mathbf{at}(B\backslash(A \wedge B))\,. \\ 7. & C = E \vee F \vee G\,. \end{array}\right\} \text{[2; } \mathit{D26}\text{]}$$

$$\left.\begin{array}{ll} 8. & H\,\varepsilon\,\boldsymbol{\Delta}(AB)\,. \\ 9. & K\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A \vee B))\,. \\ 10. & D = H \vee K\,. \end{array}\right\} \text{[3; } \mathit{D26}\text{]}$$

11. $E \wedge H\,\varepsilon\,\mathbf{el}(C)$. [1; 7; *T17*; *T19*; *T4*]
12. $E \wedge H\,\varepsilon\,\mathbf{el}(D)$. [1; 10; *T18*; *T20*; *T4*]

13. $G\,\varepsilon\,\mathbf{el}(C)\,.$ [6; 7; *T20*]

14. $G\,\varepsilon\,\mathbf{ex}(D)\,.$ [6; 8; 9; 10; *T39*; *T30*; *T267*; *T263*; *T37*]

15. $K\,\varepsilon\,\mathbf{el}(D)\,.$ [9; 10; *T19*]

16. $K\,\varepsilon\,\mathbf{ex}(C)\,.$ [4; 5; 6; 7; 9; *T30*; *T37*; *T39*; *T263*; *T267*]

$C\,\varepsilon\,\mathbf{lnk}(D)$ [11; 12; 13; 14; 15; 16; *T31*]

T286 $[ABC] : C\,\varepsilon\,\mathbf{\Delta}(AB) * \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B))\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(A)$

PR $[ABC] : \mathrm{Hp}(1)\ .\supset.$

$[\exists DEFG]\,.$

$$\left.\begin{array}{ll} 2. & D\,\varepsilon\,\mathbf{\Delta}(AB)\,. \\ 3. & E\,\varepsilon\,\mathbf{at}(A\backslash(A \wedge B))\,. \\ 4. & F\,\varepsilon\,\mathbf{at}(B\backslash(A \wedge B))\,. \\ 5. & C = D \vee E \vee F\,. \end{array}\right\} \quad [1;\ D26]$$

6. $D\,\varepsilon\,\mathbf{el}(C)\,.$ [5; *T19*]

7. $D\,\varepsilon\,\mathbf{el}(A)\,.$ [2; *AD7*; *T4*]

$$\left.\begin{array}{ll} 8. & G\,\varepsilon\,\mathbf{at}(A \wedge B)\,. \\ 9. & D\,\varepsilon\,(A \wedge B)\backslash G\,. \end{array}\right\} \quad [2;\ AD7]$$

10. $G\,\varepsilon\,\mathbf{el}(A)\,.$ [8; *D10*; *T17*; *T4*]

11. $G\,\varepsilon\,\mathbf{ex}(C)\,.$ [3; 4; 9; *T262*; *T263*; *D11*; *T30*; *T37*]

12. $F\,\varepsilon\,\mathbf{el}(C)\,.$ [4; *T19*]

13. $F\,\varepsilon\,\mathbf{ex}(A)\,.$ [4; *D11*]

$C\,\varepsilon\,\mathbf{lnk}(A)$ [6; 7; 10; 11; 12; 13; *T31*]

T287 $[ABC] : C\,\varepsilon\,\mathbf{\Delta}(AB) * \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B))\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(B)$ [*T286*]

T288 $[ABC] : C\,\varepsilon\,\mathbf{\Delta}(AB) * \mathbf{at}(\mathbf{Cm}(A \vee B))\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(A)$

PR $[ABC] : \mathrm{Hp}(1)\ .\supset.$

$[\exists DEF]\,.$

$$\left.\begin{array}{ll} 2. & D\,\varepsilon\,\mathbf{\Delta}(AB)\,. \\ 3. & E\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A \vee B))\,. \\ 4. & C = D \vee E\,. \end{array}\right\} \quad [1;\ D26]$$

5. $D\,\varepsilon\,\mathbf{el}(A)\,.$ [2; *AD7*; *T4*; *T17*]

6. $D\,\varepsilon\,\mathbf{el}(C)\,.$ [4; *T19*; 2]

7. $E\,\varepsilon\,\mathbf{el}(C)\,.$ [4; *T20*; 3]

8. $E\,\varepsilon\,\mathbf{ex}(A)\,.$ [3; *D10*; *D6*]

$$\left.\begin{array}{ll} 9. & F\,\varepsilon\,\mathbf{at}(A \wedge B)\,. \\ 10. & D = (A \wedge B)\backslash F\,. \end{array}\right\} \quad [2;\ AD7]$$

11. $F\,\varepsilon\,\mathbf{el}(A)\,.$ [9; *D10*; *T17*; *T4*]

12. $F\,\varepsilon\,\mathbf{ex}(C)\,.$ [10; *D11*; *T265*; 3; *T30*; *T37*]

$C\,\varepsilon\,\mathbf{lnk}(A)$ [5; 6; 7; 8; 11; 12; *T31*]

T289 $[ABC] : C\,\varepsilon\,\mathbf{\Delta}(AB) * \mathbf{at}(\mathbf{Cm}(A \vee B))\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(B)$ [*T288*]

The definition which follows, *AD8*, and those like it, *AD9-AD15*, are merely introduced as abbreviations for otherwise cumbersome notation.

AD8 $[ABC] : C\,\varepsilon\,\mathbf{T}_1(AB)\ .\equiv.\ C\,\varepsilon\,A \cup B \cup (\mathbf{\Delta}(AB) * \mathbf{at}(A\backslash(A \wedge B)) *$
$\mathbf{at}(B\backslash(A \wedge B))) \cup (\mathbf{\Delta}(AB) * \mathbf{at}(\mathbf{Cm}(A \vee B)))$

T290 $[ABan] :: \mathbf{sfc}(ABan)\,.\,\boldsymbol{\delta}(AB) \geqslant 3\,.\,\boldsymbol{\alpha}(AB) \geqslant 2\,.\,\boldsymbol{\beta}(AB) \geqslant 2\,.$
$\mathbf{Cm}(A \vee B)\,\varepsilon\,\mathbf{Cm}(A \vee B)\ .\supset\therefore\ [\exists b] \therefore \mathrm{Cd}\,\{b\} \geqslant n\,.\,b \subset a\,.\,A\,\varepsilon\,b\,.$
$B\,\varepsilon\,b : [CD] : C\,\varepsilon\,b\,.\,D\,\varepsilon\,b\,.\,C \neq D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABan]::\mathrm{Hp}(5)\ .\supset\therefore$

6\. $[EF]:E\,\varepsilon\,\mathbf{T}_1(AB)\,.\,F\varepsilon\,\mathbf{T}_1(AB)\,.\,E\neq F\ .\supset.\ E\,\varepsilon\,\mathbf{lnk}(F):$
[2; 3; 4; 5; *T282*; *T283*; *T284*; *T285*; *T286*; *T287*; *T288*; *T289*]

7\. $A\cap B\circ\Lambda.$ [2; 3; 4; *T283*]

8\. $A\cap(\boldsymbol{\Delta}(AB)*\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B)))\circ\Lambda.$ [3; *D26*; *AD7*; *T30*]

9\. $B\cap(\boldsymbol{\Delta}(AB)*\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B)))\circ\Lambda.$ [4; *D26*; *AD7*; *T30*]

10\. $A\cap\boldsymbol{\Delta}(AB)*\mathbf{at}(\mathbf{Cm}(A\vee B))\circ\Lambda.$ [*D26*; *AD7*; *D6*; *T30*]

11\. $B\cap\boldsymbol{\Delta}(AB)*\mathbf{at}(\mathbf{Cm}(A\vee B))\circ\Lambda.$ [*D26*; *AD7*; *D6*; *T30*]

12\. $\boldsymbol{\Delta}(AB)*\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B))\cap\boldsymbol{\Delta}(AB)*\mathbf{at}(\mathbf{Cm}(A\vee B))\circ\Lambda.$
[*T262*; *T263*; *T264*; *T265*; *T266*; *T267*; *T30*]

13\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}=\mathsf{Cd}\,\{A\}+\mathsf{Cd}\,\{B\}+\mathsf{Cd}\,\{\boldsymbol{\Delta}(AB)*\mathbf{at}(\mathbf{Cm}(A\vee B))\}+$
$\mathsf{Cd}\,\{\boldsymbol{\Delta}(AB)*\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B))\}.$
[7; 8; 9; 10; 11; 12; *AD8*; **ON**]

14\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}=1+1+\boldsymbol{\delta}(AB)\cdot\boldsymbol{\gamma}(AB)+\boldsymbol{\delta}(AB)\cdot\boldsymbol{\alpha}(AB)\cdot\boldsymbol{\beta}(AB).$
[13; 7; 8; 9; 10; 11; 12; *T219*; *T7*]

15\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}\geqslant 2+\boldsymbol{\gamma}(AB)+\boldsymbol{\delta}(AB)+\boldsymbol{\alpha}(AB)+\boldsymbol{\beta}(AB).$ [14; *048*; **ON**]

16\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}\geqslant n\therefore$ [*AD3*; *AD4*; *AD5*; *AD6*; 1; 15; *T268*; *T228*]
$[\exists b]\therefore b\subset a\,.\,\mathsf{Cd}\,\{b\}\geqslant n\,.\,A\,\varepsilon\,b\,.\,B\,\varepsilon\,b:[CD]:C\,\varepsilon\,b\,.\,D\,\varepsilon\,b\,.$
$C\neq D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$ [*AD8*; 16; 1; 6]

T291 $[ABan]::\mathbf{sfc}(ABan)\,.\,\boldsymbol{\alpha}(AB)\geqslant 2\,.\,\boldsymbol{\beta}(AB)\geqslant 2\,.\,\boldsymbol{\delta}(AB)\geqslant 3\,.$
$\mathbf{Cm}(A\vee B)\circ\Lambda.\supset\therefore[\exists b]\therefore\mathsf{Cd}\,\{b\}\geqslant n\,.\,b\subset a\,.\,A\,\varepsilon\,b\,.\,B\,\varepsilon\,b:[CD]:C\,\varepsilon\,b\,.$
$D\,\varepsilon\,b\,.\,C\neq D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABan]::\mathrm{Hp}(5)\ .\supset\therefore$

6\. $A\,\varepsilon\,\mathbf{lnk}(B):$ [2; 3; 4; *T283*]

7\. $[EF]:E\,\varepsilon\,\mathbf{T}_1(AB)\,.\,F\,\varepsilon\,\mathbf{T}_1(AB)\,.\,E\neq F\ .\supset.\ E\,\varepsilon\,\mathbf{lnk}(F):$
[*AD8*; 6; *T282*; *T284*; *T285*; *T286*; *T287*; *T288*; *T289*]

8\. $\boldsymbol{\gamma}(AB)=0.$ [5]

9\. $A\cap B\circ\Lambda.$ [6; *T33*]

10\. $A\cap(\boldsymbol{\Delta}(AB)*\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B)))\circ\Lambda.$ [2; *D26*; *AD7*; *T30*]

11\. $B\cap(\boldsymbol{\Delta}(AB)*\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B)))\circ\Lambda.$ [3; *D26*; *AD7*; *T30*]

12\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}=\mathsf{Cd}\,\{A\}+\mathsf{Cd}\,\{B\}+\mathsf{Cd}\,\{\Delta(AB)*\mathbf{at}(A\backslash(A\wedge B))*$
$\mathbf{at}(B\backslash(A\wedge B))\}.$ [8; 9; 10; 11; **ON**]

13\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}=1+1+\boldsymbol{\delta}(AB)\cdot\boldsymbol{\alpha}(AB)\cdot\boldsymbol{\beta}(AB).$ [12; *T219*; *T268*; *T279*]

14\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}\geqslant 2+\boldsymbol{\delta}(AB)+\boldsymbol{\alpha}(AB)+\boldsymbol{\beta}(AB).$ [13; **ON**]

15\. $\mathsf{Cd}\,\{\mathbf{T}_1(AB)\}\geqslant n\therefore$ [*AD2*; 1; 14; *T268*; *T228*]
$[\exists b]\therefore\mathsf{Cd}\,\{b\}\geqslant n\,.\,b\subset a\,.\,A\,\varepsilon\,b\,.\,B\,\varepsilon\,b:[CD]:C\,\varepsilon\,b\,.\,D\,\varepsilon\,b\,.$
$C\neq D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$ [1; *AD8*; 7; 15]

T292 $[ABan]::\mathbf{sfc}(ABan)\,.\,\boldsymbol{\alpha}(AB)\geqslant 2\,.\,\boldsymbol{\beta}(AB)\geqslant 2\,.\,\boldsymbol{\delta}(AB)\geqslant 3\ .\supset\therefore$
$[\exists b]\therefore b\subset a\,.\,\mathsf{Cd}\,\{b\}\geqslant n\,.\,A\,\varepsilon\,b\,.\,B\,\varepsilon\,b:[CD]:C\,\varepsilon\,b\,.\,D\,\varepsilon\,b\,.\,C\neq$
$D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$ [*T290*; *T291*]

T293 $[ABCD]:\boldsymbol{\delta}(AB)\geqslant 3\,.\,C\,\varepsilon\,\boldsymbol{\Delta}(AB)*\mathbf{at}(B\backslash(A\wedge B))*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.$
$D\,\varepsilon\,\boldsymbol{\Delta}(AB)*\mathbf{at}(B\backslash(A\wedge B))*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\,C\neq D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABCD]::\mathrm{Hp}(4)\ .\supset::$
$[\exists EFGHKM]\therefore$

$$\left.\begin{array}{ll}5. & E\,\varepsilon\,\boldsymbol{\Delta}(AB)\,.\\ 6. & F\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B))\,.\\ 7. & G\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\\ 8. & C=E\vee F\vee G\,.\end{array}\right\}\quad[2;\ D26]$$

9. $H \, \varepsilon \, \mathbf{\Delta}(AB)$.
10. $K \, \varepsilon \, \mathbf{at}(B \backslash (A \wedge B))$.
11. $M \, \varepsilon \, \mathbf{at}(\mathbf{Cm}(A \vee B))$.
12. $D = H \vee K \vee M$: [3; *D26*]
13. $E \neq H$ .v. $F \neq K$ .v. $H \neq M$: [4; 8; 12; *AD7*; *T37*]
14. $E \neq H$.⊃. $E \, \varepsilon \, \mathbf{lnk}(H)$: [1; 5; 9; *T281*]
15. $E \neq H$.⊃. $C \, \varepsilon \, \mathbf{lnk}(D)$:
[*T262*; *T263*; *T264*; *T265*; *T266*; *T267*; 14; *T280*; 5; 6; 7; 8; 9; 10; 11; 12; 14; *T30*]
16. $E \wedge H \, \varepsilon \, \mathbf{el}(C)$. [1; 5; 9; 8; *T19*; *T17*; *T4*]
17. $E \wedge H \, \varepsilon \, \mathbf{el}(D)$. [1; 5; 9; 12; *T19*; *T17*; *T4*]
18. $F \, \varepsilon \, \mathbf{el}(C)$. [8; *T20*]
19. $K \, \varepsilon \, \mathbf{el}(D)$: [12; *T20*]
20. $F \neq K$.⊃. $F \, \varepsilon \, \mathbf{ex}(D)$: [6; 10; *T263*; *T265*; *T37*; *T30*]
21. $F \neq K$.⊃. $K \, \varepsilon \, \mathbf{ex}(C)$: [10; 6; *T263*; *T265*; *T37*; *T30*]
22. $F \neq K$.⊃. $C \, \varepsilon \, \mathbf{lnk}(D)$: [16; 17; 18; 19; 20; 21; *T31*]
23. $G \neq M$.⊃. $G \, \varepsilon \, \mathbf{ex}(D)$:
[7; 9; 10; 11; 12; *T263*; *T265*; *T37*; *T30*]
24. $G \neq M$.⊃. $M \, \varepsilon \, \mathbf{ex}(C)$:
[5; 6; 7; 8; 11; *T263*; *T265*; *T37*; *T30*]
25. $G \, \varepsilon \, \mathbf{el}(C)$. [7; 8; *T20*]
26. $M \, \varepsilon \, \mathbf{el}(D)$: [11; 12; *T20*]
27. $G \neq M$ ⁚⊃. $C \, \varepsilon \, \mathbf{lnk}(D)$ ∴ [16; 17; 23; 24; 25; 26; *T31*]

$C \, \varepsilon \, \mathbf{lnk}(D)$ [13; 15; 22; 27]

T294 $[ABC] : C \, \varepsilon \, \mathbf{\Delta}(AB) * \mathbf{at}(B \backslash (A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B))$.⊃. $C \, \varepsilon \, \mathbf{lnk}(A)$

PR $[ABC] :$ Hp(1) .⊃.
$[\exists DEFG]$.

2. $D \, \varepsilon \, \mathbf{\Delta}(AB)$.
3. $E \, \varepsilon \, \mathbf{at}(B \backslash (A \wedge B))$.
4. $F \, \varepsilon \, \mathbf{at}(\mathbf{Cm}(A \vee B))$.
5. $C = D \vee E \vee F$. [1; *D26*]
6. $D \, \varepsilon \, \mathbf{el}(C)$. [2; 5; *T19*]
7. $D \, \varepsilon \, \mathbf{el}(A)$. [2; *AD7*; *T4*]
8. $F \, \varepsilon \, \mathbf{el}(C)$. [4; 5; *T20*]
9. $F \, \varepsilon \, \mathbf{ex}(A)$. [4; *T37*]
10. $G \, \varepsilon \, \mathbf{at}(A \wedge B)$.
11. $D = (A \wedge B) \backslash G$. [2; *AD7*]
12. $G \, \varepsilon \, \mathbf{el}(A)$. [10; *D10*; *T17*; *T4*]
13. $G \, \varepsilon \, \mathbf{ex}(C)$. [10; 11; 3; 4; 5; *T263*; *T265*; *T30*]

$C \, \varepsilon \, \mathbf{lnk}(A)$ [6; 7; 8; 9; 12; 13; *T31*]

T295 $[ABC] : C \, \varepsilon \, \mathbf{\Delta}(AB) * \mathbf{at}(B \backslash (A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B))$.⊃. $C \, \varepsilon \, \mathbf{lnk}(B)$ [*T294*]

AD9 $[ABC] : C \, \varepsilon \, \mathbf{T}_2(AB)$.≡. $C \, \varepsilon \, A \cup B \cup \mathbf{\Delta}(AB) * \mathbf{at}(B \backslash (A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B))$

T296 $[ABan] :: \mathbf{sfc}\,\{ABan\} . \boldsymbol{\delta}(AB) \geqslant 2 . \boldsymbol{\alpha}(AB) = 1 . \beta(AB) \geqslant 2 . \mathbf{Cm}(A \vee B) \, \varepsilon$ $\mathbf{Cm}(A \vee B)$.⊃∴ $[\exists b] ∴ b \subset a . \mathrm{Cd}\,\{b\} \geqslant n . A \, \varepsilon \, b . B \, \varepsilon \, b : [CD] : C \, \varepsilon \, b . D \, \varepsilon \, b .$ $C \neq D$.⊃. $C \, \varepsilon \, \mathbf{lnk}(D)$

PR $[ABan] ::$ Hp(5) .⊃∴

6. $A \varepsilon \mathsf{lnk}(B)$: [2; 3; 4; *T283*]

7. $[EF]: E \varepsilon \mathsf{T}_2(AB) . F \varepsilon \mathsf{T}_2(AB) . E \neq F .\supset. E \varepsilon \mathsf{lnk}(F)$:
[6; *T293*; *T294*; *T295*]

8. $A \cap B \circ \Lambda$. [6; *T33*]

9. $A \cap \Delta(AB) * \mathsf{at}(B\backslash(A \wedge B)) * \mathsf{at}(\mathsf{Cm}(A \vee B)) \circ \Lambda$. [5; *AD9*; *D26*; *T30*]

10. $B \cap \Delta(AB) * \mathsf{at}(B\backslash(A \wedge B)) * \mathsf{at}(\mathsf{Cm}(A \vee B)) \circ \Lambda$. [5; *AD9*; *D26*; *T30*]

11. $\mathsf{Cd}\,\{\mathsf{T}_2(AB)\} = \mathsf{Cd}\,\{A\} + \mathsf{Cd}\,\{B\} + \mathsf{Cd}\,\{\Delta(AB) * \mathsf{at}(B\backslash(A \wedge B)) *$
$\mathsf{at}(\mathsf{Cm}(A \vee B))$. [8; 9; 10; **ON**]

12. $\mathsf{Cd}\,\{\mathsf{T}_2(AB)\} = 1 + 1 + \boldsymbol{\gamma}(AB) \cdot \boldsymbol{\beta}(AB) \cdot \boldsymbol{\delta}(AB)$.
[*T219*; *T228*; 11; *AD3*; *AD5*; *AD6*]

13. $\mathsf{Cd}\,\{\mathsf{T}_2(AB)\} \geqslant 2 + \boldsymbol{\gamma}(AB) + \boldsymbol{\beta}(AB) + \boldsymbol{\delta}(AB)$. [12; **ON**]

14. $\mathsf{Cd}\,\{\mathsf{T}_2(AB)\} \geqslant n \therefore$ [1; 3; 13; *T268*; *T228*]
$[\exists b] \therefore b \subset a . \mathsf{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b .$
$C \neq D .\supset. C \varepsilon \mathsf{lnk}(D)$ [1; *AD9*; 7; 14]

AD10 $[ABC] : C \varepsilon \mathsf{T}_3(AB) .\equiv. C \varepsilon A \cup B \cup \Delta(AB) * \mathsf{at}(A\backslash(A \wedge B)) * \mathsf{at}(B\backslash(A \wedge B))$

T297 $[ABan] :: \mathsf{sfc}\,\{ABan\} . \boldsymbol{\delta}(AB) \geqslant 3 . \boldsymbol{\alpha}(AB) = 1 . \boldsymbol{\beta}(AB) \geqslant 2 . \mathsf{Cm}(A \vee B) \circ \Lambda .$
$\supset \therefore [\exists b] \therefore b \subset a . \mathsf{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D .\supset.$
$C \varepsilon \mathsf{lnk}(D)$

PR $[ABan] :: \mathrm{Hp}(5) .\supset \therefore$

6. $A \varepsilon \mathsf{lnk}(B)$: [2; 3; 4; *T283*]

7. $[EF] : E \varepsilon \mathsf{T}_3(AB) . F \varepsilon \mathsf{T}_3(AB) . E \neq F .\supset. E \varepsilon \mathsf{lnk}(F)$:
[6; *T284*; *T286*; *T287*]

8. $A \cap B \circ \Lambda$. [6; *T33*]

9. $A \cap \Delta(AB) * \mathsf{at}(A \backslash(A \wedge B)) * \mathsf{at}(B\backslash(A \wedge B)) \circ \Lambda$. [*T263*; *T264*; *T265*; *T30*]

10. $B \cap \Delta(AB) * \mathsf{at}(A\backslash(A \wedge B)) * \mathsf{at}(B\backslash(A \wedge B)) \circ \Lambda$. [*T263*; *T264*; *T265*; *T30*]

11. $\mathsf{Cd}\,\{\mathsf{T}_3(AB)\} = \mathsf{Cd}\,\{A\} + \mathsf{Cd}\,\{B\} + \mathsf{Cd}\,\{\Delta(AB) * \mathsf{at}(A\backslash(A \wedge B)) *$
$\mathsf{at}(B\backslash(A \wedge B))\}$. [*AD10*; 8; 9; 10; **ON**]

12. $\mathsf{Cd}\,\{\mathsf{T}_3(AB)\} = 1 + 1 + \boldsymbol{\delta}(AB) \cdot \boldsymbol{\alpha}(AB) \cdot \boldsymbol{\beta}(AB)$.
[11; *T263*; *T264*; *T265*; *T219*; *T228*]

13. $\mathsf{Cd}\,\{\mathsf{T}_3(AB)\} \geqslant 2 + \boldsymbol{\delta}(AB) + \boldsymbol{\beta}(AB)$. [12; 2; 3; 4; **ON**]

14. $\boldsymbol{\gamma}(AB) = 0$. [5]

15. $\mathsf{Cd}\,\{\mathsf{T}_3(AB)\} \geqslant n \therefore$ [*T268*; 1; 3; 14; 13; *T218*]
$[\exists b] \therefore b \subset a . \mathsf{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b .$
$C \neq D .\supset. C \varepsilon \mathsf{lnk}(D)$. [1; *AD10*; 7; 15]

T298 $[ABan] :: \mathsf{sfc}\,\{ABan\} . \boldsymbol{\delta}(AB) \geqslant 3 . \boldsymbol{\alpha}(AB) = 1 . \boldsymbol{\beta}(AB) \geqslant 2 .\supset \therefore [\exists b] \therefore$
$b \subset a . \mathsf{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D .\supset. C \varepsilon \mathsf{lnk}(D)$
[*T296*; *T297*]

Notice that we can make appropriate changes in the * names constructed to get similar results with $\boldsymbol{\alpha}(AB) \geqslant 2$ and $\boldsymbol{\beta}(AB) = 1$. We will merely state the results as follows:

T299 $[ABan] :: \mathsf{sfc}\,\{ABan\} . \boldsymbol{\delta}(AB) \geqslant 3 . \boldsymbol{\alpha}(AB) \geqslant 2 . \boldsymbol{\beta}(AB) = 1 .\supset \therefore [\exists b] \therefore$
$b \subset a . \mathsf{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D .\supset. C \varepsilon \mathsf{lnk}(D)$
[*T298*]

T300 $[ABC] : C \varepsilon \mathsf{at}(A\backslash(A \wedge B) * \mathsf{at}(B\backslash(A \wedge B)) * \mathsf{at}(\mathsf{Cm}(A \vee B)) .\supset. C \varepsilon \mathsf{lnk}(A)$

PR $[ABC] : \mathrm{Hp}(1) .\supset.$

$[\exists DEF]$.

$$\left.\begin{array}{ll} 2. & D\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)).\\ 3. & E\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B)).\\ 4. & F\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B).\\ 5. & C = D\vee E\vee F.\end{array}\right\}\quad [1;\ \mathit{D26}]$$

6. $D\,\varepsilon\,\mathbf{el}(C)$. [5; *T19*; 2]

7. $D\,\varepsilon\,\mathbf{el}(A)$. [2; *D10*; *D11*]

8. $E\,\varepsilon\,\mathbf{el}(C)$. [5; *T20*; 3]

9. $E\,\varepsilon\,\mathbf{ex}(A)$. [3; *D10*; *D11*; *D5*]

10. $A\wedge B\,\varepsilon\,\mathbf{el}(A)$. [2; *D10*; *D11*; *D5*; *T17*]

11. $A\wedge B\,\varepsilon\,\mathbf{ex}(C)$. [2; 3; 4; *T30*]

$C\,\varepsilon\,\mathbf{lnk}(A)$ [6; 7; 8; 9; 10; 11; *T31*]

T301 $[ABC]: C\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{at}(\mathbf{Cm}(A\vee B))\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(B)$ [*T300*]

T302 $[ABCDan]: \mathbf{sfc}\,\{ABan\}.\boldsymbol{\alpha}(AB) = 1.\boldsymbol{\beta}(AB) = 1.C\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) *$
$\mathbf{at}(B\backslash(A\wedge B)) * \mathbf{at}(\mathbf{Cm}(A\vee B)).D\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) *$
$\mathbf{at}(\mathbf{Cm}(A\vee B)).\ C\neq D\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$

PR $[ABCDan]: \mathrm{Hp}(6)\ .\supset.$
$[\exists EFGHKM]$.

$$\left.\begin{array}{ll} 7. & E\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)).\\ 8. & F\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B)).\\ 9. & G\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B)).\\ 10. & C = E\vee F\vee G.\end{array}\right\}\quad [3;\ \mathit{D26}]$$

$$\left.\begin{array}{ll} 11. & H\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)).\\ 12. & K\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B)).\\ 13. & M\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B)).\\ 14. & D = H\vee K\vee M.\end{array}\right\}\quad [4;\ \mathit{D26}]$$

15. $E = A\backslash(A\wedge B)$. [1; 7; 2; *T210*]

16. $F = B\backslash(A\wedge B)$. [1; 8; 3; *T210*]

17. $H = A\backslash(A\wedge B)$. [1; 12; 2; *T210*]

18. $K = B\backslash(A\wedge B)$. [1; 13; 3; *T210*]

19. $E = H$. [16; 18]

20. $F = K$. [17; 19]

21. $G\neq M$. [6; 10; 15; 20; 21; 9]

22. $G\,\varepsilon\,\mathbf{el}(C)$. [10; *T20*; 9]

23. $G\,\varepsilon\,\mathbf{ex}(D)$. [*T30*; *T37*; 9; 12; 13; 14; 15; *T266*; *T267*]

24. $M\,\varepsilon\,\mathbf{el}(D)$. [15; 14; *T20*]

25. $M\,\varepsilon\,\mathbf{ex}(C)$. [*T30*; *T37*; 14; 7; 8; 9; 10; *T266*; *T267*]

26. $E\,\varepsilon\,\mathbf{el}(C)$. [7; 10; *T19*]

27. $E\,\varepsilon\,\mathbf{el}(D)$. [20; 12; 15; *T19*]

$C\,\varepsilon\,\mathbf{lnk}(D)$ [23; 24; 25; 26; 27; 28; *T31*]

T303 $[AB].\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{at}(\mathbf{Cm}(A\vee B) \cap \boldsymbol{\Delta}(AB) *$
$\mathbf{at}(B\backslash(A\wedge B)) * \mathbf{at}(\mathbf{Cm}(A\vee B)) \circ \Lambda$
[*D26*; *AD7*; *T262*; *T263*; *T264*; *T265*; *T266*; *T267*]

T304 $[ABCD]: C\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{at}(\mathbf{Cm}(A\vee B)).$
$D\,\varepsilon\,\boldsymbol{\Delta}(AB) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{at}(\mathbf{Cm}(A\vee B)).\boldsymbol{\alpha}(AB) = 1.\boldsymbol{\beta}(AB) =$
$1\ .\supset.\ C\,\varepsilon\,\mathbf{lnk}(D)$.

PR $[ABCD] : \mathrm{Hp}(4) \;.\supset.$

$[\exists EFGHKM]\,.$

$$\left.\begin{array}{ll} 5. & E \,\varepsilon\, \mathbf{at}(A\backslash(A \wedge B))\,. \\ 6. & F \,\varepsilon\, \mathbf{at}(B\backslash(A \wedge B))\,. \\ 7. & G \,\varepsilon\, \mathbf{at}(\mathbf{Cm}(A \vee B))\,. \\ 8. & C = E \vee F \vee G\,. \end{array}\right\} \quad [1;\ D26]$$

9\. $E = A\backslash(A \wedge B)\,.$ [5; 3; *T210*]

10\. $F = B\backslash(A \wedge B)\,.$ [6; 4; *T210*]

11\. $H \,\varepsilon\, \boldsymbol{\Delta}(AB)\,.$

$$\left.\begin{array}{ll} 12. & K \,\varepsilon\, \mathbf{at}(B\backslash(A \wedge B))\,. \\ 13. & M \,\varepsilon\, \mathbf{at}(\mathbf{Cm}(A \vee B))\,. \\ 14. & D = H \vee K \vee M\,. \end{array}\right\} \quad [2;\ D26]$$

15\. $K = B\backslash(A \wedge B)\,.$ [12; 4; *T210*]

16\. $F = K\,.$ [10; 15]

17\. $F \,\varepsilon\, \mathbf{el}(C)\,.$ [8; *T20*; 6]

18\. $F \,\varepsilon\, \mathbf{el}(D)\,.$ [16; 14; *T19*]

19\. $E \,\varepsilon\, \mathbf{ex}(D)\,.$ [*T266*; 5; *T37*; 11; 12; 13; 14; *T262*; *T263*]

20\. $E \,\varepsilon\, \mathbf{el}(C)\,.$ [8; *T19*; 5]

21\. $H \,\varepsilon\, \mathbf{el}(D)\,.$ [14; *T19*; 11]

22\. $H \,\varepsilon\, \mathbf{ex}(C)\,.$ [*T262*; *T263*; *T264*; 5; 6; 7; 11; *T30*]

$C \,\varepsilon\, \mathbf{lnk}(D)$ [17; 18; 19; 20; 21; 22; *T31*]

AD11 $[ABC] : C \,\varepsilon\, \mathbf{T}_4(AB) \;.\equiv.\; C \,\varepsilon\, A \cup B \cup \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \cup \boldsymbol{\Delta}(AB) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B))$

T305 $[ABan] :: \mathbf{sfc}\,\{ABan\} \,.\, \boldsymbol{\delta}(AB) \geqslant 3 \,.\, \boldsymbol{\alpha}(AB) = 1 \,.\, \boldsymbol{\beta}(AB) = 1 \,.\, \mathbf{Cm}(A \vee B) \,\varepsilon\, \mathbf{Cm}(A \vee B) \;.\supset\therefore\; [\exists b] \therefore b \subset a \,.\, \mathrm{Cd}\,\{b\} \geqslant n \,.\, A \,\varepsilon\, b \,.\, B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b \,.\, D \,\varepsilon\, b \,.\, C \neq D \;.\supset.\; C \,\varepsilon\, \mathbf{lnk}(D)$

PR $[ABan] :: \mathrm{Hp}(5) \;.\supset\therefore$

6\. $A \,\varepsilon\, \mathbf{lnk}(B) :$ [2; 3; 4; *T283*]

7\. $[EF] : E \,\varepsilon\, \mathbf{T}_4(AB) \,.\, F \,\varepsilon\, \mathbf{T}_4(AB) \,.\, E \neq F \;.\supset.\; E \,\varepsilon\, \mathbf{lnk}(F) :$ [5; 6; *T294*; *T295*; *T300*; *T301*; *T302*; *T304*]

8\. $A \cap B \circ \Lambda.$ [6; *T33*]

9\. $A \cap \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [3; 6; *D11*; *D6*; *T30*]

10\. $B \cap \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [4; 6; *D11*; *D6*; *T30*]

11\. $A \cap \boldsymbol{\Delta}(AB) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [5; *AD9*; *D26*; *T30*]

12\. $B \cap \boldsymbol{\Delta}(AB) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [5; *AD9*; *D26*; *T30*]

13\. $\mathrm{Cd}\,\{\mathbf{T}_4(AB)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{\mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B))\} + \mathrm{Cd}\,\{\boldsymbol{\Delta}(AB) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B))\}\,.$ [8; 9; 10; 11; 12; *T303*; **ON**]

14\. $\mathrm{Cd}\,\{\mathbf{T}_4(AB)\} = 1 + 1 + \boldsymbol{\gamma}(AB) + \boldsymbol{\delta}(AB) \cdot \boldsymbol{\gamma}(AB)\,.$ [13; *T262*; *T263*; *T264*; *T265*; *T266*; *T267*; *T269*; 3; 4]

15\. $\mathrm{Cd}\,\{\mathbf{T}_4(AB)\} \geqslant 2 + \boldsymbol{\gamma}(AB) + \boldsymbol{\delta}(AB)\,.$ [14; 5]

16\. $\mathrm{Cd}\,\{\mathbf{T}_4(AB)\} \geqslant n \therefore$ [15; 3; 4; *T228*]

$[\exists b] \therefore b \subset a \,.\, \mathrm{Cd}\,\{b\} \geqslant n \,.\, A \,\varepsilon\, b \,.\, B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b \,.\, D \,\varepsilon\, b \,.\, C \neq D \;.\supset.\; C \,\varepsilon\, \mathbf{lnk}(D)\,.$ [1; *AD11*; 7; 16]

T306 $[ABan] :: \mathbf{sfc}\,\{ABan\} . \boldsymbol{\delta}(AB) \geqslant 3 . \boldsymbol{\alpha}(AB) = 1 . \boldsymbol{\beta}(AB) = 1 . \mathbf{Cm}(A \vee B) . \circ \Lambda .$
$\supset \therefore [\exists b] \therefore b \subset a . \mathrm{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b .$
$C \neq D . \supset . C \varepsilon \mathbf{lnk}(D) .$

PR $[ABan] :: \mathrm{Hp}(5) . \supset \therefore$

6. $A \varepsilon \mathbf{lnk}(B) :$ [2; 3; 4; *T283*]

7. $[EF] : E \varepsilon \mathbf{T}_3(AB) . F \varepsilon \mathbf{T}_3(AB) . E \neq F . \supset . E \varepsilon \mathbf{lnk}(F) :$
[*T284*; *T286*; *T287*; 6]

8. $\mathrm{Cd}\,\{\mathbf{T}_3(AB)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{\boldsymbol{\Delta}(AB) * \mathbf{at}(A \backslash (A \wedge B)) *$
$\mathbf{at}(B \backslash (A \wedge B))\} .$ [Proof as in *T297*]

9. $\mathrm{Cd}\,\{\mathbf{T}_3(AB)\} = 1 + 1 + \boldsymbol{\delta}(AB) .$ [8; 3; 4; *T219*; 1]

10. $\mathrm{Cd}\,\{\mathbf{T}_3(AB)\} \geqslant n \therefore$ [9; 1; 5; *T228*; *T268*]
$[\exists b] \therefore b \subset a . \mathrm{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b .$
$C \neq D . \supset . C \varepsilon \mathbf{lnk}(D)$ [1; *AD10*; 10; 7]

T307 $[ABan] :: \mathbf{sfc}\,(ABan) . \boldsymbol{\delta}(AB) \geqslant 3 . \boldsymbol{\alpha}(AB) = 1 . \boldsymbol{\beta}(AB) = 1 . \supset \therefore [\exists b] \therefore$
$b \subset a . \mathrm{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D . \supset . C \varepsilon \mathbf{lnk}(D)$
[*T305*; *T306*]

T308 $[ABan] :: \mathbf{sfc}\,(ABan) . \boldsymbol{\delta}(AB) \geqslant 3 . \boldsymbol{\alpha}(AB) \geqslant 1 . \boldsymbol{\beta}(AB) \geqslant 1 . \supset \therefore [\exists b] \therefore$
$b \subset a . \mathrm{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D . \supset . C \varepsilon \mathbf{lnk}(D)$
[*T292*; *T298*; *T299*; *T307*]

T309 $[ABCDE] : C \varepsilon \mathbf{at}(A \wedge B) . D \varepsilon \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * C .$
$E \varepsilon \mathbf{at}(A \backslash (A \wedge B)) * C . D \neq E . \supset . D \varepsilon \mathbf{lnk}(E)$

PR $[ABCDE] \therefore \mathrm{Hp}(4) . \supset :$
$[\exists FGHK] :$

5. $F \varepsilon \mathbf{at}(A \backslash (A \wedge B)) .$
6. $G \varepsilon \mathbf{at}(B \backslash (A \wedge B)) .$ [1; 2; *D26*]
7. $D = F \vee G \vee C .$
8. $H \varepsilon \mathbf{at}(A \backslash (A \wedge B)) .$
9. $K \varepsilon \mathbf{at}(B \backslash (A \wedge B)) .$ [3; 1; *D26*]
10. $E = H \vee K \vee C .$
11. $C \varepsilon \mathbf{el}(D) .$ [1; 7; *T20*]
12. $C \varepsilon \mathbf{el}(E) .$ [1; 10; *T20*]
13. $F \varepsilon \mathbf{el}(D) .$ [5; 7; *T19*]
14. $H \varepsilon \mathbf{el}(E) :$ [8; 10; *T19*]
15. $F \neq H . \vee . G \neq K :$ [4; 7; 10]
16. $F \neq H . \supset . F \varepsilon \mathbf{ex}(E) :$ [1; 5; 8; 9; 10; *T30*; *T37*; *T39*]
17. $F \neq H . \supset . H \varepsilon \mathbf{ex}(D) :$ [1; 5; 6; 7; 8; *T30*; *T37*; *T39*]
18. $F \neq H . \supset . D \varepsilon \mathbf{lnk}(E) :$ [11; 12; 13; 14; 16; 17; *T31*]
19. $G \neq K . \supset . D \varepsilon \mathbf{lnk}(E) :$ [6; 9; 18]

$D \varepsilon \mathbf{lnk}(E)$ [15; 18; 19]

T310 $[ABCD] : C \varepsilon \mathbf{at}(A \wedge B) . D \varepsilon \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * C .$
$\boldsymbol{\alpha}(AB) \geqslant 2 . \supset . A \varepsilon \mathbf{lnk}(D)$

PR $[ABCD] : \mathrm{Hp}(3) . \supset .$
$[\exists EFG] .$

4. $E \varepsilon \mathbf{at}(A \backslash (A \wedge B)) .$
5. $F \varepsilon \mathbf{at}(B \backslash (A \wedge B)) .$ [2; 1; *D26*]
6. $D = E \vee F \vee C .$
7. $E \varepsilon \mathbf{el}(A) .$ [4; *D11*; *D10*]

8. $E\,\varepsilon\,\mathbf{el}(D)\,.$ [6; *T19*; 4]

$$\left.\begin{array}{ll} 9. & G\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))\,. \\ 10. & G \neq E\,. \end{array}\right\}\quad [3;\ 4]$$

11. $G\,\varepsilon\,\mathbf{el}(A)\,.$ [9; *D10*; *D11*]

12. $G\,\varepsilon\,\mathbf{ex}(D)\,.$ [1; 4; 5; 6; 9; 10; *T30*; *T37*; *T39*]

13. $F\,\varepsilon\,\mathbf{el}(D)\,.$ [6; *T20*; 5]

14. $F\,\varepsilon\,\mathbf{ex}(A)\,.$ [5; *T39*]

$A\,\varepsilon\,\mathbf{lnk}(D)$ [7; 8; 11; 12; 13; 14; *T31*]

T311 $[ABCD]:C\,\varepsilon\,\mathbf{at}(A\wedge B)\,.\,D\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B))*C\,.\,\boldsymbol{\beta}(AB)\geqslant 2\,.\supset.\,B\,\varepsilon\,\mathbf{lnk}(D)$ [*T310*]

T312 $[ABCDE]:C\,\varepsilon\,\mathbf{at}(A\wedge B)\,.\,D\,\varepsilon\,C*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\,E\,\varepsilon\,C*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\,D\neq E\,.\supset.\,D\,\varepsilon\,\mathbf{lnk}(E)$

PR $[ABCDE]:\mathrm{Hp}(4)\,.\supset.$

$[\exists FG]\,.$

5. $F\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B))\,.$ [1; 2; *D26*]

6. $D = C\vee F\,.$ [1; 2; *D26*]

$$\left.\begin{array}{ll} 7. & G\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B))\,. \\ 8. & F = C\vee G\,. \end{array}\right\}\quad [1;\ 3;\ \mathit{D26}]$$

9. $F\neq G\,.$ [4; 5; 6; 7; 8]

10. $C\,\varepsilon\,\mathbf{el}(D)\,.$ [1; 6; *T19*]

11. $C\,\varepsilon\,\mathbf{el}(E)\,.$ [1; 8; *T19*]

12. $F\,\varepsilon\,\mathbf{el}(D)\,.$ [5; 6; *T20*]

13. $F\,\varepsilon\,\mathbf{ex}(E)\,.$ [1; 5; 7; 8; 9; *T30*; *T37*]

14. $G\,\varepsilon\,\mathbf{el}(E)\,.$ [7; 8; *T20*]

15. $G\,\varepsilon\,\mathbf{ex}(D)\,.$ [1; 5; 6; 7; 9; *T30*; *T37*]

$D\,\varepsilon\,\mathbf{lnk}(E)$ [10; 11; 12; 13; 14; 15; *T31*]

T313 $[ABCD]:C\,\varepsilon\,\mathbf{at}(A\wedge B)\,.\,D\,\varepsilon\,C*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\,\boldsymbol{\alpha}(AB)\geqslant 1\,.\supset.\,D\,\varepsilon\,\mathbf{lnk}(A)$

PR $[ABCD]:\mathrm{Hp}(3)\,.\supset.$

$[\exists EF]\,.$

$$\left.\begin{array}{ll} 4. & E\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B))\,. \\ 5. & D = C\vee E\,. \end{array}\right\}\quad [2;\ \mathit{D26}]$$

6. $F\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))\,.$ [3; *AD4*]

7. $C\,\varepsilon\,\mathbf{el}(A)\,.$ [1; *T17*; *D10*; *T4*]

8. $C\,\varepsilon\,\mathbf{el}(D)\,.$ [1; 5; *T19*]

9. $F\,\varepsilon\,\mathbf{el}(A)\,.$ [8; *D10*; *D11*]

10. $F\,\varepsilon\,\mathbf{ex}(D)\,.$ [1; 4; 5; 6; *T30*; *T39*]

11. $E\,\varepsilon\,\mathbf{el}(D)\,.$ [4; 5; *T20*]

12. $E\,\varepsilon\,\mathbf{ex}(A)\,.$ [4; *D6*]

$D\,\varepsilon\,\mathbf{lnk}(A)$ [7; 8; 9; 10; 11; 12; *T31*]

T314 $[ABCD]:C\,\varepsilon\,\mathbf{at}(A\wedge B)\,.\,D\,\varepsilon\,C*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\,\boldsymbol{\beta}(AB)\geqslant 1\,.\supset.\,D\,\varepsilon\,\mathbf{lnk}(B)$ [*T313*]

T315 $[ABCDE]:C\,\varepsilon\,\mathbf{at}(A\wedge B)\,.\,D\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))*\mathbf{at}(B\backslash(A\wedge B))*C\,.\,E\,\varepsilon\,C*\mathbf{at}(\mathbf{Cm}(A\vee B))\,.\supset.\,D\,\varepsilon\,\mathbf{lnk}(E)$

PR $[ABCDE]:\mathrm{Hp}(3)\,.\supset.$

$[\exists FGH]\,.$

$$\left.\begin{array}{ll} 4. & F\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))\,. \\ 5. & G\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B))\,. \\ 6. & D = F\vee G\vee C\,. \end{array}\right\}\quad [1;\ 2;\ \mathit{D26}]$$

$$\left.\begin{array}{ll} 7. & H \,\varepsilon\, \mathbf{at}(\mathbf{Cm}(A \vee B))\,. \\ 8. & E = C \vee H\,. \end{array}\right\} \qquad [1;\ 3;\ \mathit{D26}]$$

9. $C \,\varepsilon\, \mathbf{el}(D)\,.$ [6; *T20*; 1]

10. $C \,\varepsilon\, \mathbf{el}(E)\,.$ [8; *T19*; 1]

11. $F \,\varepsilon\, \mathbf{el}(D)\,.$ [4; 6; *T19*]

12. $F \,\varepsilon\, \mathbf{ex}(E)\,.$ [1; 4; 7; 8; *T30*; *T37*]

13. $H \,\varepsilon\, \mathbf{el}(E)\,.$ [7; 8; *T20*]

14. $H \,\varepsilon\, \mathbf{ex}(D)\,.$ [1; 4; 5; 6; 7; *T30*; *T37*]

$D \,\varepsilon\, \mathbf{lnk}(E)$ [9; 10; 11; 12; 13; 14; *T31*]

AD12 $[ABC] : D \,\varepsilon\, \mathbf{T}_5(ABC) \;.{\equiv}.\; D \,\varepsilon\, A \cup B \cup \mathbf{at}(A \backslash (A \wedge B)) *$
$\mathbf{at}(B \backslash (A \wedge B)) * C \cup C * \mathbf{at}(\mathbf{Cm}(A \vee B))\,.\, C \,\varepsilon\, \mathbf{at}(A \wedge B)$

T316 $[ABan] :: \mathbf{sfc}\,\{ABan\}\,.\,\boldsymbol{\delta}(AB) = 2\,.\,\boldsymbol{\alpha}(AB) \geqslant 2\,.\,\boldsymbol{\beta}(AB) \geqslant 2 \;.{\supset}{\therefore}\; [\exists b] \therefore$
$b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.\,C \neq D \;.{\supset}.\; C \,\varepsilon\, \mathbf{lnk}(D)$

PR $[ABan] :: \mathrm{Hp}(4) \;.{\supset}{\therefore}$

5. $A \,\varepsilon\, \mathbf{lnk}(B)\,.$ [2; 3; 4; *T283*]

6. $A \cap B \circ \Lambda \therefore$ [5; *T33*]

$[\exists C] \therefore$ [5; *T309*; *T310*; *T311*; *T312*; *T313*; *T314*; *T315*]

7. $C \,\varepsilon\, \mathbf{at}(A \wedge B) :$ [2]

8. $[DE] : D \,\varepsilon\, \mathbf{T}_5(ABC)\,.\,E \,\varepsilon\, \mathbf{T}_5(ABC)\,.\,D \neq E \;.{\supset}.\; D \,\varepsilon\, \mathbf{lnk}(E) :$
[5; 7; *AD12*; *T309*; *T310*; *T311*; *T312*; *T313*; *T314*; *T315*]

9. $A \cap \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * C \circ \Lambda.$
[*T263*; *T264*; *T265*; 7; 3]

10. $B \cap \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * C \circ \Lambda.$
[*T263*; *T264*; *T265*; 7; 4]

11. $A \cap C * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [7; *D6*]

12. $B \cap C * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [7; *D6*]

13. $\mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * C \cap C * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda\,.$
[*T262*; *T263*; *T264*; *T265*; *T266*; *T267*; 7]

14. $\mathrm{Cd}\,\{\mathbf{T}_5(ABC)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{C * \mathbf{at}(\mathbf{Cm}(A \vee B))\} +$
$\mathrm{Cd}\,\{\mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * C\}.$ [7; 9; 10; 11; 12; 13; **ON**]

15. $\mathrm{Cd}\,\{\mathbf{T}_5(ABC)\} = 1 + 1 + \boldsymbol{\alpha}(AB) \cdot \boldsymbol{\beta}(AB) + \bar{\boldsymbol{\gamma}}(AB)\,.$
[14; *T219*; *AD3*; *AD4*; *AD5*]

16. $\mathrm{Cd}\,\{\mathbf{T}_5(ABC)\} \geqslant 2 + \boldsymbol{\alpha}(AB) + \boldsymbol{\beta}(AB) + \bar{\boldsymbol{\gamma}}(AB)\,.$ [3; 4; 15; **ON**]

17. $\mathrm{Cd}\,\{\mathbf{T}_5(ABC)\} \geqslant n \therefore$ [16; *T268*]

$[\exists b] \therefore b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.$
$C \neq B \;.{\supset}.\; C \,\varepsilon\, \mathbf{lnk}(D)$ [1; *AD12*; 8; 17]

T317 $[ABCDE] : C \,\varepsilon\, \mathbf{at}(A \backslash (A \wedge B))\,.\,D \,\varepsilon\, \mathbf{at}(A \wedge B) * C * \mathbf{at}(B \backslash (A \wedge B))\,.$
$E \,\varepsilon\, \mathbf{at}(A \wedge B) * C * \mathbf{at}(B \backslash (A \wedge B))\,.\,D \neq E \;.{\supset}.\; D \,\varepsilon\, \mathbf{lnk}(E)$

PR $[ABCDE] :: \mathrm{Hp}(4) \;.{\supset}{\therefore}$

$[\exists FGHK] \therefore$

$$\left.\begin{array}{ll} 5. & F \,\varepsilon\, \mathbf{at}(A \wedge B)\,. \\ 6. & G \,\varepsilon\, \mathbf{at}(B \backslash (A \wedge B))\,. \\ 7. & D = F \vee C \vee G\,. \end{array}\right\} \qquad [1;\ 2;\ \mathit{D26}]$$

$$\left.\begin{array}{ll} 8. & H \,\varepsilon\, \mathbf{at}(A \wedge B)\,. \\ 9. & K \,\varepsilon\, \mathbf{at}(B \backslash (A \wedge B))\,. \\ 10. & E = H \vee C \vee K : \end{array}\right\} \qquad [1;\ 3;\ \mathit{D26}]$$

11. $F \neq H . \mathrm{v} . G \neq K :$ [4; 5; 6; 7; 8; 9; 10]
12. $C \, \varepsilon \, \mathbf{el}(D) .$ [1; 7; *T20*]
13. $C \, \varepsilon \, \mathbf{el}(E) .$ [1; 10; *T20*]
14. $F \, \varepsilon \, \mathbf{el}(D) .$ [5; 7; *T19*]
15. $H \, \varepsilon \, \mathbf{el}(E) :$ [8; 10; *T19*]
16. $F \neq H . \supset . F \, \varepsilon \, \mathbf{ex}(E) :$ [*T262*; *T264*; 1; 5; 8; 9; 10; *T30*; *T37*]
17. $F \neq H . \supset . H \, \varepsilon \, \mathbf{ex}(D) :$ [*T262*; *T264*; 1; 5; 6; 7; 8; *T30*; *T37*]
18. $F \neq H . \supset . D \, \varepsilon \, \mathbf{lnk}(E) :$ [12; 13; 14; 15; 16; 17; *T31*]
19. $G \neq K . \supset . G \, \varepsilon \, \mathbf{ex}(E) :$ [*T263*; *T264*; 1; 6; 8; 9; 10; *T30*; *T37*]
20. $G \neq K . \supset . K \, \varepsilon \, \mathbf{ex}(D) :$ [*T263*; *T264*; 1; 5; 6; 7; 9; *T30*; *T37*]
21. $G \neq K . \supset . D \, \varepsilon \, \mathbf{lnk}(E) \therefore$ [12; 13; 14; 15; 19; 20; *T31*]

$D \, \varepsilon \, \mathbf{lnk}(E)$ [11; 18; 21]

T318 $[ABC] : C \, \varepsilon \, A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) . \boldsymbol{\alpha}(AB) \geqslant 1 . \boldsymbol{\delta}(AB) \geqslant 1 . \supset . C \, \varepsilon \, \mathbf{lnk}(A)$

PR $[ABC] : \mathrm{Hp}(3) . \supset .$

$[\exists DE] .$

4. $D \, \varepsilon \, \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
5. $C = (A \wedge B) \vee D .$ [1; 3; *D26*]
6. $E \, \varepsilon \, \mathbf{at}(A \backslash (A \wedge B)) .$ [2; *AD4*]
7. $A \wedge B \, \varepsilon \, \mathbf{el}(A) .$ [3; *T17*]
8. $A \wedge B \, \varepsilon \, \mathbf{el}(C) .$ [3; 5; *T19*; *T4*]
9. $D \, \varepsilon \, \mathbf{el}(C) .$ [4; 5; *T20*]
10. $D \, \varepsilon \, \mathbf{ex}(A) .$ [4; *D10*; *T27*]
11. $E \, \varepsilon \, \mathbf{el}(A) .$ [6; *D11*]
12. $E \, \varepsilon \, \mathbf{ex}(C) .$ [1; 4; 6; *T30*; *T37*]

$C \, \varepsilon \, \mathbf{lnk}(A)$ [7; 8; 9; 10; 11; 12; *T31*]

T319 $[ABC] : C \, \varepsilon \, A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) . \boldsymbol{\beta}(AB) > 0 . \boldsymbol{\delta}(AB) > 0 . \supset . C \, \varepsilon \, \mathbf{lnk}(B)$ [*T318*]

T320$_a$ $[ABCDE] : C \, \varepsilon \, \mathbf{at}(A \backslash (A \wedge B)) . D \, \varepsilon \, \mathbf{at}(A \wedge B) * C * \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
$E \, \varepsilon \, A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) . \boldsymbol{\delta}(AB) = 2 . \supset . D \, \varepsilon \, \mathbf{lnk}(E)$

PR $[ABCDE] : \mathrm{Hp} . \supset .$

$[\exists FGHK] .$

5. $F \, \varepsilon \, \mathbf{at}(A \wedge B) .$
6. $G \, \varepsilon \, \mathbf{at}(\mathbf{Cm}(A \vee B)) .$ [2; *D26*]
7. $D = F \vee C \vee G .$
8. $H \, \varepsilon \, \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
9. $E = (A \wedge B) \vee H .$ [3; *D26*]
10. $K \, \varepsilon \, \mathbf{at}(A \wedge B) .$
11. $F \neq K .$ [4; 5]
12. $F \, \varepsilon \, \mathbf{el}(A \wedge B) .$ [5; *D10*]
13. $F \, \varepsilon \, \mathbf{el}(E) .$ [9; 12; *T19*; *T4*]
14. $F \, \varepsilon \, \mathbf{el}(D) .$ [7; 5; *T19*]
15. $C \, \varepsilon \, \mathbf{el}(D) .$ [1; 7; *T20*]
16. $C \, \varepsilon \, \mathbf{ex}(E) .$ [1; 8; 9; *T30*; *T37*]
17. $K \, \varepsilon \, \mathbf{el}(A \wedge B) .$ [10; *D10*]
18. $K \, \varepsilon \, \mathbf{el}(E) .$ [9; 17; *T19*; *T4*]
19. $K \, \varepsilon \, \mathbf{ex}(D) .$ [11; 10; *T37*; 1; 5; 6; *T30*]

$D \, \varepsilon \, \mathbf{lnk}(E)$ [13; 14; 15; 16; 18; 19; *T31*]

$T320_b$ $[ABCD] : C \varepsilon A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) . D \varepsilon A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
$C \neq D .\supset. C \varepsilon \mathbf{lnk}(D)$

PR $[ABCD] : \mathrm{Hp}(3) .\supset.$
$[\exists EF] .$

4. $E \varepsilon \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
5. $C = (A \wedge B) \vee E .$ [1; *D26*]
6. $F \varepsilon \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
7. $D = (A \wedge B) \vee F .$ [2; *D26*]
8. $E \neq F .$ [3; 5; 7]
9. $A \wedge B \varepsilon \mathbf{el}(C) .$ [1; *D26*; 5; *T19*]
10. $A \wedge B \varepsilon \mathbf{el}(D) .$ [2; *D26*; 7; *T19*]
11. $E \varepsilon \mathbf{el}(C) .$ [4; 5; *T20*]
12. $E \varepsilon \mathbf{ex}(D) .$ [*T265*; 8; *T37*; *T30*]
13. $F \varepsilon \mathbf{el}(D) .$ [6; 7; *T20*]
14. $F \varepsilon \mathbf{ex}(C) .$ [*T265*; 8; *T37*; *T30*]

$C \varepsilon \mathbf{lnk}(D)$ [9; 10; 11; 12; 13; 14; *T31*]

$T320_c$ $[ABCD] : C \varepsilon A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) . D \varepsilon \mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) *$
$\mathbf{at}(B \backslash (A \wedge B)) .\supset. C \varepsilon \mathbf{lnk}(D)$

PR $[ABCD] : \mathrm{Hp}(2) .\supset.$
$[\exists EFGH] .$

3. $E \varepsilon \mathbf{at}(A \wedge B) .$
4. $F \varepsilon \mathbf{at}(A \backslash (A \wedge B)) .$
5. $G \varepsilon \mathbf{at}(B \backslash (A \wedge B)) .$
6. $D = E \vee F \vee G .$ [2; *D26*]
7. $H \varepsilon \mathbf{at}(\mathbf{Cm}(A \vee B)) .$
8. $C = (A \wedge B) \vee H .$ [1; *D26*]
9. $E \varepsilon \mathbf{el}(A \wedge B) .$ [3; *D10*]
10. $E \varepsilon \mathbf{el}(D) .$ [3; 6; *T19*]
11. $E \varepsilon \mathbf{el}(C) .$ [9; 8; *T19*; *T4*]
12. $F \varepsilon \mathbf{el}(D) .$ [4; 6; *T20*]
13. $F \varepsilon \mathbf{ex}(C) .$ [4; 7; 8; *T266*; *T262*; *T39*; *T30*]
14. $H \varepsilon \mathbf{el}(C) .$ [7; 8; *T20*]
15. $H \varepsilon \mathbf{ex}(D) .$ [3; 4; 5; 6; 7; *T265*; *T266*; *T267*; *T39*; *T30*]

$C \varepsilon \mathbf{lnk}(D)$ [10; 11; 12; 13; 14; 15; *T31*]

AD13 $[ABC] : C \varepsilon \mathbf{T}_6(AB) .\equiv. C \varepsilon A \cup B \cup \mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) *$
$\mathbf{at}(B \backslash (A \wedge B)) \cup A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B))$

T321 $[ABan] :: \mathbf{sfc}\,\{ABan\} . \boldsymbol{\delta}(AB) = 2 . \boldsymbol{\alpha}(AB) = 1 . \boldsymbol{\beta}(AB) \geqslant 2 .\supset\therefore [\exists b] \therefore$
$b \subset a . \mathrm{Cd}\,\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D .\supset. C \varepsilon \mathbf{lnk}(D) .$

PR $[ABan] :: \mathrm{Hp}(4) .\supset\therefore$

5. $A \varepsilon \mathbf{lnk}(B) :$ [2; 3; 4; *T283*]
6. $[CD] : C \varepsilon \mathbf{T}_6(AB) . D \varepsilon \mathbf{T}_6(AB) . C \neq D .\supset. C \varepsilon \mathbf{lnk}(D) :$
[5; *AD13*; $T320_a$; $T320_b$; *T317*; *T318*; *T319*]
7. $A \cap B \circ \Lambda .$ [5; *T33*]
8. $A \cap \mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) \circ \Lambda .$
[1; 2; 3; 4; *T262*; *T263*; *T264*; *T30*]

9. $B \cap \mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) \circ \Lambda.$

[1; 2; 3; 4; *T262*; *T263*; *T264*; *T30*]

10. $A \cap A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [1; 2; 3; 4; *T265*; *T30*]

11. $B \cap A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [1; 2; 3; 4; *T265*; *T30*]

12. $A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) \cap \mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) \circ \Lambda.$

[*T262*; *T263*; *T264*; *T265*; *T30*]

13. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{\mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) *$

$\mathbf{at}(B \backslash (A \wedge B)) + \mathrm{Cd}\,\{A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B))\}.$ [7; 8; 9; 10; 11; 12; **ON**]

14. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = 1 + 1 + 2 \cdot 1 \cdot \boldsymbol{\beta}(AB) + 1 \cdot \boldsymbol{\gamma}(AB)\,.$ [13; 2; 3; *T216*]

15. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} \geqslant 2 + \boldsymbol{\alpha}(AB) + \boldsymbol{\beta}(AB) + \boldsymbol{\gamma}(AB)\,.$ [14; 3; 4]

16. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} \geqslant n \therefore$ [15; 1; 2; *T268*]

$[\exists b] \therefore b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.$

$C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)$ [1; *AD13*; 6; 16]

T322 $[ABan] :: \mathbf{sfc}\,\{ABan\}\,.\,\boldsymbol{\delta}(AB) = 2\,.\,\boldsymbol{\alpha}(AB) \geqslant 2\,.\,\boldsymbol{\beta}(AB) = 1 \,.\!\supset\therefore [\exists b] \therefore$

$b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)$

[*T321*]

T323 $[ABan] :: \mathbf{sfc}\,\{ABan\}\,.\,\boldsymbol{\delta}(AB) = 2\,.\,\boldsymbol{\alpha}(AB) = 1\,.\,\boldsymbol{\beta}(AB) = 1 \,.\!\supset.\, [\exists b] \therefore$

$b \subset a\,.\,\mathrm{Cd}\,\{b\} = n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)\,.$

PR $[ABan] :: \mathrm{Hp}(4) \,.\!\supset\therefore$

5. $A \,\varepsilon\, \mathbf{Ink}(B) :$ [2; 3; 4; *T283*]

6. $[CD] : C \,\varepsilon\, \mathbf{T}_6(AB)\,.\,D \,\varepsilon\, \mathbf{T}_6(AB)\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D) :$

[5; *AD13*; $T320_a$; $T320_b$; *T317*; *T318*; *T319*]

7. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{\mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) *$

$\mathbf{at}(B \backslash (A \wedge B))\} + \mathrm{Cd}\,\{A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B))\}.$ [Proof as in *T321*]

8. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = 1 + 1 + 2 + \boldsymbol{\gamma}(AB)\,.$ [7; 2; 3; 4; *T219*]

9. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = n \therefore$ [2; 3; 4; 8; *T268*]

$[\exists b] \therefore b \subset a\,.\,\mathrm{Cd}\,\{b\} = n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.$

$C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)\,.$ [1; *AD13*; 6; 9]

T324 $[ABan] :: \mathbf{sfc}\,\{ABan\}\,.\,\boldsymbol{\delta}(AB) = 2\,.\,\boldsymbol{\alpha}(AB) \geqslant 1\,.\,\boldsymbol{\beta}(AB) \geqslant 1 \,.\!\supset\therefore [\exists b] \therefore$

$b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)$

[*T316*; *T321*; *T322*; *T323*]

T325 $[ABan] :: \mathbf{sfc}\,\{ABan\}\,.\,\boldsymbol{\delta}(AB) = 1\,.\,\boldsymbol{\alpha}(AB) \geqslant 2\,.\,\boldsymbol{\beta}(AB) \geqslant 2 \,.\!\supset\therefore [\exists b] \therefore$

$b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)$

PR $[ABan] :: \mathrm{Hp}(4) \,.\!\supset\therefore$

5. $A \,\varepsilon\, \mathbf{Ink}(B) :$ [2; 3; 4; *T283*]

6. $[CD] : C \,\varepsilon\, \mathbf{T}_6(AB)\,.\,D \,\varepsilon\, \mathbf{T}_6(AB)\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D) :$

[5; *AD13*; $T320_a$; $T320_b$; *T317*; *T318*; *T319*]

7. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{\mathbf{at}(A \wedge B) * \mathbf{at}(A \backslash (A \wedge B)) *$

$\mathbf{at}(B \backslash (A \wedge B))\} + \mathrm{Cd}\,\{A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B))\}.$ [Proof of *T321*]

8. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} = 1 + 1 + 1 \cdot \boldsymbol{\alpha}(AB) \cdot \boldsymbol{\beta}(AB) + 1 \cdot \boldsymbol{\gamma}(AB)\,.$ [7; 2; 3; 4; *T219*]

9. $\mathrm{Cd}\,\{\mathbf{T}_6(AB)\} \geqslant n \therefore$ [2; 3; 4; 8; *048*; *T268*]

$[\exists b] \therefore b \subset a\,.\,\mathrm{Cd}\,\{b\} \geqslant n\,.\,A \,\varepsilon\, b\,.\,B \,\varepsilon\, b : [CD] : C \,\varepsilon\, b\,.\,D \,\varepsilon\, b\,.$

$C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)$ [1; *AD13*; 6; 9]

T326 $[ABCD] : D \,\varepsilon\, \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * \mathbf{Cm}(A \vee B)\,.$

$C \,\varepsilon\, \mathbf{at}(A \backslash (A \wedge B)) * \mathbf{at}(B \backslash (A \wedge B)) * \mathbf{Cm}(A \vee B)\,.\,C \ddagger D \,.\!\supset.\, C \,\varepsilon\, \mathbf{Ink}(D)$

PR $[ABCD] :: \mathrm{Hp}(3) \,.\!\supset\therefore$

$[\exists FGH] \therefore$

4.	$E\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))$.	[1; *D26*]
5.	$F\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B))$.	
6.	$D = E \vee F \vee \mathbf{Cm}(A\vee B)$.	
7.	$G\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))$.	[2; *D26*]
8.	$H\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B))$.	
9.	$C = G \vee H \vee \mathbf{Cm}(A\vee B)$:	
10.	$E \neq G$ .v. $F \neq H$:	[3; 4; 5; 6; 7; 8; 9; *T213*; *T266*; *T267*]
11.	$\mathbf{Cm}(A\vee B)\,\varepsilon\,\mathbf{el}(C)$.	[*D26*; 9; 2; *T20*]
12.	$\mathbf{Cm}(A\vee B)\,\varepsilon\,\mathbf{el}(D)$.	[*D26*; 6; 1; *T20*]
13.	$E\,\varepsilon\,\mathbf{el}(D)$.	[4; 6; *T19*]
14.	$G\,\varepsilon\,\mathbf{el}(C)$:	[7; 9; *T19*]
15.	$E \neq G$.⊃. $E\,\varepsilon\,\mathbf{ex}(C)$:	[4; 7; 8; 9; *T264*; *T266*; *T37*; *T30*]
16.	$E \neq G$.⊃. $G\,\varepsilon\,\mathbf{ex}(D)$:	[4; 5; 6; 7; *T264*; *T266*; *T37*; *T30*]
17.	$E \neq G$.⊃. $C\,\varepsilon\,\mathbf{lnk}(D)$:	[11; 12; 13; 14; 15; 16; *T31*]
18.	$F \neq H$.⊃. $C\,\varepsilon\,\mathbf{lnk}(D)$ ∴	[17]
	$C\,\varepsilon\,\mathbf{lnk}(D)$	[10; 17; 18]
T327	$[ABC]$: $\boldsymbol{\delta}(AB) \geqslant 1$. $C\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{Cm}(A\vee B)$.⊃. $A\,\varepsilon\,\mathbf{lnk}(C)$	
PR	$[ABC]$: Hp(2) .⊃.	
	$[\exists DE]$.	
3.	$D\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))$.	[2; *D26*]
4.	$E\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B))$.	
5.	$C = D \vee E \vee \mathbf{Cm}(A\vee B)$.	
6.	$D\,\varepsilon\,\mathbf{el}(A)$.	[3; *D10*; *D11*]
7.	$D\,\varepsilon\,\mathbf{el}(C)$.	[3; 5; *T19*]
8.	$E\,\varepsilon\,\mathbf{el}(C)$.	[4; 5; *T20*]
9.	$E\,\varepsilon\,\mathbf{ex}(A)$.	[4; *D11*]
10.	$A\wedge B\,\varepsilon\,\mathbf{el}(A)$.	[1; *T17*]
11.	$A\wedge B\,\varepsilon\,\mathbf{ex}(C)$.	[*T262*; *T263*; *T265*; 3; 4; 5; *T30*]
	$A\,\varepsilon\,\mathbf{lnk}(C)$	[6; 7; 8; 9; 10; 11; *T31*]
T328	$[ABC]$: $\boldsymbol{\delta}(AB) \geqslant 1$. $C\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{Cm}(A\vee B)$.⊃. $B\,\varepsilon\,\mathbf{lnk}(C)$	[*T327*]
T329	$[ABCD]$: $C\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B)) * \mathbf{at}(B\backslash(A\wedge B)) * \mathbf{Cm}(A\vee B)$. $D\,\varepsilon\,A\wedge B * \mathbf{at}(\mathbf{Cm}(A\vee B))$.⊃. $C\,\varepsilon\,\mathbf{lnk}(D)$	
PR	$[ABCD]$: Hp(2) .⊃.	
	$[\exists EFG]$.	
3.	$E\,\varepsilon\,\mathbf{at}(A\backslash(A\wedge B))$.	[1; *D26*]
4.	$F\,\varepsilon\,\mathbf{at}(B\backslash(A\wedge B))$.	
5.	$C = E \vee F \vee \mathbf{Cm}(A\vee B)$.	
6.	$G\,\varepsilon\,\mathbf{at}(\mathbf{Cm}(A\vee B))$.	[2; *D26*]
7.	$D = (A\wedge B)\vee G$.	
8.	$G\,\varepsilon\,\mathbf{el}(\mathbf{Cm}(A\vee B))$.	[6; *D10*]
9.	$G\,\varepsilon\,\mathbf{el}(C)$.	[5; 8; *T20*; *T4*]
10.	$G\,\varepsilon\,\mathbf{el}(D)$.	[6; 7; *T20*]
11.	$A\wedge B\,\varepsilon\,\mathbf{el}(D)$.	[2; *D26*; 7; *T19*; *D26*]
12.	$A\wedge B\,\varepsilon\,\mathbf{ex}(C)$.	[*T265*; 3; 4; 5; *T30*; *T262*; *T263*]
13.	$E\,\varepsilon\,\mathbf{el}(C)$.	[3; 5; *T19*]

14. $E \varepsilon \mathbf{ex}(D).$ [*T262*; *T265*; 3; 8; *T30*; *T37*]
$C \varepsilon \mathbf{lnk}(D)$ [9; 10; 11; 12; 13; 14; *T31*]

AD14 $[ABC]: C \varepsilon \mathbf{T}_7(AB) .\equiv. C \varepsilon A \cup B \cup \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) *$
$\mathbf{Cm}(A \vee B) \cup A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B))$

T330 $[ABan] :: \mathbf{sfc}\{ABan\} . \boldsymbol{\delta}(AB) = 1 . \boldsymbol{\alpha}(AB) = 1 . \boldsymbol{\beta}(AB) \geqslant 2 .\supset\therefore [\exists b] \therefore$
$b \subset a . \mathrm{Cd}\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D .\supset. C \varepsilon \mathbf{lnk}(D)$

PR $[ABan] :: \mathrm{Hp}(4) .\supset\therefore$

5. $A \varepsilon \mathbf{lnk}(B) :$ [2; 3; 4; *T283*]

6. $[CD] : C \varepsilon \mathbf{T}_7(AB) . D \varepsilon \mathbf{T}_7(AB) . C \neq D .\supset. C \varepsilon \mathbf{lnk}(D) :$
[5; *T326*; *T327*; *T328*; *T329*; *T312*; *T318*; *T319*]

7. $A \cap B \circ \Lambda.$ [5; *T33*]

8. $A \cap \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{Cm}(A \vee B) \circ \Lambda.$
[*T264*; *T266*; *T267*; 1; *T30*]

9. $B \cap \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{Cm}(A \vee B) \circ \Lambda.$
[*T264*; *T266*; *T267*; 1; *T30*]

10. $A \cap A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [*T265*; 1; *T30*; 5]

11. $B \cap A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$ [*T265*; 1; *T30*; 5]

12. $\mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{Cm}(A \vee B) \cap (A \wedge B) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \circ \Lambda.$
[*T262*; *T263*; *T264*; *T265*; *T266*; *T267*; 1; *T30*]

13. $\mathrm{Cd}\{\mathbf{T}_7(AB)\} = \mathrm{Cd}\{A\} + \mathrm{Cd}\{B\} + \mathrm{Cd}\{\mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) *$
$\mathbf{Cm}(A \vee B)\} + \mathrm{Cd}\{A \wedge B * \mathbf{at}(\mathbf{Cm}(A \vee B))\}.$ [7; 8; 9; 10; 11; 12; **ON**]

14. $\mathrm{Cd}\{\mathbf{T}_7(AB)\} = 1 + 1 + 1 \cdot \boldsymbol{\beta}(AB) \cdot 1 + 1 \cdot \boldsymbol{\gamma}(A).$ [13; *T219*; 2; 3; 4]

15. $\mathrm{Cd}\{\mathbf{T}_7(AB)\} = n \therefore$ [14; 2; 3; *T268*]
$[\exists b] \therefore b \subset a . \mathrm{Cd}\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b .$
$C \neq D .\supset. C \varepsilon \mathbf{lnk}(D)$ [1; *AD14*; 6; 15]

T331 $[ABan] :: \mathbf{sfc}\{ABan\} . \boldsymbol{\delta}(AB) = 1 . \boldsymbol{\alpha}(AB) \geqslant 2 . \boldsymbol{\beta}(AB) = 1 .\supset\therefore [\exists b] \therefore$
$b \subset a . \mathrm{Cd}\{b\} \geqslant n . A \varepsilon b . B \varepsilon b : [CD] : C \varepsilon b . D \varepsilon b . C \neq D .\supset. C \varepsilon \mathbf{lnk}(D).$
[*T330*]

T332 $[ABCD] : C \varepsilon A \wedge B * \mathbf{Cm}(A \vee B) . D \varepsilon \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) *$
$\mathbf{at}(\mathbf{Cm}(A \vee B)) .\supset. C \varepsilon \mathbf{lnk}(D)$

PR $[ABCD] : \mathrm{Hp}(2) .\supset.$
$[\exists EFG].$

3. $E \varepsilon \mathbf{at}(A\backslash(A \wedge B)).$
4. $F \varepsilon \mathbf{at}(B\backslash(A \wedge B)).$ [2; *D26*]
5. $G \varepsilon \mathbf{at}(\mathbf{Cm}(A \vee B)).$
6. $D = E \vee G \vee G.$ [2; *D26*]
7. $C = (A \wedge B) \vee \mathbf{Cm}(A \vee B).$ [1; *D26*]
8. $G \varepsilon \mathbf{el}(\mathbf{Cm}(A \vee B)).$ [5; *D10*]
9. $G \varepsilon \mathbf{el}(C).$ [7; 8; *T20*; *T4*]
10. $G \varepsilon \mathbf{el}(D).$ [5; 6; *T20*]
11. $A \wedge B \varepsilon \mathbf{ex}(D).$ [1; *D26*; 3; 4; 6; 8; *T39*; *T30*]
12. $A \wedge B \varepsilon \mathbf{el}(C).$ [7; 11; *T19*]
13. $E \varepsilon \mathbf{el}(D).$ [3; 6; *T19*]
14. $E \varepsilon \mathbf{ex}(C).$ [3; 7; *T39*; *T30*]

$C \varepsilon \mathbf{lnk}(D)$ [9; 10; 11; 12; 13; 14; *T31*]

T333 $[ABC]: C\,\varepsilon\,(A \wedge B) \vee \mathbf{Cm}(A \vee B) \,.\, A\,\varepsilon\,\mathbf{Ink}(B) \,.\, \mathbf{Cm}(A \vee B)\,\varepsilon\,\mathbf{Cm}(A \vee B) \;.\!\supset\!.$
$A\,\varepsilon\,\mathbf{Ink}(C)$
PR $[ABC]: \mathrm{Hp}(3) \;.\!\supset\!.$
4. $A \wedge B\,\varepsilon\,A \wedge B\,.$ [2; *D8*; *D5*]
5. $A \wedge B\,\varepsilon\,\mathbf{el}(A)\,.$ [4; *T17*]
6. $A \wedge B\,\varepsilon\,\mathbf{el}(C)\,.$ [1; 4; *T19*]
7. $A\backslash(A \wedge B)\,\varepsilon\,\mathbf{el}(A)\,.$ [2; *D11*]
8. $A\backslash(A \wedge B)\,\varepsilon\,\mathbf{ex}(C)\,.$ [1; *D11*; 7; *T30*]
9. $\mathbf{Cm}(A \vee B)\,\varepsilon\,\mathbf{el}(C)\,.$ [1; 3; *T20*]
10. $\mathbf{Cm}(A \vee B)\,\varepsilon\,\mathbf{ex}(A)\,.$ [*D6*; *T30*]
$A\,\varepsilon\,\mathbf{Ink}(C)$ [5; 6; 7; 8; 9; 10; *T31*]

T334 $[ABC]: C\,\varepsilon\,(A \wedge B) \vee \mathbf{Cm}(A \vee B) \,.\, A\,\varepsilon\,\mathbf{Ink}(B) \,.\, \mathbf{Cm}(A \vee B)\,\varepsilon\,\mathbf{Cm}(A \vee B) \;.\!\supset\!.$
$B\,\varepsilon\,\mathbf{Ink}(C)$ [*T333*]

AD15 $[ABC]: C\,\varepsilon\,\mathbf{T}_8(AB) \;.\!\equiv\!.\; C\,\varepsilon\,A \cup B \cup \mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) *$
$(\mathbf{Cm}(A \vee B)) \cup A \wedge B * \mathbf{Cm}(A \vee B)$

T335 $[ABan] :: \mathbf{sfc}\,\{ABan\} \,.\, \boldsymbol{\delta}(AB) = 1 \,.\, \boldsymbol{\alpha}(AB) = 1 \,.\, \boldsymbol{\gamma}(AB) > 0 \,.\, \boldsymbol{\beta}(AB) = 1 \;.\!\supset\therefore$
$[\exists b] \therefore b \subset a \,.\, \mathrm{Cd}\,\{b\} \geqslant n \,.\, A\,\varepsilon\,b \,.\, B\,\varepsilon\,b : [CD] : C\,\varepsilon\,b \,.\, D\,\varepsilon\,b \,.\, C \neq D \;.\!\supset\!.$
$C\,\varepsilon\,\mathbf{Ink}(D)$
PR $[ABan] :: \mathrm{Hp}(5) \;.\!\supset\therefore$
6. $A\,\varepsilon\,\mathbf{Ink}(B) :$ [2; 3; 4; *T283*]
7. $[CD] : C\,\varepsilon\,\mathbf{T}_8(AB) \,.\, D\,\varepsilon\,\mathbf{T}_8(AB) \,.\, C \neq D \;.\!\supset\!.\; C\,\varepsilon\,\mathbf{Ink}(D) :$
[5; 6; *T332*; *T333*; *T334*; *T300*; *T301*; *T302*]
8. $A \cap (A \wedge B) * \mathbf{Cm}(A \vee B) \circ \Lambda.$ [3; *D26*; *T30*]
9. $B \cap (A \wedge B) * \mathbf{Cm}(A \vee B) \circ \Lambda.$ [4; *D26*; *D30*]
10. $\mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) * \mathbf{at}(\mathbf{Cm}(A \vee B)) \cap (A \wedge B) * \mathbf{Cm}(A \vee B) \circ \Lambda.$
[*T262*; *T263*; *T264*; *T265*; *T266*; *T267*; *D26*; *T30*]
11. $\mathrm{Cd}\,\{\mathbf{T}_8(AB)\} = \mathrm{Cd}\,\{A\} + \mathrm{Cd}\,\{B\} + \mathrm{Cd}\,\{\mathbf{at}(A\backslash(A \wedge B)) * \mathbf{at}(B\backslash(A \wedge B)) *$
$\mathbf{at}(\mathbf{Cm}(A \vee B))\} + \mathrm{Cd}\,\{(A \wedge B) * \mathbf{Cm}(A \vee B)\}.$
[8; 9; 10; proof as in *T305*; *AD15*]
12. $\mathrm{Cd}\,\{\mathbf{T}_8(AB)\} = 1 + 1 + 1 \cdot 1 \cdot \boldsymbol{\gamma}(AB) + 1.$ [11; 3; 4; *AD15*]
13. $\mathrm{Cd}\,\{\mathbf{T}_8(AB)\} = n \therefore$ [12; *T268*; 2; 3; 4]
$[\exists b] \therefore b \subset a \,.\, \mathrm{Cd}\,\{b\} \geqslant n \,.\, A\,\varepsilon\,b \,.\, B\,\varepsilon\,b : [CD] : C\,\varepsilon\,b \,.\, D\,\varepsilon\,b \,.\, C \neq D \;.\!\supset\!.$
$C\,\varepsilon\,\mathbf{Ink}(D)$ [*AD15*; 1; 7; 13]

T336 $[ABan] :: \mathbf{sfc}\,\{ABan\} \,.\, \boldsymbol{\alpha}(AB) \geqslant 1 \,.\, \boldsymbol{\beta}(AB) \geqslant 1 \,.\, \boldsymbol{\delta}(AB) = 1 \;.\!\supset\therefore [\exists b] \therefore$
$b \subset a \,.\, \mathrm{Cd}\,\{b\} \geqslant n \,.\, A\,\varepsilon\,b \,.\, B\,\varepsilon\,b : [CD] : C\,\varepsilon\,b \,.\, D\,\varepsilon\,b \,.\, C \neq D \;.\!\supset\!.\; C\,\varepsilon\,\mathbf{Ink}(D)$
[*T325*; *T330*; *T331*; *T335*]

T337 $[ABan] : \mathbf{sfc}\,\{ABan\} \,.\, A\,\varepsilon\,\mathbf{Ink}(B) \;.\!\supset\!.\; \boldsymbol{\alpha}(AB) \geqslant 1 \,.\, \boldsymbol{\beta}(AB) \geqslant 1 \,.\, \boldsymbol{\delta}(AB) \geqslant 1$
PR $[ABan] : \mathrm{Hp}(2) \;.\!\supset\!.$
$[\exists CDE]\,.$

$$\left.\begin{array}{ll} 3. & C\,\varepsilon\,\mathbf{el}(A)\,. \\ 4. & C\,\varepsilon\,\mathbf{el}(B)\,. \\ 5. & D\,\varepsilon\,\mathbf{el}(A)\,. \\ 6. & D\,\varepsilon\,\mathbf{ex}(B)\,. \\ 7. & E\,\varepsilon\,\mathbf{el}(B)\,. \\ 8. & E\,\varepsilon\,\mathbf{el}(A)\,. \end{array}\right\} \quad [2;\ D8]$$

9. $A \wedge B\,\varepsilon\,a\,.$ [3; 4; 1; *AD2*; *D5*]

10. $A\backslash(A \wedge B)\, \varepsilon\, a\, .$ [5; 6; 1; *AD2*; 9; *D11*]

11. $B\backslash(A \wedge B)\, \varepsilon\, a\, .$ [7; 8; 1; *AD2*; 9; *D11*]

$\boldsymbol{\alpha}(AB) \geqslant 1 \,.\, \boldsymbol{\beta}(AB) \geqslant 1 \,.\, \boldsymbol{\delta}(AB) \geqslant 1$ [1; 9; 10; 11; *T206*]

T338 $[ABan] :: \mathsf{sfc}\, \{ABan\} \,.\, A\, \varepsilon\, \mathsf{lnk}(B) \;.\supset\therefore\; [\exists b] \therefore\; b \subset a \,.\, \mathsf{Cd}\, \{b\} \geqslant n \,.\, A\, \varepsilon\, b\, .$
$B\, \varepsilon\, b : [CD] : C\, \varepsilon\, b \,.\, D\, \varepsilon\, b \,.\, C \neq D \;.\supset.\; C\, \varepsilon\, \mathsf{lnk}(D)$
[*T308*; *T324*; *T336*; *T337*]

T339 $[ABan] : \mathsf{sfc}\, \{ABan\} \,.\, A\, \varepsilon\, \mathsf{pr}\,(B) \,.\, n \geqslant 2 \;.\supset.\; [\exists b] \,.\; b \subset a \,.\, \mathsf{Cd}\, \{b\} \geqslant n\, .$
$A\, \varepsilon\, b \,.\, B\, \varepsilon\, b \,.\, \mathsf{cl}\,(nb)$ [*T261*; *D27*; *D29*; *DS1*]

T340 $[ABan] : \mathsf{sfc}\, \{ABan\} \,.\, A\, \varepsilon\, \mathsf{lnk}(B) \;.\supset.\; n \geqslant 2 \,.\, [\exists b] \,.\, b \subset a \,.\, \mathsf{Cd}\, \{b\} \geqslant 2\, .$
$A\, \varepsilon\, b \,.\, B\, \varepsilon\, b \,.\, \mathsf{cl}\,(nb)$ [*T202*; *T338*; *D28*; *D29*; *DS1*]

T341 $[ABan] : \mathsf{sfc}\, \{ABan\} \,.\, B\, \varepsilon\, \mathsf{pr}\,(A) \,.\, n \geqslant 2 \;.\supset.\; [\exists b] \,.\, b \subset a \,.\, \mathsf{Cd}\, \{b\} \geqslant n\, .$
$A\, \varepsilon\, b \,.\, B\, \varepsilon\, b \,.\, \mathsf{cl}\,(nb)$ [*T339*]

T342 $[ABan] : \mathsf{sfc}\, \{ABan\} \,.\, n \geqslant 2 \,.\, A \neq B \,.\, \sim(A\, \varepsilon\, \mathsf{ex}(B)) \;.\supset.\; [\exists b] \,.\, b \subset a\, .$
$\mathsf{Cd}\, \{b\} \geqslant n \,.\, A\, \varepsilon\, b \,.\, B\, \varepsilon\, b \,.\, \mathsf{cl}\,(nb)$ [*T33*; *T339*; *T340*; *T341*]

T343 $[ABan] :: \mathsf{sfc}\, \{ABan\} \,.\, n \geqslant 2 \;.\supset\therefore\; A\, \varepsilon\, \mathsf{ex}(B) \;.\equiv:\; [b] : b \subset a\, .$
$\mathsf{Cd}\, \{b\} \geqslant n \,.\, A\, \varepsilon\, b \,.\, B\, \varepsilon\, b \,.\, A \neq B \;.\supset.\; \sim(\mathsf{cl}\,(nb))$ [*T255*; *T342*]

Hence, *T343* gives the desired result. We have, for each cardinal number greater than or equal to 2, a term which is primitive if and only if it is defined on a subsystem of cardinality at least $2^n - 1$.

REFERENCE

[19] Welsh, P. J., "Primitivity in mereology. I," *Notre Dame Journal of Formal Logic*, vol. XIX (1978), pp. 25-62.

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