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SOME NOTES ON "A DEDUCTION THEOREM FOR RESTRICTED GENERALITY"

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In [2] the deduction theorem for Ξ:

If X_0, $X \vdash Y$, *and* $X_0 \vdash \mathbf{L}([\mathbf{u}]\mathbf{X})$ *where* u *is not involved in* X_0, *then* $X_0 \vdash X \supset_u Y$,[1]

was proved using the following axioms.

Axiom 2. $\vdash \mathbf{L}x \supset_x \Xi xx$.
Axiom 3. $\vdash \mathbf{L}x \supset_{x,y} : xu \supset_u . \ yuv \supset_v xu$.
Axiom 4. $\vdash \mathbf{L}x \supset_{x,t} : xu \supset_u yu(tu) \supset_y . \ (xu \supset_u (yuv \supset_v zuv)) \supset_x (xu \supset_u zu(tu))$.
Axiom 5. $\vdash \mathbf{L}x \supset_x \Xi x\ (\mathbf{WQ})$.
Axiom 6. $\vdash \Xi \mathbf{IH}$.
Axiom 7. $\vdash \mathbf{LH}$.

Of these, $\vdash \mathbf{LH}$ as it restricts the system to obs which satisfy

$$\mathbf{A}u \vdash \mathbf{H}(\mathbf{H}u),$$

is a somewhat unsatisfying axiom. In particular with $\mathbf{E} = \mathbf{A}$ it is inconsistent with the others (see [1]).

Also the rules obtained by applying Rule Ξ once to each of the remaining axioms are consistent. This was shown in an unpublished paper by H. B. Curry and the author. Curry in [3] proved that for an equivalent system no nonpropositions are provable and Seldin in [4] has shown consistency in a stronger sense.

We show here that the deduction theorem for Ξ can be proved without $\vdash \mathbf{LH}$. We achieve this by taking $\mathbf{L}$ as primitive (rather than as defined by $\mathbf{L} \equiv \mathbf{FAH}$) and we define $\mathbf{H}$ as $\mathbf{BLK}$. Axiom 3 leads to the rule:

$$\mathbf{L}x,\ xu \vdash yuv \supset_v xu$$

1. In [2] $\mathbf{L} \equiv \mathbf{FAH}$. $X \supset_u Y$ is an alternative notation for $\Xi([u]X)([u]Y)$.

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so with $\mathbf{K}Y$ for x and $\mathbf{K}X$ for y we obtain

$$\mathbf{BLK}Y, Y \vdash Xv \supset_v Y.$$

Axiom 6 then allows us to derive the rule that was used in Case 2 of the proof in [2], which was the only case in which $\vdash\mathbf{LH}$ was used.

$\vdash\mathbf{LH}$ was also used in deriving $\mathbf{L}x \vdash \Xi xx$ from the other axioms. This can still be done so strictly Axiom 2 is not needed. From Axiom 4 we obtain:

$$\mathbf{L}x,\ xu \supset_u yu(tu),\ xu \supset_u (yuv \supset_v zuv) \vdash xu \supset_u zu(tu).$$

With $z = \mathbf{K}x$ and $y = \mathbf{K}([u].\ zuv \supset_v xu)$, we have by Axiom 3:

$$\mathbf{L}x \vdash xu \supset_u (yuv \supset_v zuv)$$

and

$$\mathbf{L}x \vdash xu \supset_u yu(tu)$$

so that

$$\mathbf{L}x \vdash \Xi xx$$

follows. Thus the deduction theorem for Ξ can be proved on the basis of Axioms 3, 4, 5, and 6. (In a system without equality Axiom 5 is also unnecessary).

REFERENCES

[1] Bunder, M. W., "A paradox in illative combinatory logic," *Notre Dame Journal of Formal Logic*, vol. XI (1970), pp. 467-470.

[2] Bunder, M. W., "A deduction theorem for restricted generality," *Notre Dame Journal of Formal Logic*, vol. XIV (1973), pp. 341-346.

[3] Curry, H. B., "The consistency of a system of combinatory restricted generality," *The Journal of Symbolic Logic*, vol. 38 (1973), pp. 489-492.

[4] Seldin, J. P., "The **Q**-consistency of $\mathfrak{F}_{22}$," to be published in *Notre Dame Journal of Formal Logic*.

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