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MATERIAL IMPLICATION, CONFIRMATION, AND COUNTERFACTUALS

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1 Students of truth-functional logic frequently regard material implication to be patently absurd. Most of us who teach elementary logic have encountered intelligent students who frustratedly exclaimed something to the effect that: Any logic which pronounces true a sentence such as, "If the moon is green cheese, John F. Kennedy was 35th President of the United States," is illogical. A great deal of printer's ink has been spilled in the attempt to rationalize away the paradoxes of material implication: if a proposition p is false then, whatever proposition q may be, the proposition *if* p *then* q is true; and again if q is a true proposition then, whatever proposition p may be, the proposition *if* p *then* q is true. Although I have contributed to this effort myself,[1] I am at last inclined to throw in the towel and admit the endeavor is fruitless, that the paradoxes and problems generated by material implication are intolerable embarrassments. I am encouraged in my attitude of intolerance by the fruits this act of pruning will provide, as the essay proceeds.

Implicational propositions may be categorized, with respect to the determinability of the meaning and truth-value of the sentences which express them, as follows. In the first place, we have a proposition whose meaning and whose truth-value are determinate, such as:

(1) If John F. Kennedy was 35th President of the United States, he was assassinated in Dallas.

Next we need a proposition whose meaning is indeterminate as far as its audient or reader is concerned, and whose truth-value is, *a fortiori*, also indeterminate. Two types of propositions immediately come to mind. There are amphibolous constructions, such as the fatal oracle received by Croesus; and there are propositions expressed in an unfamiliar language, which one has reason to trust are serious and correct. A remark about amphibolous statements might be in order here. A wiser king than Croesus

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might have demanded clarification from the priestess. But a clarification such as: "If you attack Persia, Persia or Lydia will perish," substitutes a new proposition for the old, and actually fits under the fourth category yet to be discussed. On the assumption that few who read this are familiar with Irish, let me merely say that "má" is a word for "if" and that the following sentence is grammatically correct:

(2) Má scríobtar lasán lasfaidh sé.

Third we need a proposition whose meaning is unclear, but whose truth-value is determinate. How can one hope to determine the truth-value of a proposition whose meaning he does not understand? One type of proposition occurs to me: a proposition to which one assigns a truth-value because he assents to (or denies) it on the basis of authority. Such a proposition might be an article of faith, such as:

(3) If the Church is infallible, there are three Persons in one God.

Finally we need a proposition whose meaning is determinate, but whose truth-value is indeterminate because, for instance, the truth-values of its components are not known. Thus in the presence of a covered bird cage which he is informed contains a raven, one might assert:

(4) If that is a raven, it is black.

On the assumption that (4) expresses an empirical hypothesis, we have to admit that it may be false because mutations such as that which has produced a race of blue bullfrogs in the vicinity of Fort Knox, Kentucky, are not unknown to zoologists.[2] Since (4) may be deemed false (though more probably would prove true) when the cover is removed, its value as it stands is indeterminate. Another historically interesting type of proposition which fits into this category is the future contingent proposition. To sum up, if we form a box over which we place *M* and *T* (for Meaning and Truth-Value), to the left of which we place a column of 1, 2, 3, 4; and inside of which we signigy by + or - whether the former is determinable of the latter, we have:

	M	*T*
1	+	+
2	-	-
3	-	+
4	+	-

Now the purpose of the remarks made thus far has been to recapitulate a position well known since Aristotle's *De Interpretatione*, namely, that it is possible for a proposition to be indeterminable as to truth-value.

Since Łukasiewicz's 1920 paper,[3] the accepted tactic for reducing problems arising from indeterminability of truth-value has been to construct a 3-valued logic. However, the 3-valued truth-functional logics which have been constructed so far do not contribute to the solution of the problems I am concerned with. I find myself attracted to the system of Kleene, which adds an I-value to the system of Łukasiewicz for $p \rightarrow q$ (viz., when p and q are both I); but to solve the paradoxes of material implication we shall have to extend his initiative and introduce even more I-values, and in fact I-values of a different kind. Following the notational conventions found in Rescher's *Many-Valued Logic*, we shall use "$\neg$" for "not", "$\wedge$" for "and", "$\vee$" for "or", "$\rightarrow$" for "if", and "$\leftrightarrow$" for "if and only if"; we shall refer to a 3-valued logic by printing the first letter of the author's surname in bold type, with a "3" subscripted to the right. Thus Łukasiewicz's system is $\mathbf{L}_3$, Kleene's is $\mathbf{K}_3$, and the system to be proposed in this paper is $\mathbf{F}_3$.

The principal deviation of $\mathbf{F}_3$ from earlier three-valued logics consists in its interpretation of "I" as "inappropriate" rather than as "indeterminate". I do believe that there is an appropriate place for the notion of indeterminacy in truth-functional logic, and I shall make use of this notion later in anticipating a possible objection to $\mathbf{F}_3$. However, I prefer to symbolize indeterminacy (whenever it is necessary to symbolize it) as T/F. In doing so, I am committing myself to the view that there are only two, and not an infinite number, of truth-values for a proper proposition. A proposition such as, "The next roll of the dice will come up snake-eyes" may be evaluated as *probably false*, in the sense in which Toulmin explained probability[4]—i.e., I would say it will be false, but I am reserving judgement since I recognize a one-in-thirty-six chance of its turning out true. The opportunity to put a numerical measure on the degree of indeterminacy is, of course, accidental to the notion of indeterminacy itself. What is essential is simply the recognition that an indeterminate proposition lies "somewhere between" the poles of truth and falsehood. The symbolism T/F has the advantage of immediately making clear the validity of the Law of the Excluded Middle, thus:

p	$(p$	$\vee$	$\neg p)$
T	T	T	F
T/F	T/F	T/T	F/T
F	F	T	T

Keeping in mind, then, that I-value signifies inappropriate, it turns out that $\mathbf{F}_3$ agrees with $\mathbf{L}_3$ in its interpretation of the functors "$\neg$", "$\wedge$", and "$\vee$". The critical difference occurs with respect to "$\rightarrow$", and of course "$\leftrightarrow$". The propositional functors of $\mathbf{F}_3$ are interpreted as follows:

p	$\neg p$
T	F
I	I
F	T

	$p \wedge q$			$p \vee q$			$p \rightarrow q$			$p \leftrightarrow q$		
p/q	T	I	F	T	I	F	T	I	F	T	I	F
T	T	I	F	T	T	T	T	I	F	T	I	F
I	I	I	F	T	I	I	I	I	F	I	I	F
F	F	F	F	T	I	F	I	I	I	F	F	I

An explanation of the linguistic rationale for these truth-tables is in order here. The functor "$\neg$" has straightforward interpretation. If p is true, $\neg p$ is false, and vice versa; but if p is I, it is inappropriate to assign truth-value to it, in which case it is also inappropriate to assign truth-value to its negation. $\mathbf{L}_3$ maintains that conjunction takes the weakest value of either of its components, and $\mathbf{F}_3$ concurs. It might be worth noting in passing, however, that this means we cannot interpret conjunction as asserting that both of the conjoined propositions are true. For then, if either of the conjuncts is inappropriate, we would have to evaluate the entire conjunction as false; for the condition of truth would be violated in that it would not be possible for both conjuncts to be true. After considering just such an interpretation, I have not found it useful for two reasons. First, the propositional function $p \leftrightarrow p$ could not be a tautology, for two of its values would be F, viz., for p is I and for p is F. Second, I do not think this interpretation mirrors the way we ordinarily use the language. Consider the following fictional situation. Two Athenians are discussing the outlook of the war. A Spartan fleet of 150 ships is lying off the Piraeus, and the Athenians have only 50 ships with which to challenge their blockade. The speaker, however, has information that an ally of 200 ships is rapidly approaching the area. He says: "We are sorely pressed here, and a great city-state is about to be defeated." His statement consists of a true conjunct, and an amphibolous one (which is inappropriate use of language). Now it may be that his audient would fail to recognize the amphibolous character of the second conjunct, but if it were noticed, e.g., by a party who had also been apprised of the approach of the ally, I do not think that this party would call the statement false. I think it would be considered, at worst, a perversity of discourse, inappropriate behavior for a serious discussion. Hence the conjunction receives a value of I. In the light of what has been said about conjunction, the interpretation of "$\vee$" is unproblematical. A disjunction is true if either disjunct is true, false if both are false, and otherwise inappropriate.

The innovative functor of $\mathbf{F}_3$ is "$\rightarrow$". The functor "$\leftrightarrow$" is a mere product of the former and "$\wedge$", being understood thus $p \leftrightarrow q$ for $(p \rightarrow q) \wedge (q \rightarrow p)$.[5] It is now incumbent on me to justify the values I have chosen for $p \rightarrow q$. In accordance with the discussion concerning the functor "$\wedge$", I believe that if one of the arguments of a conditional is true and the other inappropriate, the whole conditional receives I-value. $T \rightarrow T$, $T \rightarrow F$, and $I \rightarrow I$ are unproblematical. That leaves conditionals whose antecedent is false, and those whose antecedent is inappropriate and whose consequent is false. As I said at the outset of this paper, I regard propositions of the form $F \rightarrow T$ and, *pari passu*, $F \rightarrow F$ to be intolerable linguistic usages. Obviously then, if the antecedent of a conditional is or may be false, the whole conditional must be inappropriate, and must receive an I-value. The traditional interpretation handed down from the Stoics to the effect that a false antecedent yields a true conditional is the Gordian knot that has tied up the application of logistic analysis to scientific and other forms of rational discourse, and it must be severed. My justification is essentially simple. As I understand the language, when I state a conditional proposition in serious discourse, I am saying: "I give you this antecedent; I lay it down before you as the foundation of my conclusion; and I tell you that with it given my consequent (conclusion) can be trusted in that it follows from it whether by strict implication or causal relation or empirical correlation or whatever." This being my understanding of the meaning of the word "if", I regard as most inappropriate any utterance which pretends to give a basis for implication and at the very same time undercuts that basis by presenting an antecedent *known* to be false. Note the emphasis on "*known* to be false." Someone may object that we want to be able to assert that q on the assumption that p, but without committing ourselves to the truth of that assumption; and so must allow for true conditionals which turn out to have false antecedents. This contingency, however, is amply provided for by the indeterminate value mentioned earlier. The proposition "q on the assumption that p" might be analyzed:

p	q	$p \rightarrow q$
T/F	T	T/I

which, as we will see, does not interfere with the thesis-status of any formula for which it occurs. This leaves the $I \rightarrow F$ as F evaluation to be explained yet, but we must postpone this discussion briefly until further considerations prepare the way.

There is another irregular but not unheard of convention to be introduced into $\mathbf{F}_3$, which is that any wff is a thesis on condition that it has, for all possible combinations of genuine values ($T \& F$) of its arguments, at least one T-value and no F-value. I am assuming that serious discussion should eschew consideration of atomic propositions which are known to be inappropriate, and hence I-value does not figure into the combinations of values for the atomic propositions. Again, a contradiction occurs when there is at least one F-value and no T-value. That is to say, a thesis is any wff all of whose genuine values are T and which has at least one genuine value; and vice versa for a contradiction. A contingency will have at least one T-value and one F-value. Finally, any wff all of whose values are I is inept. To demand the traditional strong criterion for thesishood: viz., that the value of the formula be T for every combination of values of the arguments, would eliminate many intuitively reasonable rules of inference from truth-functional logic, such as Modus Ponens. Besides, the weaker condition guarantees the Law of Deduction: viz., that a valid argument is such that it cannot have true premisses and a false conclusion. Any wff which may have true premisses and a false conclusion for some combination of truth-values of its arguments will receive at least one F-value in $\mathbf{F}_3$, just as it should.

Now let us pose some objections to $\mathbf{F}_3$, and, hopefully, eliminate them. In the first place, $\mathbf{F}_3$ disqualifies many time-honored rules of inference from the status of thesishood. Clavius' Law, $(\neg p \wedge p) \rightarrow p$, fails by receiving I-value for both values of p. The Law of Duns Scotus, $p \rightarrow (\neg p \rightarrow q)$, fails for the same ineptitude. But then 3-valued logics generally exclude some venerable formulas from the status of thesishood, and I must say that I am perfectly delighted to dance upon the grave of these paradox-laden formulas. Two other important casulties are the Law of Contraposition and Modus Tollens; but we shall see later how to resurrect what was valuable in the former (and the means to revive the latter will then be obvious). On the other hand, $\mathbf{F}_3$ admits to thesishood an interesting and intuitively valid formula, $\neg(p \to \neg p)$, whose rejection by the traditional two-valued logic should count seriously against the latter's claim to be the most apt instrument for codifying the relevant rules of reasoning.

The most serious objection to $\mathbf{F}_3$ which I have had to overcome, however, concerns a classical fallacy which threatened to become tautologous. I am referring to the Fallacy of Affirming the Consequent. Originally, I had considered $I \to F$ to be I, and on that basis the following truth-table disconcertingly revealed that $((p \to q) \wedge q) \to p$ is a thesis in $\mathbf{F}_3$.

p	q	$((p \to q)$	$\wedge\ q)$	$\to p$
T	T	T	T	T
T	F	F	F	I
F	T	I	I	I
F	F	I	F	I

This was not the only problem: as would be expected by exportation, $(p \to q) \to (q \to p)$ turned out to be a thesis, and Denial of the Antecedent turned out to be merely an ineptitude, not a fallacious contingency. What was I to make of this? The value of $\mathbf{F}_3$, as I had conceived it, lay in its returning the analytical use of truth-functional logic to the firm ground of common sense. It got rid of the paradox of material implication, and, as we shall see, it solved both the paradox of confirmation and the problem of counterfactual conditionals. Yet, let all this be so, what would it profit a man to resolve all manner of paradoxes, if he should suffer assent to fallacy?

Were it not for the problems presented by the exportation of affirmation of the consequent and by denial of the antecedent, I might be willing to go along with $((p \to q) \wedge q) \to p$. For what is wrong with affirming the consequent is that more than one antecedent may imply the same consequent, and the truth of the consequent does not permit us to conclude that one of the two contrary antecedents is true. Suppose some should assert: "If Newton's theory of gravity is true, then a lead ball when dropped from the Tower of London falls; and a lead ball when dropped from the Tower of London does fall; so Newton's theory of gravity is true." One would immediately reply: "Well, Einstein's theory of gravity also correctly predicts the behavior of such a ball; and Einstein's theory contradicts Newton's theory; so you have no right from this instance of confirmation to conclude that Newton's theory is the true one." Well and good, but you see the problem here is that more than one theory is known to be available for explaining the phenomenon, and that fact makes it inappropriate for one to claim conclusive verification of a hypothesis which is so opposed on the basis of confirmation instances. When such an argument is put forward, one might argue, the initial $p \to q$ premiss might even receive, on extralogical grounds, a value of "inappropriate" from the scientific community. On the other hand, if only one hypothesis is available, each single confirmation instance does give some evidence for that hypothesis—it being only remembered that no product of human reason is ever entitled to immunity from challenge. A formula which serves the purpose of the fallacy of affirming the consequent is available in $\mathbf{F}_3$ for the case when two alternative hypotheses are recognized: viz., $(((p \vee r) \rightarrow q) \wedge q) \rightarrow p$, which is F where p, q, and r are F, T, and T respectively.

However, given the other problems, and given the danger that admission of $((p \rightarrow q) \wedge q) \rightarrow p$ might well leave us like the man from Industan who concluded, blindly, that an elephant is a snake, another solution which abrogates affirmation of the consequent would be desirable. And such a solution has been found. It consists simply in evaluating $I \rightarrow F$ as F. The justification promised earlier for this part of the truth-table of $p \rightarrow q$ is essentially *ad hoc*. It works. When p is F and q is T, affirmation of the consequent, its exportation, and denial of the antecedent all are valued false. What is more, perhaps this is just what the practice of scientific logic would lead us to expect. For suppose a theory entails numerous observation statements and suppose all of the latter are true, and then suppose that the scientific community rejects this theory as false, perhaps because it fails to cohere with a broader, more powerful theory. Would we not want the truth-table for an affirmation of the consequent argument concerning this theory and its entailed observation statements to come out false, thus:

T	O_i	$((T \rightarrow O_i) \wedge O_i) \rightarrow T$
F	T	$I \quad I \quad F\,F$

This concludes the presentation and defense of $\mathbf{F}_3$. It has already proven its merit in eliminating the paradoxes of material implication, and validating the rule of reasoning that it is not the case that a proposition entails its contradictory. This alone should qualify it for serious consideration as an alternative system of truth-functional logic. Moreover, we shall now discover that it also unsticks two very important investigations of logistic analysis—namely, Carl Hempel's paradox of confirmation and Nelson Goodman's problem of counterfactual conditionals.

2 Carl Hempel's "Studies in the Logic of Confirmation" considered, and rejected, Jean Nicod's 1930 analysis of confirmation:

> Consider the formula or the law: *A entails B*. How can a particular proposition, or more briefly, a fact, affect its probability? If this fact consists of the presence of *B* in a case of *A*, it is favorable to the law '*A* entails *B*'; on the contrary, if it consists of the absence of *B* in a case of *A*, it is unfavorable to this law. It is conceivable that we have here the only two direct modes in which a fact can influence the probability of a law . . . Thus, the entire influence of particular truths or facts on the probability of universal propositions or laws would operate by means of these two elementary relations which we shall call *confirmation* and *invalidation*.[6]

Hempel understood Nicod to maintain that the law:

$$(x)[P(x) \supset Q(x)]$$

is confirmed by an object a if a is P and Q, disconfirmed if a is P but not Q; and if a is not P, a is neutral, or irrelevant, with respect to the law.

Hempel found Nicod's criterion inadequate, in the first place, on the ground that it provided "no standards of confirmation for existential hypotheses (such as 'There exists organic life on other stars' or 'Poliomyelitis is caused by some virus') or for hypotheses whose explicit formulation calls for the use of both universal and existential quantifiers (such as 'Every human being dies some finite number of years after his birth'). . . ."[7] This is an inadequacy, at most, not a fatal objection; it demands the extension of Nicod's criterion to these important and more complex cases. However, the following objection was taken to be fatal, even to the simple case of a universal lawlike statement. Hempel considered two propositions:

(S_1) $(x)[\text{Raven}(x) \supset \text{Black}(x)]$
(S_2) $(x)[\sim\text{Black}(x) \supset \sim\text{Raven}(x)]$

He took these two propositions to be logically equivalent, as they certainly are in 2-valued logic, and laid down an Equivalence Condition: "Whatever confirms (disconfirms) one of two equivalent sentences, also confirms (disconfirms) the other."[8] This condition, he argued, and I agree, is necessary because:

> Otherwise, the question as to whether certain data confirm a given hypothesis would have to be answered by saying: "That depends on which of the different equivalent formulations of the hypothesis is considered"—which appears absurd. Furthermore—and this is a more important point than an appeal to a feeling of absurdity—an adequate definition of confirmation will have to do justice to the way in which empirical hypotheses function in theoretical scientific contexts such as explanations and predictions; but when hypotheses are used for purposes of explanation and prediction, they serve as premises in a deductive argument whose conclusion is a description of the event to be explained or predicted. The deduction is governed by the principles of formal logic, and according to the latter, a deduction which is valid will remain so if some or all of the premises are replaced by different but equivalent statements. . . .[9]

On the supposition that S_1 and S_2 are equivalent, Hempel shows that the equivalence condition is violated by Nicod's criterion. He considers four objects such that:

a is a raven and is black
b is a raven but is not black
c is not a raven but is black
d is neither a raven nor is black

Object *a* confirms S_1 but is irrelevant to the equivalent S_2; *b* disconfirms both S_1 and S_2; *c* is irrelevant to both S_1 and S_2; and *d* is irrelevant to S_1 but confirms S_2. In other words, the objects which confirm S_1 are irrelevant to S_2, and vice versa.

In $\mathbf{F}_3$, however, the critical supposition does not hold; for the Law of Contraposition is not a thesis. The propositions $p \to q$ and $\neg q \to \neg p$, far from being equivalent, do not even imply each other. Their truth-tables are reproduced below:

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	T	I
T	F	F	F
F	T	I	I
F	F	I	T

Notice that, when p stands for "x is a raven" and q for "x is black", the first row of the truth-value combinations corresponds to the object a above, the second to b, third to c, and fourth to d. Since the assumed equivalence of S_1 and S_2 was based on the assumption of the equivalence of $p \rightarrow q$ and $\neg q \rightarrow \neg p$, and since these latter are *not equivalent*, the objection to Nicod's construal of confirmation vanishes. Indeed, the truth-table above shows that the results which troubled Hempel are just what should be expected. Object a turns out to confirm S_1 (T-value) and be irrelevant to S_2 (I-value); object b disconfirms both S_1 and S_2; object c is irrelevant to S_1 and S_2; and object d is irrelevant to S_1 but confirms S_2.

The paradox of confirmation, which Hempel next considered, also dissolves in $\mathbf{F}_3$. We do not have to recognize red pencils, green leaves, yellow cows, or any and all black objects as confirming the hypothesis that all ravens are black, since objects which confirm S_2 are indeed irrelevant to S_1. What then are we to say of Hempel's ingenious effort to show that the paradox of confirmation is not objectively founded, i.e., is a psychological illusion. I am dissatisfied with this effort, just as Hempel himself was dissatisfied with other confusions of logical and psychological issues.[10] Moreover, Hempel rejected two attempts to solve the paradoxes (viz., supplementing the customary universal conditional by an existential clause, and supplementing it by indicating a specific "field of application" of the hypothesis) on the grounds that neither reflects the procedure of science. The same objection is applicable to any attempt to rehabilitate the paradoxes. In genuine scientific discourse, yellow cows simply do not confirm hypotheses about black ravens!

3 Nelson Goodman described the problem of counterfactual conditionals as the need "to define the circumstances under which a given counterfactual holds while the opposing conditional with the contradictory consequent fails to hold."[11] He illustrated with two examples (which I modify slightly as to temperature, for reasons of convenience which will appear later):

(5) If that piece of butter had been heated to 32.2°C, it would have melted.

(6) If that piece of butter had been heated to 32.2°C, it would not have melted.

He argued that since the antecedents of counterfactual conditionals are false, they must all be true, irrespective of the values of their consequents, and even if their consequents are contradictory. In the light of what has been discovered with $\mathbf{F}_3$, one might be tempted to argue that the false antecedents of counterfactual conditionals require that they receive I-values. Such an approach would leave us worse off than we were with Goodman; for there is obviously something very appropriate about this type of proposition, which is utilized so extensively in both ordinary and scientific discourse.

The solution, strangely enough, lies in the recognition that the antecedents of true counterfactual conditionals are *true*! I say this without facetious intent, as the following considerations will show. Let us begin with the particular conditional:

(7) If that piece of butter has not melted, it has not been heated to 32.2°C.

If the Law of Contraposition were valid in $\mathbf{F}_3$, we might derive from (7) the following:

(8) If that piece of butter has been heated to 32.2°C, it has melted.

This looks much like Goodman's (5), *except* that the arguments are stated in the indicative mood. As such, the values for its arguments, noting that (7) is true, are F and F. However, the crucial difference between (8) and (5) is the use of the subjunctive mood in (5). Thus (8) is not a counterfactual conditional; it is an inappropriate conditional.

The contrary-to-fact subjunctive construction introduces an important *modality* which must be considered in the logical analysis of counterfactual conditionals. Just as the alethic modality:

> *It is necessary that* the sum of the interior angles of a Euclidean triangle are equal to 180°.

adds an important factor to:

> The sum of the interior angles of a Euclidean triangle are equal to 180°.

so the counterfactual modality:

> If that piece of butter *had been* heated to 32.2°C, it *would have* melted.

adds an important modal factor to:

> If that piece of butter *has been* heated to 32.2°C, it *has* melted.

The use of this modality has a similar (though not identical) effect as the use of the double negative in the indicative mood. That is, it serves to cancel out the falsehood of the contraposed proposition to which it is attached.

This counterfactual modality provides us with a means to save what was valuable in the Law of Contraposition. In proper grammatical usage, a contrapositive argument ought not to be stated entirely in the indicative. Thus:

> From the fact that if this match is scratched it will light, if follows that if it does not light it was not scratched.

is awkward. We should say:

> From the fact that if this match is scratched it will light, it follows that if it *were* not to light, it *would* not have been scratched.

In other words, in contraposition we encounter arguments of mixed modality. Since squares and diamonds have been appropriated already in modal logic, perhaps we should use some figure for the symbolization of this counterfactual modality (maybe spades, or clubs?). For the present, I prefer simply to subscript an italicized *cf* to the right of the sentential symbol so modified. With this in mind, I might take as axiomatic:

$$p \rightarrow q \equiv_{cf} \neg q \rightarrow_{cf} \neg p$$

It would be easy to derive the corollaries:

$$p \rightarrow \neg q \equiv_{cf} q \rightarrow_{cf} \neg p \qquad \text{and} \qquad \neg p \rightarrow q \equiv_{cf} \neg q \rightarrow_{cf} p$$

It is obvious from ordinary linguistic usage that a conditional proposition, stated in the counterfactual modality, is true exactly when its corresponding indicative contrapositive is true. Thus Goodman's (5) is true, because our (7) is true. As in the nonmodal logic of F_3, a conditional is true only if both antecedent and consequent are true. So, as noted earlier, the solution to the problem of counterfactual conditionals rests on the recognition that their antecedents can be true. Therefore, Goodman's troublesome (6) is false--is of the form $T \rightarrow F$—because its consequent is the negation of the true consequent of (5). Thus the problem of counterfactual conditionals is resolved.

One might wonder what effect this interpretation of the Law of Contraposition has on Hempel's paradox of confirmation. $Ra \rightarrow Ba$ is equivalent to ${}_{cf}\neg Ba \rightarrow_{cf} \neg Ra$. Therefore whatever confirms the latter should also, by the Equivalence Condition, confirm the former. True, but yellow cows still cannot figure into the confirmation of "All ravens are black" because, in science, confirmation is accomplished by prediction, and predictions are properly stated in the indicative mood. It is senseless to ask what confirms a counterfactual. When a scientist performs a "mental" experiment, expressing it as a subjunctive counterfactual, he is not committing his law or theory to the test, nor offering evidence for it; he is rather asking us to take it on faith.

John van Heijenoort wrote:

> Any given paradox rests on a number of definitions, assumptions and arguments, and we can solve it by questioning any of these. That is why the literature on paradoxes is so rich and abounds with so many solutions. . . . For the important paradoxes, the question is not of solving them by any means but of solving them by means that enlarge and strengthen our logical intuitions. It is to find, among the sometimes too numerous solutions, the one that fits our logic most smoothly and perhaps, to some extent, to adapt our logic to this solution.[12]

It is in this spirit that the F_3 system is offered to the consideration of contemporary logicians. Hempel and Goodman cannot be faulted for failing to solve the paradoxes they discovered. They were limited by the tool at their disposal. Indeed, it is their investigations which are in large measure to be credited with revealing the inadequacy of that tool. You cannot do neurosurgery with nineteenth centruy instruments. I would not claim that F_3 is the *final logic*, but it does appear to me that it does everything worthwhile

that two-valued logic did, and more besides. And isn't that what interesting discoveries are all about?

NOTES

1. R. J. Farrell, "A note on the truth-table for $p \supset q$," *Notre Dame Journal of Formal Logic*, vol. XVI (1975), pp. 301-304.

2. A specimen may be viewed at the Cincinnati Museum of Natural History.

3. J. Łukasiewicz, "On three-valued logic." Reprinted in translation in *Polish Logic, 1920-1939*, Storrs McCall (ed.), Oxford University Press, Oxford (1967), pp. 16-18.

4. S. E. Toulmin, "Probability," *Proceedings of the Aristotelian Society*, Suppl. Vol. XXIV. Reprinted in *Essays in Conceptual Analysis,* Antony Flew (ed.), St. Martin's Press (1966).

5. Note that "p if and only if q" is not the same in $\mathbf{F}_3$ as "p is equivalent to q." We might introduce a sixth functor "$\equiv$" for equivalence, if desired. An important result of this distinction is that two formulae may mutually entail each other yet not be equivalent, and hence not be mutually replaceable, e.g., $p \rightarrow q$ and $\neg p \vee q$.

6. C. Hempel, "Studies in the logic of confirmation," as reprinted in Hempel, *Aspects of Scientific Explanation*, MacMillan (1965), p. 10.

7. *Ibid*., pp. 11-12.

8. *Ibid*., p. 13.

9. *Ibid*., p. 13.

10. *Ibid*., p. 6.

11. N. Goodman, "The problem of counterfactual conditionals," in *Fact, Fiction, and Forecast*, Bobbs-Merrill (1965), p. 4.

12. J. van Heijenoort, "Logical paradoxes," in *The Encyclopedia of Philosophy*, vol. 5, Macmillan (1967), p. 51.

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