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THE ANCESTRAL RELATION WITHOUT CLASSES

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This paper[1] is an exploration of two alternative analyses of the ancestral relation. One reason for undertaking this exploration is that concern with the ancestral relation dates from the very beginnings of modern logic. A long tradition of logicians have explicated the ancestral relation in terms of classes. This tradition dates at least from Gottlob Frege's *Begriffsschrift* of 1879[2], and is to be found in the writings of C. S. Peirce, Richard Dedekind, and continues down through the 1972 edition of W. V. Quine's *Methods of Logic*.[3] In Quine's view the introduction of quantification over classes brings with it a new power of expression, which in the present instance is displayed by the ability it gives one to translate the schema 'x is an ancestor of y'. The translation that Quine gives of 'x is an ancestor of y' is 'x is a member of every class which contains y and all parents of members'. One task of this paper is to examine the character of this translation. It is seen that this translation is at best an explication or rational reconstruction, in that Professor Quine's translation of this relational term tells us something which need not at all have been obvious to one that understood the definiendum, namely, that x is thus asserted to be a member of many larger classes than just the classes of y's ancestors. The constructive portion of this paper proposes a new explication of the ancestral relation, which lacks the defect just noted in the traditional definition. The new explication is developed in terms of a relation called "generational removal". The concept of this relation is developed in such

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1. This paper was presented December 28, 1973 at the Annual Meeting of the Association for Symbolic Logic in Atlanta, Georgia. My thanks go to Prof. Jack Kaminsky for what I learned from his dissent concerning points in a previous version of this paper.

2. Gottlob Frege, *Begiffsschrift* translated in Jean van Heijenoort (ed.) *From Frege to Gödel*, Harvard University Press, Cambridge, Massachusetts (1967). *Cf.* p. 4.

3. Willard Van Orman Quine, *Methods of Logic*, Holt, Rinehart and Winston, New York (1972), pp. 235-240. All quotes will be followed by page references in parentheses.

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a way as to allow for degrees of generational removal. And a further distinction is introduced between direct and nondirect generational removal.

As has already been mentioned, the claim has been made that one of the advantages of adopting an ontology committed to a realm of classes is that the relational predicate '*x* is an ancestor of *y*' cannot be translated into any of the known logics which do not contain terms for classes. In his new third edition of his *Methods of Logic* Willard Van Orman Quine provides a brief discussion of the schema '*x* is an ancestor of *y*'. He there asserts that he is following Frege when he provides a translation of this schema into the logic of classes. He is justifying this translation when he says:

> But this power of expressing irreducibly new laws would of itself justify little interest in class theory, were it not accompanied by a corresponding increase of power on the side of application. A good example of this effect may be seen in the definition of the predicate or relative term 'ancestor' on the basis of 'parent'. (p. 237)

Having first introduced the machinery for class logic, Quine then says:

> Now the problem is to write '*x* is an ancestor of *y*' using only '*F*' and our various logical symbols. (p. 238)

'*Fxy*' here translates '*x* is a parent of *y*'. The translation that Quine gives of '*x* is the ancestor of *y*' is more of an explication or rational reconstruction (in the Carnapian sense) than a straightforward translation. For Quine's translation of this relational term tells us something which need not all have been obvious to one that understood the definiendum. His translation is again that: *x* is an ancestor of *y* if and only if *x* is a member of every class which contains *y* and all parents of members. The symbolic version of this is:

$$(\alpha)(\{y \in \alpha \ \& \ (z)(\omega)[(\omega \in \alpha \ \& \ Fz\omega) \supset (z \in \alpha)]\} \supset x \in \alpha)$$

The above schema has been translated out of Quine's dot notation into a notation of parentheses, brackets and braces. What Quine is saying in the above schema can best be seen by examining the near limiting case of a universe of discourse which has just three members. Let the names of the three members be '*a*', '*b*', and '*c*', and further specify that *b* is the father of *c*. In this limiting case, 'α' is the class that is uniquely determined by the individuals *a*, *b*, and *c* as members. Quine's schema, which in this instance is meant to be a translation of '*a* is an ancestor of *c*' then becomes:

$$(\alpha)(\{c \in \alpha \ \& \ [(c \in \alpha \ \& \ Fbc) \supset (b \in \alpha)]\} \supset a \in \alpha)$$

When the above instantiation is simplified to eliminate redundancies, what remains says:

$$(\alpha)(\{(c \in \alpha) \ \& \ [Fbc \supset (b \in \alpha)]\} \supset a \in \alpha)$$

Or, in other words, - for every class α, if *c* is a member of α and *b* is the parent of *c* only if *b* is a member of α, then *a* is a member of α. Expressed in a slightly different idiom this becomes: if someone is a member of a class which has every parent of a member as a member, then any ancestor of that one is a member. What has been done in this limiting case is to universally instantiate the '*z*' and '*w*' of the original schema for '*b*' and '*c*' respectively, and then simplify.

That Quine's translation "says" more than would usually be meant by one who said that '*a* is an ancestor of *c*', is seen from the fact that since '*a*' is universally quantified, the ancestor *a* is asserted to be the member of many other (e.g., larger) classes than just the class of *c*'s ancestors. For example, *a* is thus asserted to be a member of the *c*'s "ancestors and neckties; for, neckties being parentless, their inclusion does not disturb the fact that all parents of members are members." (p. 238) This is no surprise to Quine–nor does he consider it a serious defect in his definition. Even if Quine is correct and this feature of his definition *is not* cause for rejecting it, the absence of this feature in an alternate definition of '*a* is an ancestor of *c*' would provide grounds for choosing between them. Now we turn from Quine's translation of the ancestral to the task of translating the ancestral relation without referring to classes.

First let us determine one ordinary meaning or definition of the word 'ancestor'. One such definition provides us with the following starting point. It defines an ancestor as "One from whom a person is descended, whether on the father's or mother's side, at any distance of time." With this definition in mind we can make the simple observation that if x is an ancestor of y, then y is a descendant of x. The property of an ancestor that I wish to capture here is expressed by the inelegant but descriptive phrase "order of generational removal." What is meant by this is very simple and can best be explained by an illustration. If x is the father of y, then x is one generation removed from y, i.e., x is of the first order of "generational removal" with respect to y. I now introduce the convention that the capital letter 'G' is to abbreviate the relational predicate 'order of generational removal'. A number within parentheses following 'G' is taken as specifying the degree of a given generational removal. Thus, the generational removal that is involved in x's being the father of y is expressed in this notation as: G(1). If so desired, this notation can be supplemented by an enumeration, or ordered listing, of the individuals involved. The convention might be adopted that the leftmost variable or constant picks out the temporally earliest individual. Thus, $G(1,x,y)$' may be taken as expressing the fact that "x is of the first order of generational removal earlier than y." Or, in other words, x lived temporally one generation earlier than y. With orders greater than one, say '$G(n)$', there will by definition always be n generations separating the individuals x and y. Generational removal is thus a tertiary relation holding between a number and two other individuals.

We have not as yet uniquely captured the ancestor relation with 'G', for not everyone of the generation previous to y is an ancestor of y. All y's parent's contemporaries have a generational removal of order one from y. Thus we must take one step further and distinguish direct from nondirect generational removal, abbreviating them as 'DG' and NDG'. Using these terms there is a direct generational removal of order one between x and y

if and only if either x is a parent of y or y is a parent of x. There is a direct generational removal of order two between x and z if and only if x is a parent of a parent of z, or vice versa. There is a generational removal of order n between x and z if and only if 'parent of a' gets repeated n times between x and z.

Consider an example which illustrates this notation. Suppose that w is the great-grandfather of z. The relationship between w and z would then be expressed in the above notation as $\mathrm{DG}(3, w, z)$. This notation may be extended even further by introducing a superscripted 'x' between the first and last terms, e.g., $\mathrm{DG}(2, w, x^2, z)$. This superscripted '$x$' would specify the number of ordered individuals that occur between 'w' and 'z'. Obviously, the superscript of 'x' would always equal the first argument of 'G' (or 'DG') minus one. The first example above could thus also be expressed as '$\mathrm{DG}(1, x, z^0, y)$ and the last example as '$\mathrm{DG}(3, w, x^2, z)$'. Now how would one express the sentence "x is an ancestor of y" in the above notation? The most obvious difference between the sentence 'x is an ancestor of y' and 'x is the great-grandfather of y' is that the latter is such that the order of "generational removal" can be determined just by an inspection of the meaning of the terms involved, whereas in the former case we cannot so determine it. Thus, in asserting that 'w is an ancestor of z' all that is being expressed is that:

$$(\exists n)\mathrm{DG}(n, w, z)$$

which is to say, there is a number n such that w has a generational removal of order n from z. To ask for a more specific translation of 'w is an ancestor of z' is to ask for a more specific account of the order of generational removal, and this is to ask for information not given by the original statement itself. Therefore, we see that there is a way of translating that 'a is an ancestor of c' without having to say that a is a member of the class of "c's ancestors and neckties".

One ontological comment is called for in concluding this paper. Quine's translation of 'x is an ancestor of y' required him to quantify over classes, and by his reckoning that committed him to an ontology of classes. The alternative translation just exhibited employed the existential quantification '$(\exists n)\mathrm{DG}(n, \ldots, ---)$' where n was taken to be a number, namely the number of generations one individual was directly removed from another individual. And so this second definition, by Quine's lights, would seem to be committed to an ontology of numbers. At this point, if one takes classes to be more fundamental than numbers, or if one believes numbers to be ontologically derivative from classes, it would appear that little progress has been made. But it is by no means obvious to everyone that one must take classes as ontologically more fundamental than numbers, and so for such dissenters my alternative definition of 'x is an ancestor of y' opens a new option where none was explicitly seen before.

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