Notre Dame Journal of Formal Logic Volume XX, Number 1, January 1979 NDJFAM

## EXTENSIONAL EQUIVALENCE OF SIMPLE AND GENERAL UTILITARIAN PRINCIPLES

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In the third chapter of his Forms and Limits of Utilitarianism.<sup>1</sup> David Lyons attempts to answer the following question: For the purpose of comparing value, does it make any difference whether we assess acts according to their general utilities or tendencies rather than according to their simple utilities? In presenting his answer, Lyons first argues for causal linearity, then for utilitarian linearity, and finally for the extensional equivalence of *non-comparative*<sup>2</sup> pairs of simple and general utilitarian principles which are identical in all other respects. Since his equivalence argument depends upon considering the behaviour of others in deciding the utility of our own actions, he next argues that such considerations do not conflict in any way with the notion of general utilitarian relevance. He then completes his answer by extending his equivalence thesis to include pairs of corresponding simple and general  $comparative^2$ utilitarian principles. In the next few pages, I will examine Lyon's arguments and show that the final step leading to the conclusion that corresponding pairs of comparative utilitarian principles are equivalent fails and that this conclusion is, in fact, false. I will also present a weaker equivalence result which Lyons' argument does establish.

The notion of causal linearity which Lyons maintains is this: Let A be some act, E the effect of a single occurrence of A, and T the total effect of n occurrences of A. Then  $T = n \times E$  expresses the condition of causal linearity and  $T \neq n \times E$  expresses the corresponding condition of causal nonlinearity. Analogous to this is the notion of utilitarian linearity: Let Abe some act, S the utility of a single occurrence of A, and G the total utility of n occurrences of A. Then  $G = n \times S$  expresses the condition of utilitarian linearity and  $G \neq n \times S$  the corresponding condition of utilitarian nonlinearity. Lyons holds that a complete description of actions, taking into account threshold-related effects, will always yield  $T = n \times E$ , and a complete description of the relevant utilitarian properties of actions, taking into account threshold-related utilities, will always yield  $G = n \times S$ . It is only when thresholds are considered in evaluating one side of the equation

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but not in evaluating the other that the appearance of non-linearity arises. Thus, if we remember thresholds throughout our determinations, we will always arrive at the same conclusions as to whether actions have positive or negative value, regardless of whether we use simple or general principles of evaluation.<sup>3</sup> This immediately implies equivalence of pairs of non-comparative utilitarian principles.

To establish equivalence for comparative principles, we must show that for any two alternative actions  $A_1$  and  $A_2$ ,  $S_1 \leq S_2$  if and only if  $G_1 \leq G_2$ (where  $S_1$  and  $S_2$  are the simple utilities of  $A_1$  and  $A_2$ , and  $G_1$  and  $G_2$  are the utilities of  $A_1$  and  $A_2$  computed through application of the generalization test<sup>4</sup>). Lyons argues that  $S_1 \leq S_2$  and  $G_1 > G_2$  taken together imply a contradiction. He also claims that since  $A_1$  and  $A_2$  are open alternatives, the number of occurrences of each needed to produce any related thresholds will be constant. Either of these claims would establish the equivalence of pairs of corresponding comparative principles.

Having a sketch of Lyons' answer and of the central notions involved, let us examine his arguments in detail and see how they go astray.

Does the condition of causal linearity hold for every act? Let us suppose for the moment that for some act A and some n,  $T_A^n \neq n \times E_A$ . (The sub- and super-scripts are mine and their intention should be obvious.) Then let

$$d = |T_A^n - (n \times E_A)|.$$

Then d represents some effect which is both produced by and not produced by the same n occurrences of A. This is impossible. (There may be some problem with considering effects as pure quantities, but the point should be clear.)

Let A be stealing an apple from Smith's orchard. Now the effect of an isolated occurrence of A would be negligible, at least if we consider only the effects on Smith's finances; i.e.,  $E_A = 0$ . Now for a sufficiently large number of occurrences within a sufficiently short period of time,  $T_A^n$  could represent serious damage to Smith's bank account and  $T_A^n \neq n \times E_A$ . But for the same n, if the n occurrences were spread over many years, we would have  $T_A^n = 0 = n \times E_A$ . Within this example lies the entire answer to the question of causal linearity.

Whether or not Smith is damaged financially depends not so much on n as on the density of the n occurrences of A. Hence, in one case each occurrence of A contributes to a *threshold effect*, while in the other case it does not. But, says Lyons, this means that the efficacy of a particular occurrence of A depends upon whether it occurs in a sufficiently dense practice for threshold effects to be produced. Since belonging to such a practice is a property of occurrences of A, a description of an occurrence of A which is to be complete in all causally relevant respects should state whether or not the occurrence does or does not have this property. Then we would have  $A_1$  and  $A_2$  as sub-types of A, where  $A_1$  is an occurrence of A outside a practice sufficiently dense to produce the given threshold effect and  $A_2$  is an occurrence of A within such a practice. Now  $E_{A_1} = 0$  as

before, but  $E_{A_2} \neq 0$ , since it has a deleterious threshold-related effect in that it contributes to some significant financial harm done to Smith. Furthermore,  $T_{A_1}^n = n \times E_{A_1}$  and  $T_{A_2}^n = n \times E_{A_2}$ .

Now let us consider the question of utilitarian linearity. Is there an act A and an n such that  $G_A^n \neq n \times S_A$ ? This is a special case of the question of causal linearity where we are concerned only with the value-laden effects of occurrences of A. I actually considered an example of such a special case in my discussion of causal linearity. We can generalize Lyons' formulation in terms of the special case of value-laden effects to handle other special cases in the following way: For a particular type K of effects, any act A and any n we have  $T(K)_A^n = n \times E(K)_A$ . This immediately gives us  $G_A^n = n \times S_A$  for all acts A and all n. This means that if we always take the utilities of thresholds and the corresponding threshold effects of the actions under consideration into account, it makes no difference in our final evaluation whether we use a simple or general non-comparative utilitarian principle, for it will not be possible to get a negative value using one method and a non-negative value using the other.

Now one might argue that it is easier to use one method rather than another, and hence, on practical grounds, we would be correct more often if we used the easier method. Lyons admits this. But this does not deny the fact that, done correctly, the two kinds of principles give the same results in every application. Practicality is not in question at this point.

But the practicality of applying our principle and giving proper consideration to thresholds is a very important question when we try to develop a normative system from either kind of principle. Suppose we compute the simple and general utilities of an act A with the result that  $G_A < 0 \le S_A$ . Then we know there is some relevant property of occurrences of A which we have not properly considered in our computations. Calling this property C, we can divide occurrences of A into occurrences of Awith C, AC, and occurrences of A without C,  $A\overline{C}$ . Assuming C is the factor leading to the negative threshold utility, we have:

$$S_{AC} < 0 \leq S_{A\overline{C}}$$
 and  $G_{AC} < 0 \leq G_{A\overline{C}}$ .

Whether we use a simple or a general principle, it clearly becomes important that we try to determine whether the occurrence of A in question will be an occurrence of AC or of  $A\overline{C}$ . If we cannot make this determination with certainty and if  $S_{AC}$  is very bad while  $S_{A\overline{C}}$  is only slightly good, it may be wise not to perform this particular occurrence of A. But an unquestioned reliance upon a misconceived application of the generalization test and a systematic refusal to ask whether a particular act will have threshold-related effects are both contrary to the spirit of utilitarianism.

In describing the mistake which leads to the appearance of utilitarian non-linearity, Lyons introduces the notion of a description of an act *in vacuo*. A description *in vacuo* of an act is one in which thresholds are ignored. Lyons says we often do this when computing *simple* utilities only to include the threshold when applying the generalization test, with the result that we actually compare two different kinds of acts:  $G_{AC}^{n} \neq n \times S_{A}$ .

But I would claim an even graver mistake is being made. We do not simply *ignore* the possible threshold-related effects but implicitly *assume* there are none in computing simple utilities in such cases. For example, when computing the simple utility of stealing an apple from Smith, we implicitly assume that our action will *not* contribute to a threshold when we say  $E_A = 0$ . The equation above represents the mistake we make when we compute the simple utility of an act without making any assumption as to any related thresholds, but this is impossible since every occurrence of A is an occurrence of either AC or  $A\overline{C}$ . The mistake which we actually make is of this sort:  $G_{AC}^n \neq n \times S_{A\overline{C}}$ .

Except for the revision just suggested, I find Lyons' argument to be correct to this point. Now we must look at his argument for the extensional equivalence of *comparative* principles.

Let us suppose we have a set of open alternative actions  $A_1, A_2, \ldots, A_n$ with simple utilities  $S_1 \leq S_2 \leq \ldots \leq S_n$ . Now for each  $i \leq n$  we have  $G_{A_i}^{m_i} = m_i \times S_{A_i}$ . But do we have  $G_{A_1}^{m_1} \leq G_{A_2}^{m_2} \leq \ldots \leq G_{A_n}^{m_n}$  where  $m_1, m_2, \ldots, m_n$  are sufficiently large to insure the production of any relevant thresholds related to  $A_1, A_2, \ldots, A_n$  respectively? This will clearly depend upon the  $m_i$ 's. But, says Lyons, let us suppose that equivalence does not obtain. Then for some  $i, j \leq n, G_{A_i}^{m_i} > G_{A_j}^{m_j}$  although  $S_{A_i} \leq S_{A_j}$ . Then if the possibility of performing either  $A_i$  or  $A_j$  occurs k times,  $k \times S_{A_i}$  will be produced if we follow the general principle and  $k \times S_{A_j}$ , so the general principle does not pick the action with the greatest possible general utility, which is a contradiction.

But Lyons is making a mistake here. In the above example k is assumed to be great enough to produce any thresholds related to either  $A_i$ or  $A_i$ , i.e., k is greater than or equal to both  $m_i$  and  $m_i$ . Now  $m_i$  and  $m_i$ are not equal, for if they were, utilitarian linearity would yield equivalence in our example. Hence,  $m_i \leq m_i \leq k$ . But the  $(k - m_i)$  occurrences of  $A_i$ which follow the  $m_i$  th occurrence of  $A_i$  are not really occurrences of  $A_i$  at all, paradoxical though this may seem. An important property of  $A_i$  is that its occurrences have threshold-related effects. Once the threshold is reached, no subsequent action can have an effect related to that threshold. The result is that there can be at most  $m_i$  opportunities to perform (either  $A_i$  or  $A_j$ ). But, since  $m_j < m_i$ , this clearly gives us  $m_j \times S_{A_i} \le m_j \times S_{A_i}$  even though  $G_{A_i}^{m_i} > G_{A_i}^{m_j}$ . Although we do not get the sort of equivalence Lyons tried to establish, we do get a sort of equivalence. The total utility of everyone's doing the act with greater simple utility may not be greater than the total utility of everyone's doing the act with lesser simple utility, but the total utility of everyone's doing the act with the greater simple utility who has the opportunity of doing either it or the act with lesser simple utility is greater than the total utility of everyone's doing the act with the lesser simple utility who has the opportunity of doing either it or the act with greater simple utility.

Lyons attempts another "demonstration" of the original equivalence thesis for comparative principles which actually turns out to be a reply to my objection to his first argument. Lyons claims that since the  $A_i$ 's are open alternatives, they must be "internally related" in such a way that  $m_1 = m_2 = \ldots m_n$ . Grant him this, and the original thesis follows from utilitarian linearity, but there is no good reason to think this premise is true. Let us see if we can find a counter-example. Suppose Jones works in an office. On Monday, Jones forgets to bring any money to work with him while each of his co-workers brings two dollars to work with him. The office workers intend to take up a collection to buy the boss a Christmas gift that day-a gift certificate from a store which has a \$10 minimum on gift certificates. After lunch, and after each of Jones' co-workers has spent one of his dollars to buy his own lunch, one of Jones' co-workers notices Jones' predicament and calls the fact that Jones' has not had lunch to the attention of the office staff. At the same time, someone announces that it is time to collect money for the boss' gift. Assume that each of the workers thinks it is five times more important that the boss get a gift than that Jones have lunch that day. Then for each of Jones' co-workers we have:

A: x uses his dollar to buy Jones lunch; B: x contributes his dollar toward the boss' gift;  $G_A^1 = S_A = 1$ ;  $G_B^{10} = 5$ ; and  $S_B = \frac{1}{10} \times 5 = \frac{1}{2}$ .

Then we must conclude that  $S_A > S_B$  but  $G_A^1 < G_B^{10}$ , contradicting both Lyons' claim that necessarily  $m_A = m_B$  for open alternatives A and B, and his original equivalence thesis for comparative utilitarian principles. However, we still have the weaker equivalence I formulated above.

A critical evaluation of Lyons' discussion provides a basis for answering the question as to the importance of the generalization test. Lyons's arguments for causal and utilitarian linearity are convincing, and from this follows strict equivalence of pairs of non-comparative simple and general utilitarian principles identical in all other respects. But unfortunately, Lyons does not establish his most interesting claim: a strict equivalence between corresponding comparative utilitarian principles. Here we must settle for a weaker thesis.

## NOTES

- 1. Published by Oxford University Press, Oxford (1965).
- 2. Roughly, non-comparative utilitarian principles are those which advise us to avoid disutility while comparative principles are those which advise us to maximize utility.
- 3. Lyons offers an enlightening and generally correct account of thresholds and threshold-related effects in the earlier sections of his book. I will assume the reader's familiarity.
- 4. We employ the generalization test when we ask, "What would happen if everyone did the same?" Cf. Lyons, op. cit., p. 1.

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