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Message Semantics

DAVID HARRAH

1 Introduction There is an important class of expressions for which we should have a logic. We don't yet have a complete and satisfactory logic for these expressions, but at least a start has been made.[1] One aim of this paper is to describe a semantics for these expressions. Another aim is to compare the treatment of names in this semantics with the treatment of names in situation semantics.

The expressions in question may be called *explicitly vectored expressions.* Examples are given in Section 2. We construct a logic for these within an extensional framework. Roughly: We begin with a first-order language, add some nonlogical axioms, add apparatus to provide for speech-act sentences, add a system for reply and response, and then define the key concept of formal message. Then we extend the system by adding pronouns, so that certain kinds of seemingly informal expressions can be construed as abbreviating, and thereby expressing, the formal messages. In this way we have a logic for both the explicitly vectored formal expressions and the implicitly vectored informal expressions.

Loosely speaking, the system provides messages as the meanings of utterances. For this reason the system may be said to be a *message semantics.* (Cf. "propositional semantics," in which propositions are provided as meanings.)

2 Vectored expressions The class of expressions that we are concerned with here can be indicated well enough by the following examples.

First consider the "formal memo" (whose components we number at the left, in a scheme that will be used later):

(0) 11 March 1985
(1) From: Jane McGaugh, Dean
(2) To: John Bryant, Chairman, Philosophy
(3) Subject: Faculty workload
(4) Re: Your letters of March 6 and 7

Received May 13, 1985

(5) Assuming that no additional faculty will be hired,
(6) my reply to your questions is...

According to the logic that we describe below, (0)–(5) in this example serve to indicate the vectoring of this message. The body of the message begins with (6).

Other examples are:

(7) George, old boy, my advice is...
(8) George, on the question of..., the best advice is...
(9) I, James McHugh, being of sound mind, hereby bequeath...
(10) ...and so, patient reader, you will not be surprised to learn that...

Can all such vectored expressions be rendered as sentences in some standard form? say as performative sentences? Corresponding to the first example there are, e.g.,

(11) I, Jane McGaugh, as Dean, say to John Bryant...
(12) Jane McGaugh, as Dean, hereby says to John Bryant...

For our purposes the relevant point is that such performatives are not equivalent to nonvectored indicatives. The sentence

(13) Jane McGaugh, as Dean, says to John Bryant...

can be said truly by anyone, but (11) and (12) cannot.

3 The sentences of L Our logic of messages is called *LM*. It is based on a language *L*, which has five parts: the *d*-part (for declarative sentences), the *sa*-part (for speech-act sentences), the *r*-part (for responses), the *c*-part (for compound sentences), and the *m*-part (for messages).

The alphabet of *L* is finite, and its letters are effectively recognizable. An *expression* is a finite string of letters.

The *d*-part of *L* is a standard first-order system. It has the connectives $-$, &, $\vee$, $\supset$, $\equiv$; the quantifiers U and E; identity $=$; and descriptions $\neg xF$. The notions of term and *d*-wff are defined as usual and given the usual extensional semantics. (Our variables over the terms of *L* are t, t', . . . ; our variables over the *d*-wffs are F, G, H, ...) Deductive apparatus is based on *d*-logical axioms and the modus ponens rule. Certain *d*-sentences are chosen as nonlogical axioms, and the *normal* interpretations of *L* are those in which: (a) certain nonlogical constants are interpreted in the intended way, and (b) the axioms are true.

One set of axioms is chosen for Zermelo-Fraenkel set theory, and another for Tarski's theory of syntax. For every expression X of *L* there is a term t (specifically, the concatenation name of X) that is effectively recognizable as being the standard name of X under normal interpretations. One axiom says that there is a set of all the expressions of *L*. Thus *L* can name its own expressions, describe sets of expressions, and represent certain properties of them.

The integers are constructed within the set theory and are regarded as surrogates for times. Certain terms T, T', . . . are selected to be standard names for these. Hence there are *d*-wffs $(T < T')$ that say that the time T is earlier than the time T', and, where T and T' are standard names for times, such *d*-wffs are effectively recognizable as true (or false) under normal interpretations.

Another set of axioms is chosen to describe the communication process. These use a three-place predicate *Sends* and a three-place predicate *Recs.* Under normal interpretations *Sends*(xyz) says that x sends a token of the expression y at the time z, and *Recs*(xyz) says that x receives a token of the expression y at time z. The axioms say that, for any x:

1. x can send only one y at a time
2. x can receive only one y at a time
3. x can not send and receive different y's at the same time
4. there is an earliest time at which x sends or receives.

Certain terms N, N', . . . are selected to be standard names of communicants. Given such an N, we can effectively recognize it as being such a term; and, given a normal interpretation I, we can effectively find the communicant named by N under I. In the metalanguage we assume that each communicant has a unique signature. Given any X, we can tell whether it is a signature; if it is, we can tell who made it and when. Each signature is made by only one person, and made only once.

For the *sa*-part of L there is a special stock of *speech-act indicators.* A *bsa-wff* (basic speech-act wff) has the form $OV(Y_1, \ldots, Y_n)$, where O is a speech-act indicator, V is a string of variables (possibly empty), and each Y_i is a term or d-wff. The variables in V act as variable-binding operators whose scope is $(Y_1, \ldots, Y_n)$. Thus proper substitution and alphabetic variance can be defined for these wffs.

The *sa-wffs* are defined by recursion:

1. All d-wffs and *bsa*-wffs are *sa*-wffs.
2. If F and G are *sa*-wffs, and x is a variable, then $(F \,\&\, G)$, $(F \vee G)$, UxF, ExF are *sa*-wffs; also, $(F \supset G)$ is an *sa*-wff if F is a d-wff.

To each *sa*-wff F several entities are assigned, including a d-wff $CA(F)$ (called the *core assertion in F*) and various kinds of reply. Certain constraints are imposed on these assignments. It turns out that, given any *sa*-wff F, the concept of $CA(F)$ for that F is expressible in L, and each of the kinds of reply to that F is expressible in L.

There is a special stock of *reply-indicators.* For each type of reply there is a corresponding reply-indicator, which is effectively recognizable as such. The *br-wffs* (basic response wffs) have the form $(\rho F\text{: } G)$, where ρ is a reply-indicator, and F and G are *sa*-wffs. $CA[(\rho F\text{: } G)] = (CA(G) \,\&\, G')$, where G' says in a standard way "G is a reply to F of the kind indicated by ρ". The *r-wffs* (response-wffs) are defined by:

1. Every *br*-wff is an *r*-wff.
2. If F is an *r*-wff, and G is either an *r*-wff or an *sa*-wff, then $(F \,\&\, G)$ and $(G \,\&\, F)$ are *r*-wffs.

For *r*-wffs, $CA[(F \,\&\, G)] = (CA(F) \,\&\, CA(G))$.

The c-wffs (compound wffs) are the *sa*-wffs plus the *r*-wffs. A *d-*, *bsa-*, . . . , wff is *n-place* iff it has exactly n free variables. A *d-*, *bsa-*, . . . , *sentence* is a *d-*, *bsa-*, . . . , wff that is zero-place. Given an n-place *d-*, *bsa-*, . . . , wff F, we write either $F(t_1, \ldots, t_n)$ or $F(t_1 \ldots t_n)$ or $Ft_1 \ldots t_n$ for the result of properly substituting $t_1, \ldots, t_n$ for the free variables in F.

Example: Suppose "$?^1$" is chosen to be the indicator for one-example-of questions. Let F be $?^1x(Gx)$, where Gx is a d-wff with x free. Then F is a *bsa*-wff, $CA(F) = \mathrm{E}xGx$, and the wanted replies to F are $Gt_1, Gt_2, \ldots$. Now let F' be $(\rho F\colon Gt)$, where ρ is a reply-indicator. Then F' is an r-wff that, in response to F, conveys the reply Gt. If Gt is true, and ρ is the indicator for wanted reply, then $CA(F')$ is true. If $CA(F')$ is false, one can respond to F' via the r-wff $(\rho' F'\colon -CA(F'))$, where ρ' indicates corrective reply, which in this case is $-CA(F')$.

A system of *ca*-derivation is adopted, for content analysis of c-wffs. The rules of *ca*-derivation provide for the rearrangement and decomposition of c-wffs. Some of the rules are analogous to rules of d-logic. Examples:

$F, (F \supset G) \to G$
$(F \,\&\, G) \to F$
$\mathrm{U}xFx \to Ft$
$F \to G$, where G is any alphabetic variant of F.

Some of the rules are peculiar to *sa*-logic. Examples:

$F \to CA(F)$
$(F \vee G) \to (CA(F) \supset F)$
$\mathrm{E}xFx \to (CA(Ft) \supset Ft)$
$(\rho F\colon G) \to G$.

The use of these rules in our system is described below.[2]

4 Messages To provide for formal messages the alphabet of L has the square brackets [,], the colon :, the comma, and the letters M, O, D, S, R, A, B. We say that X is a *list of* $Y_1, \ldots, Y_n$ iff X is the result of writing $Y_1, \ldots, Y_n$ in some order, separated by commas; each Y_i here is *listed in* X.

A *status-wff* is a two-place d-wff. A *sender-specifier* is an expression (N, F, T) such that N is a standard communicant name, F is a status-wff, and T is a standard time-denoter. A *receiver-specifier* is an expression (G, F, T) such that G is a one-place d-wff, F is a status-wff, and T is a standard time-denoter.

An *m-like expression* is an expression of the form

$$[M\colon [O\colon X_1]\,[D\colon X_2]\,[S\colon X_3]\,[R\colon X_4]\,[A\colon X_5]\,[B\colon X_6]].$$

Hereafter, when referring to an m-like expression X, we may use "X_1", "X_2", . . . to refer to the respective parts of X as indicated above. Occasionally we may use "m-like" as short for "an m-like expression".

A *formal message* is an m-like expression such that:

1. In the *origin-part* $[O\colon X_1]$ the expression X_1 is a list of sender-specifiers all sharing the same time-denoter T (which denotes *the time of* the message).

2. In the *distribution-part* $[D\colon X_2]$ the expression X_2 is a list of receiver-specifiers whose times are all equal to or later than the time denoted by T.

3. In the *subject-part* $[S\colon X_3]$ the expression X_3 is a list of expressions each of which is either a closed term or a predicate or a d-sentence.

4. In the *re-part* $[R{:}X_4]$ the expression X_4 is a list of expressions each of which is either a closed term or a d-sentence. (The terms are the reference; the sentences are the explanation.)

5. In the *assumption-part* $[A{:}X_5]$ the expression X_5 is a list of d-sentences.

6. In the message *body* $[B{:}X_6]$ the expression X_6 is a list of c-sentences.

Occasionally we may use "M" as a metalinguistic variable over formal messages. In any formal message the segment from $[O{:}X_1]$ to $[A{:}X_5]$, inclusive, is the *vector-specifier*, or *vector*, for short.

To provide for informal messages, L has a stock of expressions designated as *m-pronouns*. Let us refer to these by "β_I", "β_{now}", "β_{you}", "β_{it1}", "β_{it2}", . . . ,and use "β_{it}" as a variable over the latter.

Where X is m-like, *X' comes from X by m-pronoun insertion* iff there is a finite sequence $Z_1, \ldots, Z_n$ $[n > 4]$ of expressions such that all of the following hold:

1. $Z_1 = X$, and $Z_n = X'$.

2. Either $Z_2 = X$, or else X_1 is a list of sender-specifiers all having the same sender-name N and Z_2 comes from X by replacing N by β_I at some places not in X_1 or X_2.

3. Either $Z_3 = Z_2$, or else X_1 is a list of sender-specifiers all having the same time-denoter T and Z_3 comes from Z_2 by replacing T by β_{now} at some places not in X_1.

4. Either $Z_4 = Z_3$, or else X_2 is a list of receiver-specifiers all having the same first item, this being a d-wff of the form $(x = N')$, and Z_4 comes from Z_3 by replacing N' by β_{you} at some places not in X_1 or X_2.

5. For each i from 5 to n, either $Z_i = Z_{i-1}$, or else there is a closed term t such that for no β_{it} is it the case that the expression $(\beta_{it} = t)$ occurs in Z_{i-1}, and Z_i comes from Z_{i-1} by replacing t by the i-th β_{it} at some places and replacing X_5 by the expression that comes from X_5 by adding a comma and then the expression $(\beta_{it} = t)$, where β_{it} is the i-th β_{it}.

Where X is m-like, *X' comes from X by m-truncation* iff X' comes from X by a finite nonempty sequence of deletions according to the following rules:

1. Any colon may be deleted.
2. Any of the letters O, D, S, R, A, B may be deleted.
3. Any square bracket may be deleted iff its matching bracket is also deleted.
4. The commas that are used to separate the listed items in the list X_6 may be deleted.

Where X is m-like, *X' comes from X by m-abbreviation* iff there is a Y such that (1) Y comes from X by m-pronoun insertion, and (2) X' comes from Y by m-truncation.

We say that X is an *informal message* iff X comes from some formal message by m-abbreviation, and X is a *message* iff X is either a formal message or an informal message. Where X is any message, *X represents Y* iff either

1. X is a formal message, and $Y = X$, or
2. X is an informal message, Y is a formal message, and X comes from Y by m-abbreviation.

Claim: (1) Every formal message, and every informal message, is effectively recognizable as such. (2) Every message represents exactly one entity, and that entity is a formal message. (3) Given any message, we can effectively tell which formal message it represents.

5 The content of a message In this section we assume that M is any message, and that M represents M'. Also, let the concatenation name of M be t.

Y is a *presumption of M* iff either:

1. The X_1 in M' is the list $(N_1, F_1, T), \ldots, (N_n, F_n, T)$, and Y is one of:
   a. $F_i(N_i, T)$ for some i $[1 \leq i \leq n]$, or
   b. $Sends(N_i, t, T)$ for some i $[1 \leq i \leq n]$, or
   c. $\mathrm{U}x(Sends(x, t, T) \supset x = N_1 \vee \ldots \vee x = N_n)$, where x is the alphabetically first variable.

2. There is a receiver-specifier (G, F, T) listed in the X_2 in M', and Y is $\mathrm{U}x(G(x) \supset F(x, T))$, where x is the first variable beyond the variables in F and G.

3. There is a receiver-specifier (G, F, T) listed in the X_2 in M', and Y is $\mathrm{U}x[G(x) \supset \mathrm{E}z(z \geq T \,\&\, Recs(x, t, z))]$, where x and z are the first two variables beyond those in G.

4. There is a descriptive term $\neg xF$ listed in either X_3 or X_4 in M', and Y is $\mathrm{E}y\mathrm{U}x(F \equiv x = y)$, where y is the first variable beyond those in $\neg xF$.

5. Y is a d-sentence listed in X_3, X_4, or X_5 in M'.

Note that, if M is an informal message, then M gets all its presumptions from M' except the two concerning *Sends*; M presumes that M is sent, not that M' is sent. Useful facts: (1) every presumption is a d-sentence; (2) the set of presumptions of M is nonempty, finite, and effectively constructible from M.

In the next few definitions, think of S as the receiver's belief set. Read "M/S" as "M, given S". Read "*va*" as "vector analysis" and "*ma*" as "message analysis".

A *va-derivation from M/S* is a finite nonempty sequence Z of d-wffs such that, for each F in Z, one of the following holds:

1. F is a presumption of M
2. F is a d-sentence in S
3. F is an axiom of L (logical or nonlogical)
4. F comes from preceding members of Z by modus ponens.

An *ma-derivation from M/S* is a finite nonempty sequence Z of c-wffs such that, for each F in Z, either:

(1)-(4) [as in *va*-derivation],

(5) F is a c-sentence listed in X_6 in M', or

(6) F comes from preceding members of Z by some rule of *ca*-derivation.

F is *va-derivable from M/S* iff F is a member of some *va*-derivation from M/S. (Notation: $M + S \vdash^{va} F$.) We define *ma-derivable* similarly. In terms of these concepts we can define various concepts of content, and various concepts of reply.

Very roughly: The *assertive content of M/S* is the set of all *d*-sentences F such that $M + S \vdash^{ma} F$. (Notation: $Acont(M/S)$, or $Acont(M)$ if S is empty or vacuous.) A *complete framework for reply to M/S* is a set S' such that, for some Z that is an *ma*-derivation from M/S and that is complete in a certain sense, S' consists of all the *d*- and *bsa*-sentences in Z. A *vector-challenge to M/S* is an *r*-wff (ρF: $-F$) such that $M + S \vdash^{va} F$. A *sufficient response to M/S* is an *r*-wff F such that either (1) F is a vector-challenge to M/S, or (2) for some S' that is a complete framework for reply to M/S, F gives a sufficient reply to each of the sentences in S'. The *outer erotetic content of M/S* is the set of all complete frameworks for reply to M/S. The *outer erotetic commitment of M/S* is the set of all sufficient responses to M/S.[3]

In the foregoing definitions we have used the parameter M. It is easy to define more general concepts in which, in place of a single message M, there is a set of messages (the main application being the case where all the messages come from the same group of senders). For most of these concepts, however, we must require that the set of messages is finite.

6 The meaning of a message We do not claim to have a full account of "the meaning of a message." We do claim that, if messages have meanings, the meaning of a message M is a function of both: (1) the meaning of the microcontent of M, and (2) the structure of M.

For the system *LM* the meaning of the microcontent of M is as indicated above. Roughly, the terms and *d*-wffs in $Acont(M)$ have the standard extensional denotation-and-truth-value kind of meaning, and certain *c*-wffs that are *ma*-derivable from M have various kinds of erotetic meaning (various kinds of reply are defined for them).

Example of the relevance of structure: Let M and M' be alike except that M lists F in X_5 but not in X_6, and M' lists F in X_6' but not in X_5'. Then F is in $Acont(M)$ and in $Acont(M')$, and $M \vdash^{va} F$, but (ceteris paribus) it is not the case that $M' \vdash^{va} F$, so $-F$ is a sufficient reply to M but not to M'. Thus M and M' differ in erotetic meaning.

Much of the logic of messages can be developed on the basis of our concepts of content, without referring to any entities that may be the meanings of messages. E.g., *M assertively contains M'* iff $Acont(M)$ includes $Acont(M')$. A message M is *true* iff all the *d*-sentences in $Acont(M)$ are true, and *false* iff some *d*-sentence in $Acont(M)$ is false. (Alternatively: (1) M is *true* as above. (2) M is *false* iff all the presumptions of M are true but some *d*-sentence in $Acont(M)$ is false. (3) *M has no truth value* iff some presumption of M is false.)

7 Message semantics For heuristic purposes, think of things this way: If M is any informal message, and M represents M', then the meaning of M is M', and the meaning of M' is the various content of M'. Then, because the meaning of M is M', and M' is a message, we may say that our system LM is a message semantics (it assigns messages as meanings).

The intended area of application includes explicitly vectored expressions like those described in Section 2, and in addition many expressions that, although not explicitly vectored, will on certain occasions be construed as abbreviations of vectored expressions. Example: Father, facing away from his son and daughter, says

Go feed the cow,

and the son and daughter both construe this as an abbreviation of

William, go feed the cow.

The intended area of application of message semantics could be described as the class of all utterances that are explicitly or implicitly vectored.

There can be many systems of message semantics, differing in regard to either the formal messages or the abbreviation systems that may be included. The abbreviation system described in Section 5 is well behaved, effective, with no ambiguity in the informal messages. We can extend that abbreviation system in several ways while still preserving effectiveness (e.g., allow deletion of commas from X_1 and X_2).

Other systems allow further types of abbreviator and further deletions (e.g., allow deletion of the identities ($\beta_{it} = t$) from X_5). Advantage: greater similarity to English. Disadvantage: loss of effectiveness.

Message semantics is not per se an alternative to, or rival of, other kinds of semantics. Rather it is a framework within which other kinds of semantics can be accommodated as subsystems. In LM as described above the terms and d-wffs of L have a standard extensional semantics. If we want to extend our system (e.g., to provide for modal sentences, belief sentences, . . .), we can let the terms and d-wffs of L have an intensional semantics. This need not affect the apparatus of formal messages, abbreviation, and informal messages; an extended system with an intensional semantics can still be a message semantics.

8 Effectiveness vs. efficiency Let "*BP*" abbreviate "Barwise-Perry", and let "*SH*" denote the expression "Shane Harrah". Loosely speaking, on the *BP* approach *SH* is an "efficient" expression. It means [in English] "a person called 'Shane Harrah'", and in any utterance situation where *SH* occurs we anchor *SH* to some person as seems appropriate.[4]

According to the approach presented here, in practice (in English, e.g.) we start with expressions like

(SH^+) Shane Harrah male born to Rita G. Harrah on May 18, 1956, at Riverside Community Hospital in Riverside, California

We parse this as a single term and construe it as denoting a particular individual. We do not have axioms or meaning postulates like

Shane Harrah was born in Riverside, California but we do have, e.g.,

> Shane Harrah male born . . . in Riverside, California, was born in Riverside, California.

In addition we have several abbreviation systems that allow us to shorten these expressions in different ways.

One is our efficient system. This system allows deletion of all of the "male born . . . in Riverside, California" from SH^+. This produces expressions like *SH* that are ambiguous but efficient in the *BP* sense. We use this system to construct informal messages for casual communication in certain areas of life.

Another is our effective system. It is based on an *IRS* (an index of receivers and senders). The *IRS* provides "*IRS* names", which are effective, unambiguous abbreviations of proper names. According to it, SH^+ may be abbreviated by:

Shane Harrah 568-15-5507.

We need this effective system for communication in the public areas of life, because in the public areas we need both brevity and denotation – unambiguous and effective denotation. In our effective system, besides the index of receivers and senders, there is a corresponding index of signatures, which may be handwritten signatures, fingerprints, voiceprints, or the like. Names denote at the level of types; signatures denote at the level of tokens. We put a signature on a message token so that the *Sends* presumptions can be effectively verified from the token. Why do we need sender names N in X_1, in addition to the signature on the token? So that the set of *Sends* presumptions will be nonempty, finite, and effectively specifiable.

On the *BP* approach one might say that in public communication sender names are efficient but are anchored according to well-established constraints. The problem is: exactly what constraints? Not "N is anchored to the person who signs the token" (because the token might be signed and sent by the wrong person, and we want to be able to give a vector-challenge). What then? Conjecture: Any adequate set of constraints will be based on something like the effective system described above.

NOTES

1. See primarily [4], but also [2] for additional discussion.
2. For more details on the *sa*- and *c*-parts of L, see [4] and Section 7.9 in [3].
3. For precise definitions, and further discussion, see [4].
4. In this paper we anchor the phrase "*BP* approach" to [1]. This anchoring might be unfair, either because our characterization of the approach does not fit [1] precisely enough, or because [1] is not a definitive presentation of the views of Barwise and Perry. If it is unfair, we apologize. In any case our main purpose is not to criticize one approach but to compare two approaches.

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Department of Philosophy
University of California
Riverside, California 92521