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Implication and Presupposition

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Since introduced by Strawson [5], the notion of presupposition has been widely discussed as an interpretation of categorical statements. There have been problems, however, with explicating the notion in a clean-cut formal way.[1] If this could be accomplished and if the difficulties associated with certain statements, such as those denying existence, could be overcome, we might be well on the way to resolving the incompatibility between classical and contemporary interpretations of quantification. This resolution is of no small importance, especially for the teaching of logic to talented undergraduates who are majoring in disciplines other than philosophy and mathematics. For the conflict between logics tends to disquiet the minds of these ordinary consumers of our craft, who are not much interested in symbolic gamesmanship, and cause them to be suspicious that formal logic is not very applicable to their concerns.

The theory of presupposition is interpreted in this paper to mean that categorical propositions are material conditionals: they are prefixed by a stipulation that the classes occurring in them are genuine (i.e., have existing members). That is: "All Martians are blond" is understood not as a conjunction: "If anyone is a Martian he is blond *and* there is at least one Martian" but rather as: "If there is at least one Martian *then* if anyone is a Martian he is blond". This interpretation, meant to reconcile the old logic and the new, instantly collides with the problem of conditionals with false antecedents. For suppose we understand "All John's children are asleep" as:

$$(\exists u)Cuj \supset (x)(Cxj \supset Ax) \ .$$

If John is not a parent, then the statement is true. Moreover, by traditional subalternation we also have:

$$(\exists x)(Cxj \ \& \ Ax)$$

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which implies:

$$(\exists x)Cxj \ ,$$

which contradicts the assumption. This interpretation might be alleviated, however, if the subaltern were interpreted presuppositionally as well.

A few years ago I suggested [3] that conditionals with antecedents known to be false should receive a value I, for "inappropriate", for just as $(p \,\&\, q) \supset p$ shows that F cannot be correct, so $\sim(p \supset \sim p)$ demonstrates the untenability of T. This I-value supplies the formal interpretation needed for the theory of presupposition.

Perhaps the term *inappropriate* was infelicitous. It is correct enough for textbook conditionals with antecedents about green cheese or for inept argument forms, such as Clavius' law: $(\sim p \supset p) \supset p$, whose value is I for every combination of values of its elements. But of course there is nothing wrong with uttering a conditional when the value of its antecedent is unknown. We use truth tables as decision criteria for propositional arguments precisely in order to cover all the possibilities of value for their components. Moreover, when we want to indicate explicitly uncertainty or disbelief concerning the antecedent, we use the hypothetical or counterfactual subjunctive, about which more will be said later. The point is that some combinations of values, when known, cannot be used to make informative statements. When we know the antecedent of a stipulative conditional is false, we in fact ignore the whole proposition. So perhaps a better characterization of the I-value would be *to be ignored.* In fact, what I am suggesting is really just a case of theory catching up with practice. For decades now programming languages have been accustomed to ignore conditionals whose antecedents are false—or, in the jargon of the trade, to fall through to the next executable statement.

It is common to interpret categorical sentences so that "All *a* are *b*" is equivalent to "All members of the class *a* are members of the class *b*". Recall Cantor's definition of set (i.e., class):

> By a set I understand, generally, any multiplicity which can be thought of as one, that is to say, any totality of definite elements which can be bound up into a whole by means of a law. ([4], p. 418)

Classes, then, are defined by their members in virtue of some "law": perhaps an intrinsic character, perhaps an extrinsic relation. The predicate, symbolized by an upper case token, represents the law; the variable (or constant) represents the member. Some might think that oddity, the null class, invalidates the definition; but in the view of this paper "null class" is a misnomer for no class. The law—e.g., the intersection of contraries—is not satisfiable by any entity; and no member means a condition necessary for a class is not fulfilled.

Moreover, perhaps hearkening back to Hamilton, there is no reason why the presuppostion of existence should not apply equally well to the class whose name appears on the right as to that whose name appears on the left of the copula. And to be perfectly consistent, the classes which appear in particularly quantified statements cannot be exempt either. Presupposition is a feature of all interesting categories and therefore is a requirement for the truth or falsity of categorical statements. Nongenuine categorizations, and the sentences in which they occur, are to be ignored. "All John's children are asleep" thus becomes:

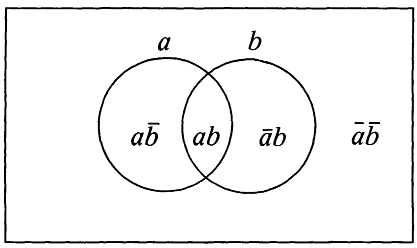
$$((\exists u)Cuj \ \& \ (\exists v)Av) \supset (x)(Cxj \supset Ax)$$

and "Some child of John is asleep" is:

$$((\exists u)Cuj \ \& \ (\exists v)Av) \supset (\exists x)(Cxj \ \& \ Ax) \ .$$

The presupposing antecedent need not always be written out, of course, just as long as we keep it in mind whenever it is of issue.

We can summarize categorical propositions relating two classes and their complements with a kind of truth table. We note the four regions of the familiar diagram:



Then using $\underline{P}$ to signify *entity present* we have an array of 16 rows exhausting the possible combinations. Rightward we attach 32 columns for all configurations of $\underline{a}$, $\underline{b}$, $\underline{\bar{a}}$, $\underline{\bar{b}}$, A, E, I, and O. Thus we obtain Table 1.[2]

Inspection of Table 1 shows that the laws of subalternation, contradiction, contrariety, and conversion are clearly valid. Obversion and contraposition, on the other hand, present a problem. There is, to be sure, no case in which a true premise implies a false conclusion; and each law has at least one $T \supset T$ combination. The $T \supset I$, $I \supset T$, $F \supset T$, $F \supset F$, and $F \supset I$ cases all get I-values. However numerous $I \supset F$ cases occur, as for instance $Aab \supset A\bar{b}\bar{a}$ in row 11.

In [3] I indicated that I would have liked to say $I \supset F$ should receive an I-value. That is, the interpretation for truth-functional implication that I would have preferred, and do now propose, should look like this:

p	q	$p \supset q$
T	T	T
T	F	F
T	I	I
F	T	I
F	F	I
F	I	I
I	T	I
I	F	I
I	I	I

Table 1.

ab	$a\bar{b}$	$\bar{a}b$	$\overline{ab}$	Aab	$Aa\bar{b}$	$A\bar{a}b$	$A\overline{ab}$	Aba	$Ab\bar{a}$	$A\bar{b}a$	$A\overline{ba}$	Eab	$Ea\bar{b}$	$E\bar{a}b$	$E\overline{ab}$
P	P	P	P	F	F	F	F	F	F	F	F	F	F	F	F
P	P	P	–	F	F	T	F	F	F	T	F	F	F	F	T
P	P	–	P	F	F	F	T	T	F	F	F	F	F	T	F
P	P	–	–	F	F	I	I	T	I	T	I	F	F	I	I
P	–	P	P	T	F	F	F	F	F	F	T	F	T	F	F
P	–	P	–	T	I	T	I	F	F	I	I	F	I	F	I
P	–	–	P	T	F	F	T	T	F	F	T	F	T	T	F
P	–	–	–	T	I	I	I	T	I	I	I	F	I	I	I
–	P	P	P	F	T	F	F	F	T	F	F	T	F	F	F
–	P	P	–	F	T	T	F	F	T	T	F	T	F	F	T
–	P	–	P	I	T	I	T	I	I	F	F	I	F	I	F
–	P	–	–	I	T	I	I	I	I	T	I	I	F	I	I
–	–	P	P	I	I	F	F	I	T	I	T	I	I	F	F
–	–	P	–	I	I	T	I	I	T	I	I	I	I	F	I
–	–	–	P	I	I	I	T	I	I	I	T	I	I	I	F
–	–	–	–	I	I	I	I	I	I	I	I	I	I	I	I

ab	$a\bar{b}$	$\bar{a}b$	$\overline{ab}$	Eba	$Eb\bar{a}$	$E\bar{b}a$	$E\overline{ba}$	Iab	$Ia\bar{b}$	$I\bar{a}b$	$I\overline{ab}$	Iba	$Ib\bar{a}$	$I\bar{b}a$	$I\overline{ba}$
P	P	P	P	F	F	F	F	T	T	T	T	T	T	T	T
P	P	P	–	F	F	F	T	T	T	T	F	T	T	T	F
P	P	–	P	F	T	F	F	T	T	F	T	T	F	T	T
P	P	–	–	F	I	F	I	T	T	I	I	T	I	T	I
P	–	P	P	F	F	T	F	T	F	T	T	T	T	F	T
P	–	P	–	F	F	I	I	T	I	T	I	T	T	I	I
P	–	–	P	F	T	T	F	T	F	F	T	T	F	F	T
P	–	–	–	F	I	I	I	T	I	I	I	T	I	I	I
–	P	P	P	T	F	F	F	F	T	T	T	F	T	T	T
–	P	P	–	T	F	F	T	F	T	T	F	F	T	T	F
–	P	–	P	I	I	F	F	I	T	I	T	I	I	T	T
–	P	–	–	I	I	F	I	I	T	I	I	I	I	T	I
–	–	P	P	I	F	I	F	I	I	T	T	I	T	I	T
–	–	P	–	I	F	I	I	I	I	T	I	I	T	I	I
–	–	–	P	I	I	I	F	I	I	I	T	I	I	I	T
–	–	–	–	I	I	I	I	I	I	I	I	I	I	I	I

ab	$a\bar{b}$	$\bar{a}b$	$\overline{ab}$	Oab	$Oa\bar{b}$	$O\bar{a}b$	$O\overline{ab}$	Oba	$Ob\bar{a}$	$O\bar{b}a$	$O\overline{ba}$
P	P	P	P	T	T	T	T	T	T	T	T
P	P	P	–	T	T	F	T	T	T	F	T
P	P	–	P	T	T	T	F	F	T	T	T
P	P	–	–	T	T	I	I	F	I	F	I
P	–	P	P	F	T	T	T	T	T	T	F
P	–	P	–	F	I	F	I	T	T	I	I
P	–	–	P	F	T	T	F	F	T	T	F
P	–	–	–	F	I	I	I	F	I	I	I
–	P	P	P	T	F	T	T	T	F	T	T
–	P	P	–	T	F	F	T	T	F	F	T
–	P	–	P	I	F	I	F	I	I	T	T
–	P	–	–	I	F	I	I	I	I	F	I
–	–	P	P	I	I	T	T	I	F	I	F
–	–	P	–	I	I	F	I	I	F	I	I
–	–	–	P	I	I	I	F	I	I	I	F
–	–	–	–	I	I	I	I	I	I	I	I

The problem with $I \supset F$ being I-value was that affirmation of the consequent and denial of the antecedent thereby became valid. The ad hoc fix, $I \supset F$ is F, seemed promising; but to restrict obversion or contraposition is unacceptable.

But suppose the fix had been applied to conjunction. One might well consider that of the three possible values T, I, and F a conjunction should take the weaker of its two arguments. The conjunction of a true statement with an amphiboly (the latter being an inappropriate use of language, to be ignored) should thus receive an I-value. This option is especially attractive if the only alternative is F. For one would not want

$$(p \,\&\, (\sim p \supset q)) \supset p$$

to turn out nontautologous. There is, to be sure, an argument for evaluating the conjunction $T \,\&\, I$ as F. Conjunction can be thought of as specifying that "both p and q are true".

This strong conjunction, however, is analogous to strong disjunction. It is a logical functor in its own right, the point of which only becomes apparent in three-value logic. When we want to explicitly signal its use in ordinary language, we use "both . . . and", just as when we want to signal strong disjunction we use "either . . . or". But returning to $T \,\&\, I$, can any semantic justification be offered for evaluating it as T? Perhaps so, especially when we think of I-value as meaning "to be ignored". In ordinary language, one might assign a T-value to:

Gibberish gibberishes and the moon is not a perfect sphere.

on the generous consideration that what is said is true as far as it is intelligible. In other words, one might *ignore* the lead conjunct and with it the conjunction function itself and evaluate only what is left. With this approach, the paradigms for weak conjunction and weak disjunction become:

p	q	$p \,\&\, q$	$p \vee q$
T	T	T	T
T	F	F	T
T	I	T	T
F	T	F	T
F	F	F	F
F	I	F	F
I	T	T	T
I	F	F	F
I	I	I	I

Weak conjunction is equivalent to "it is not the case that P is false or that q is false"; i.e., DeMorgan's law.

It is rather easy, semantically speaking, to ignore the I-side of a conjunction or disjunction. These operators connect independent clauses, statements in their own right which are independently appraisable. When it comes to implication, however, generosity has its limits. A much tighter knot is drawn by implication between its arguments than by conjunction or disjunction. This connection is shown in grammar by the fact that the antecedent is a dependent clause. By itself a sentence fragment, it cannot stand on its own, and we can write $T \supset$ __ and $F \supset$ __ only in virtue of the convention by which we know we are provisionally appraising the full indicative statement which corresponds to the antecedent. Could anyone be inclined to ignore the function in:

If gibberish gibberishes, then the moon is green cheese;

or

If the moon is largely rock, then gibberish gibberishes?

Implication is by common agreement much more complex than conjunction and disjunction. We have stipulative conditionals, hypothetical conditionals, deliberative conditionals, and entailment; and, in discussing implication, we rapidly become embroiled in the logic of the subjunctive mood.

At the level of the indicative stipulative conditional, let us at least investigate how far we get with the proposed paradigms. Truth tables demonstrate that they preserve the backbone of logic. They invalidate affirming the consequent and denying the antecedent, eliminate the transposition equivalence which generated Hempel's paradox, affirm modus ponens, modus tollens, hypothetical and disjunctive syllogisms, and of course the immediate inferences of categorical logic.[3] Exportation and absorption cease to be equivalences, but remain valid bidirectional implications. Reductio, expressed as

$$(p \supset (q \,\&\, {\sim}q)) \supset {\sim}p$$

fails; but in a reduction to absurdity we conjoin the proposition to be disproved with others that are not in dispute, and expressed as

$$(((p \,\&\, q) \supset (r \,\&\, {\sim}r)) \,\&\, q) \supset {\sim}p$$

it is valid. The constructive dilemma, finally, expressed as

$$(((p \supset q) \,\&\, (r \supset s)) \,\&\, (p \vee r)) \supset (q \vee s)$$

is also invalid (for instance, when p is T, q is I, r is F, and s is F). However, when written in the alternative format

$$((p \supset q) \,\&\, (r \supset s)) \supset ((p \vee r) \supset (q \vee s)) \ ,$$

the problem is cleared.[4]

I have been referring to material implication, which always takes the indicative mood, as the stipulative conditional. It most commonly occurs in intentional contexts such as:

If the day is sunny, we will go on a picnic.

But it is also found in predictions:

If we hold on fourth down, we win.

and in qualifications:

If it is clear, crystalline, and scratches corundum, it's a diamond.

When spoken, the truth-value of the antecedent may not be known; but the speaker has stipulated a condition which, if not fulfilled, voids his point. It is not ordinarily appropriate to stipulate a condition one knows to be false, so that the truth or falsity of stipulative conditionals is normally determined by the value of their consequents. If the day is sunny and Dad takes the family out, he has kept his word. But if the offense fumbles in the end zone, what can the coach say?

The use of the stipulative conditional of particular interest to logicians is its occurrence in argumentation. By the law of deduction, a valid argument is such that *on condition that* the premises are true so is the conclusion. Otherwise, although valid, it is moot. Since the truth of the conclusions of immediate inferences and syllogisms has nothing to do with the content of their predicates, which are represented by variables anyway, but is entirely a function of their logical operators, these argument forms, like propositional ones, can be decided by mechanical means. When the variables are fleshed out to make statements, we suppose that the premises are true: they are the givens on which the conclusions rest.

The temptation to appraise conditionals with false antecedents true, I suspect, derives especially from qualificational conditionals, which are closely related to hypothetical conditionals. If someone produces a quartz crystal, we would still want to say:

> If it could scratch corundum, it would be a diamond.

We would want to because of our confidence in Mohs scale, which underpins the universal categorical statement:

> Every clear crystal that scratches corundum is a diamond.

Whenever a universal categorical statement is true, the corresponding hypothetical conditional, properly formulated in the present or perfect subjunctive, and the counterfactual conditional, properly formulated in the imperfect or pluperfect subjunctive, are certainly true, irrespective of the value of the indicative counterparts of their antecedents. In other words, while their mood adds something to their connotations, the subjunctive conditionals can be decided on the basis of some corresponding universal categorical proposition.

Although we are not as careful about the use of the subjunctive as the classical authors, it is clear that we use it to connote a subjective element of uncertainty, doubt, or even disbelief in antecedents and consequents. Often the uncertainty as to the antecedent is heightened by combining the present subjunctive with the indicative, as in: "Even if they *should* let us down, we *will* do our part". Some combinations, however, are improper. We don't use the imperfect subjunctive in the antecedent and the present indicative in the consequent. Neither do we use the present indicative in the antecedent and the present subjunctive in the consequent. Moreover, one combination I would call attention to is this: when we use 'if' in the stipulative sense, as is conveyed by the phrase "on condition that", we don't use the subjunctive mood. "If that were so" and "if that should be so" are quite common, but "on condition that that were so" and "on condition that that should be so" sound odd. In this fact of language we find the evidence that it is not the stipulative conditional that is at issue when we speak of wanting to issue a conditional that asserts q on the assumption that p but without committing ourselves to the truth of p. Such conditionals, our linguistic lassitude notwithstanding, are properly expressed in the subjunctive. When they are qualificational conditionals, they are decided with reference to universals; when hypothetical conditionals, the decision is rendered vis-a-vis predictions.

The concept of the *I*-value was initially proposed as a response to Hempel's paradox of confirmation, motivated by its author's desire to rehabilitate the logistic analysis of scientific discourse. The law of transposition, on which the paradox, depended, fails in the proposed three-value logic. While some other lost laws, such as that of Duns Scotus, spawn no regret, transposition is not as easy to abrogate. It is true that it has been known to be problematic independently of Hempel's problem: when $p \supset q$ is interpreted causally, it is hardly plausible to relate $\sim q$ causally to $\sim p$. Nevertheless, there is certainly some sense in which transposition is valid. This is where the logic of the subjunctive is required, for transposition is certainly valid when one of its sides is read hypothetically or counterfactually: if p implies q then if q should be false so would be p. The interesting examples that immediately present themselves are universal statements of the form: if every P-thing is a Q-thing, then if anything should not be Q it would not be P. As Table 2 shows, there is a nonequivalent but mutually implicational relation between the two sides of the transposition. The law is partially saved, thus explaining its intuitiveness; but the critical assumption of Hempel's paradox—the equivalence—is eliminated.

The concept of the *I*-value has further proved its worth in resolving the bifurcation between classical and contemporary logic. By providing a formal interpretation for the theory of presupposition, we are no longer confronted by two logics: one which does and one which does not support subalteration. Moreover its utility is not limited to the problems discussed in this essay. One can easily show that the problem which drove Carnap to contrive reduction sentences in order to express dispositional definitions dissolves under this concept. In those cases where the test is not, has not, or even never will be performed, the mutual implication:

$$(\exists u)(\exists v)(\exists w)(Du \,\&\, Tv \,\&\, Rw) \supset (x)((Dx \supset (Tx \supset Rx)) \,\&\, ((Tx \supset Rx) \supset Dx))$$

is to be ignored. The "region of indeterminateness" (see [2], p. 296) and potential for meaninglessness thus disappears.

One last problem with the notion of presupposition demands clarification. If categorical statements presuppose existence, what happens to propositions whose subjects are empty? Such propositions occur in denials of existence: "No unicorns exist" and in vacuous tautologies such as: "All square circles are square". The presuppositional account of the latter can be easily saved, since the subject is obviously a complex (intersection) whose elements can be separated:

$$((\exists x)Su \,\&\, (\exists v)Cv) \supset (x)\ ((Sx \,\&\, Cx) \supset Sx)\ .$$

We might apply the same solution to the former, breaking down unicorn into a conjunction of defining characters. But another approach suggests itself.

The presuppositional account requires that $\exists u(\phi u)$ be true for any category ϕ in a statement. We have become accustomed to regard the particular

Table 2.

$(\exists x)Px \,\&\, Qx$	$(\exists x)Px \,\&\, {\sim}Qx$	$(\exists x){\sim}Px \,\&\, Qx$	$(\exists x){\sim}Px \,\&\, {\sim}Qx$
T	*T*	*T*	*T*
T	*T*	*T*	*F*
T	*T*	*F*	*T*
T	*T*	*F*	*F*
T	*F*	*T*	*T*
T	*F*	*T*	*F*
T	*F*	*F*	*T*
T	*F*	*F*	*F*
F	*T*	*T*	*T*
F	*T*	*T*	*F*
F	*T*	*F*	*T*
F	*T*	*F*	*F*
F	*F*	*T*	*T*
F	*F*	*T*	*F*
F	*F*	*F*	*T*
F	*F*	*F*	*F*

$((\exists u)Pu \,\&\, (\exists v)Qv) \supset (x)$		$((Px \supset Qx) \supset ({\sim}Qx \supset {\sim}Px))$		
T	*I*	*F*	*I*	*F*
T	*I*	*F*	*I*	*F*
T	*I*	*F*	*I*	*F*
T	*I*	*F*	*I*	*F*
T	*T*	*T*	*T*	*T*
T	*I*	*T*	*I*	*I*
T	*T*	*T*	*T*	*T*
T	*I*	*T*	*I*	*I*
T	*I*	*F*	*I*	*F*
T	*I*	*F*	*I*	*F*
F	*I*	*F*	*I*	*F*
F	*I*	*F*	*I*	*F*
F	*I*	*I*	*I*	*T*
F	*I*	*I*	*I*	*I*
F	*I*	*I*	*I*	*T*
F	*I*	*I*	*I*	*I*

quantifier as existential, thereby merging two notions: individuation and existence. There is potential for paradox here, which comes when one overlooks the parameter of range. In ordinary contexts, such as talk about John's children, we assume that the range is restricted to real entities, so that individuation implies and is implied by the affirmation of existence. In such contexts, ${\sim}(\exists x)\phi x$ amounts to a denial of reality. However, we commonly distinguish between real existence and existence in the mind. When the range is broadened to include both, as it is when we are speaking about unicorns, kings of France, and many more interesting entities, we cannot allow individuation and existence to be confounded. To say $(\exists x)\phi x$ means there is some ϕ-entity within the specified range, but that does not mean the entity is real. Affirmations and denials of reality must be accomplished by means of that predicate by which we distinguish a real hundred dollar bill from an imaginary one.

Strawson proposed the theory of presuppostion in order to relieve logic of its embarrassment in the face of the bald monarch of France. It was clear to him then that the existence of its subject is a necessary condition for a proposition's being true or false. The use of uniquely referring expression implies, "in some sense of 'imply' " ([5], p. 331), the existence of the referent. A formal explication of this sense of *imply* has not been achieved in the context of two-value logic. Sensing, perhaps, the discomfiture that waits on those who try, Strawson fell back on the informal notion of *signal*, and this in turn was at least partially responsible for his skeptical concession "ordinary language has no exact logic". In providing a three-valued formal interpretation of this sense of 'imply', this paper does not dispute the abundant evidence he provided for the informality of the *sentences* of ordinary language. But it is self-defeating for logicians to subscribe to this thesis with respect to the propositions contained within them. If our discipline is to have any pragmatic value at all, then we must insist, of statements, that the logic is there, like ore in a mine, or else discourse is barren.

NOTES

1. van Fraasen's [6] attempt to explicate presupposition as an indicative implication foundered on modus tollens, and forced him to contrive the semantic relation of necessitation. Bunch [1] argued that presupposition is essentially a subjunctive concept because it is dispositional in nature, and disposition terms can't be adequately expressed except by the subjunctive. While acknowledging a close relationship between presupposition and the subjunctive, this paper sets forth an indicative interpretation which is adequate for all the important theses of propositional logic.

2. A similar table can be set up with a middle term for syllogistic logic. It has 256 rows and 96 columns, a bit more than can be conveniently reproduced.

3. Three truth tables are here reproduced for modus tollens, disjunctive syllogism, and affirming the consequent:

p	q	$((p \supset q)\ \&\ {\sim}q) \supset {\sim}p$				$((p \vee q)\ \&\ {\sim}p) \supset q$			$((p \supset q)\ \&\ q) \supset p$		
T	T	T	F	I	F	T	F	I	T	T	T
T	F	F	F	I	F	T	F	I	F	F	I
T	I	I	I	I	F	T	F	I	I	I	I
F	T	I	F	I	T	T	T	T	I	T	F
F	F	I	T	T	T	F	F	I	I	F	I
F	I	I	I	I	T	F	F	I	I	I	I
I	T	I	F	I	I	T	T	T	I	T	I
I	F	I	T	I	I	F	F	I	I	F	I
I	I	I	I	I	I	I	I	I	I	I	I

4. One might think that if $((p \supset q)\ \&\ (r \supset s)) \supset ((p \vee r) \supset (q \vee s))$ is valid, then by exportation—which this paper affirms is valid—$(((p \supset q)\ \&\ (r \supset s))\ \&\ (p \vee r)) \supset (q \vee s)$ should be valid as well. It is important to keep in mind that in this three-value logic, the column of values under the main connector of a thesis need not (and usually is not) composed of all T's. Therefore the fact that a certain implicational formula is a thesis does not guarantee that its consequent is never F. It only ensures

that the consequent is never F when the antecedent is T. Therefore, taking exportation $(p \supset (q \supset r)) \supset ((p \& q) \supset r)$ and substituting $(p \supset q) \& (r \supset s)$ for p, $p \vee r$ for q, and $q \vee s$ for r, we get the valid constructive dilemma implying the contingent one. The implication itself, of course, is valid. Wherever the consequent is F, one finds that the antecedent is I or F. In other words, the use of a thesis to validly imply a formula does not guarantee that the latter is a thesis. It is affirmable only for those rows (truth-value combinations) in which the antecedent thesis is true, and thus detachable.

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