

Predication in the Logic of Terms

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Abstract The paper contrasts modern predicate logic (MPL) and term/functor logic (TFL) on predication. A predication in TFL consists of two terms and a “logical copula” that has formal properties such as symmetry or transitivity. The I-functor in ‘PiS’ (the old form of ‘(some) S is P’) is symmetrical, behaving like the plus sign of high school algebra; TFL transcribes ‘PiS’ as ‘ $P + S$ ’. The transitive A-functor in ‘PaS’ (every S is P) is minus-like: ‘ $P - S = -((-P) + S)$ ’ represents the equivalence of ‘PaS’ to ‘not ((-P)iS)’. In propositional logic ‘ $q + p$ ’ transcribes ‘ $p \& q$ ’ and ‘ $q - p$ ’ transcribes ‘ q if p ’; thus ‘ $q - p = -((-q) + p)$ ’ is the algebraic form of ‘ $p \rightarrow q = -(p \& (-q))$ ’. TFL applies to relational statements of any complexity. E.g., to show the inconsistency of ‘every A is B and something R to an A is not R to a B’ we add ‘ $-(R + B) + (R + A)$ ’ to ‘ $B - A$ ’ to get the contradiction ‘ $-(R + B) + (R + B)$ ’. The predicative functors are shown to give TFL a slight advantage over MPL in expressive and inference power when dealing with singular statements.

The copula has no place in the language of modern logic. It will be shown that a significant price in the hard currency of inference power is being paid because of its absence. A properly formulated term logic, extended to handle relational inference, is both syntactically simpler and inferentially more powerful. But, historically, term logic took a wrong turn and we begin with that.

I Traditional syllogistic logic with its A, E, I, and O classification of categorical statements has a distinctive syntax that was not properly understood by its practitioners. Confusion arose because most syllogists favored a parsing of

*I am grateful to Michael Lockwood, Aris Noah, George Engebretsen, Jerry Samet, John Bacon, and David Kelley for much helpful discussion and to the anonymous referee whose acute and meticulous observations were responsible for considerable improvements over an earlier version.

the categorical sentence into a noun phrase subject and a verb phrase predicate, dividing, say, ‘some S is P ’ into a particular subject, ‘some S ’, and an affirmative predicate, ‘is P ’. The noun/verb analysis survives in modern predicate logic (MPL) at the level of singular atomic sentences: we shall see that even there it is an abiding source of logistic weakness. But for traditional (pre-Fregean) logic the linguistic NP/VP parsing for both singular and general sentences was blighting, since it barred the way to a logically effective treatment of relational sentences.

Logical grammar requires that a sentence be parsed by distinguishing its categorematic and syncategorematic parts, dividing ‘some S is P ’ into its *material* elements, the terms S and P , and its *formative* elements, in this case the binary I-functor, ‘some (is a)’, which connects the two terms to form the statement. A logic of terms whose syntax is term/functor will be called a *term/functor logic* (TFL).¹

The term/functor style of analysis may be said to go back to Aristotle, who formulated sentences in a way that placed the functor *between* the terms. Aristotle did this by putting the predicate term first and reading the functor predicatively as one of the expressions ‘belongs to some’ or ‘belongs to every’. Thus Aristotle would represent ‘some S is a P ’ as ‘ P belongs to some S ’. This, however, left room for misunderstanding, since one could read it as ‘ P belongs to / some S ’, which again divides the statement in the logically ineffectual subject/predicate way. The proper TFL reading is ‘ P belongs-to-some S ’ or ‘ P some S ’, with ‘some’ a binary functor, a logical copula, that predicatively joins P to S . Medieval logicians who represented this as ‘ PiS ’ had the right idea but even there the NP/VP analysis remained tempting, since the classification of a statement as A or I need not be taken in a term/functor way. For most syllogists, ‘I’ was read as a mnemonic sign; an “I statement” was simply a statement whose subject was particular and whose predicate was affirmative, a definition that is blind to the role of ‘I’ as a term connective in its own right.

Having failed to take a straightforward term/functor approach, traditional logic was unable to develop a logic of terms beyond the confines of simple categoricals. It has been shown ([10], [11], [19], and [20]) that term/functor logic can be naturally extended to handle any argument within the scope of MPL. The present concern is to go further, by pointing to areas of logic where MPL is inferior in inference and expressive power to TFL, which can provide a logical account of certain inferences that MPL cannot represent.

1.1 Term functor syntax An elementary sentence of TFL consists of two terms and a functor that connects them. Let ‘+’ stand for the I-functor ‘some’ taken as a binary term connective. Then ‘ $S + P$ ’ represents ‘some S is a P ’ or ‘ PiS ’.

Singular sentences are also of this form. According to Leibniz, what distinguishes ‘Cicero is a senator’ from ‘some Roman is a senator’ is the fact that ‘Cicero’ denotes no more than a single individual. Moreover, because of this, ‘some Cicero is a senator’ entails ‘no Cicero is a nonsenator’ or ‘every Cicero is a senator’. Where ‘some’ entails ‘every’, neither word of quantity is actually used in discourse. Let ‘-’ be the unary functor representing negative particles such

as ‘un’, ‘non’, and ‘not’ that join with a single term to form its contrary or with a single statement to form its contradictory. If NN is a proper name, ‘ NN is P ’ transcribes as ‘ $NN + P$ ’ and it entails ‘ $-(NN + (-P))$ ’.

Leibniz’s treatment of singular sentences as I-sentences recommends itself over the more standard scholastic treatment of ‘Cicero is a senator’ as an A-sentence, ‘every Cicero is a senator’. By interpreting singular sentences as particular sentences that entail their own universal generalizations, one explains their logical powers (see [20], Chapters 1, 2, and 5). Semantically as well, singular and particular general propositions are alike in their truth claims. Universal propositions claim absence; for example, ‘every S is P ’ claims the nonexistence of things that are non- P and S . Singular and particular propositions, on the other hand, claim presence. Just as ‘some Roman is not a senator’ claims that there exists a nonsenator who is a Roman, so ‘Cicero is not a senator’ is existentially positive in claiming the existence of a nonsenator who is Cicero. So construed, the sentence is of the form ‘ $NN + (-P)$ ’. By contrast, ‘Homer did not write the *Iliad* and the *Odyssey*’, asserted by someone who gives as his reason that no such person as Homer existed, is of the form ‘ $-(NN + P)$ ’.

The basic sentences of TFL are of two kinds: particular-positive and universal-negative. In treating ‘Cicero is a senator’ as having the same form as ‘some Roman is a senator’, the tendency is to assimilate the former to the latter but not the other way around. But there is no reason not to think *neutrally* of both as simple predications. Particular quantity is then simply one form of positive predication; a predication is positive (of the form ‘ $X + Y$ ’) or negative (of the form ‘ $-(X + Y)$ ’).

[The more common threefold classification of statements into singular, particular, and universal is associated with the idea that ‘some’ and ‘every’ are unary operators qualifying a subject noun phrase as particular or universal. In TFL, ‘some’ is *binary*; as such it is a term connective, a kind of positive copula that ties the two parties of predication in the elementary sentences of TFL. We shall see below that ‘every’ is definable as a negative copula (Section 5).]

In any case, the term/functor parsing of singular sentences is in sharp contrast to their treatment in MPL. Names and predicates are the material elements and such sentences are viewed as atomic, altogether lacking in formative elements. The custom of representing the material parts of atomic sentences by letters from different fonts of type suits their special character.² For example, the formula ‘Sc’ (representing ‘Cicero is a senator’) consists entirely of two syntactically distinct (noninterchangeable) categorematic expressions, a subject name and a predicate verb, which are not mediated by a syncategorematic sign.

Terms in TFL are grammatically interchangeable, and as long as we are restricted to the I-functor the order of terms does not matter. Łukasiewicz favored the I-functor as the primitive binary term connective for an axiomatic treatment of syllogistic logic. And indeed, two functors—one binary and one unary—suffice for a logic of terms and sentences. We are using ‘+’ for the binary functor and ‘-’ for the unary functor. The reason for adopting ‘+’ and ‘-’ as logical functors is that they behave in the logic of terms just as they behave in high school algebra [20]. That the I-functor and ‘and’ are plus-like is illustrated in Sections 2, 3, and 4. The algebraic behavior of the minus sign is illustrated later (Section 5).

2 The functor ‘+’ is commutative. Indeed, its formal properties as a binary term connective are just those of ‘and’. We exploit this by using ‘+’ to represent ‘and’ as well as ‘some’, taking care that context disambiguates these interpretations. To ensure this we could use angular brackets whenever ‘and’ connects terms to form a compound term; we should then transcribe ‘some farmer is a gentleman and scholar’ as ‘F + ⟨G + S⟩’. The compound term ‘⟨G + S⟩’ would then be distinct from the sentence ‘G + S’ (‘some gentleman is a scholar’). Of course ‘+’ in its conjunctive meaning is also used for a binary connective between sentences. Here too we must introduce and adhere to some conventions in order to avoid ambiguity. For example, we could use only lower case letters for sentences and only upper case letters for terms, reading ‘*p* + *q*’ as ‘*p* and *q*’ and ‘*P* + *Q*’ as ‘some *P* is *Q*’.

The predicative and conjunctive use of ‘+’ is systematically ambiguous. Such systematic ambiguity is logistically efficient provided that a sign having different interpretations has the same formal properties for logical reckoning. An example of the logistic advantage of using ‘+’ with double interpretation is seen in the application of the Law of Association to account for the equivalence

some officer is a gentleman and scholar
 = some officer and gentleman is a scholar.

This transcribes as

$$O + \langle G + S \rangle = \langle O + G \rangle + S$$

in which predicative and conjunctive occurrences of ‘+’ crisscross.³

The formal affinity of conjunction and predication extends also to relative clauses like ‘gentleman who is a scholar’ (‘gentleman and scholar’) and to compound phrases like ‘German scientist’ (‘German who is a scientist’, ‘German and scientist’), all of which are transcribed as ‘⟨G + S⟩’. The laws of commutation, association, and simplification apply to such phrases as well. Thus ‘some German scientist is a physicist’ is transcribed as ‘⟨G + S⟩ + P’ and it entails such sentences as ‘⟨S + G⟩ + P’ (‘some scientist who is a German is a physicist’), ‘G + ⟨S + P⟩’ (‘some German is a scientist who is a physicist’), and ‘S + ⟨G + P⟩’ (‘some scientist is a German and a physicist’). Chomsky has argued that a phrase like ‘wise man’ in a sentence like ‘a wise man is honest’ should be understood as having the sentence ‘some man is wise’ in its “deep structure” and he commends the Port Royal logicians for perceiving that ‘a man is wise’ is a subsentence of ‘a wise man is honest’. Dyadic parsing, exploiting the formal properties of the binary term connective, shows that a subsentence equivalent to ‘a man is wise’ is embedded in ‘a wise man is honest’. Thus ‘⟨M + W⟩ + H’ (‘a man who is wise is honest’) is equivalent to ‘⟨W + M⟩ + H’ (‘a wise man is honest’); by simplification it entails ‘M + W’ (‘a man is wise’). We may similarly derive ‘G + P’ (‘some German is a physicist’) from ‘S + ⟨G + P⟩’.

The dyad, consisting of two material components (two terms or two sentences) and a connecting binary functor, is the basic syntactic unit of term logic. It is a fundamental terminist thesis that any sentence or compound expression can be parsed as a dyad whose material components may themselves be dyads. For example, the compound sentence ‘*p* & *q* & *r*’ may be freely transcribed as

a triad, ' $p + q + r$ ', and rewritten dyadically as ' $p + (q + r)$ ' whose right conjunct is itself a dyad. Similarly, 'some farmer is a Republican who is a gentleman and a scholar' might first be transcribed as ' $F + \langle R + G + S \rangle$ ' and then reparsed dyadically as ' $F + \langle R + \langle G + S \rangle \rangle$ '.

3 Relational statements in TFL The terminist logician also analyses relational sentences dyadically. According to Leibniz, the dyadic parts of relational sentences are themselves sentences or subsentences of the whole. Thus Leibniz remarks that 'Paris loves Helen' is to be understood as 'Paris loves and *eo ipso* Helen is loved'. Leibniz's idea is that the relational expression 'loves' is to be construed as Janus-faced, 'lover/loved', turning one face to 'Paris' and the other to 'Helen'. To render this perspicuous we index the terms being paired in the sentence, transcribing 'Paris is a lover of Helen' as ' $P1 + L12 + H2$ '. The term pairs implicit in this sentence are:

P1, L12	Paris, lover
H2, L12	Helen, loved
P1, (L12,H2)1	Paris, lover of Helen
H2, (L12,P1)2	Helen, loved by Paris.

We are treating 'Paris loves Helen' on all fours with 'some Pole loves some Hungarian', both of which are represented as ' $P1 + L12 + H2$ '. This relates to the considerations adduced earlier for putting 'Cicero is a senator' and 'some Roman is a senator' into the same category of predications. The plus sign is commutative and associative, so ' $P1 + (L12 + H2)$ ' is equivalent to ' $P1 + (H2 + L12)$ ' ('some Pole is what some Hungarian is loved by', 'Paris is what Helen is loved by'), and to ' $(P1 + L12) + H2$ ' (someone that a Pole loves is a Hungarian', 'someone that Paris loves is Helen').

A proper term pair consists of two terms that have a numerical index in common. In what follows, 'term pair' will always mean 'proper term pair'. The terms of a well-formed dyad must be a proper pair: the common index signifies that the terms codenote the same thing. Henceforth 'dyad' will mean 'well-formed dyad'. In a simple, two term sentence, numerical indices will be omitted. Thus 'some Spaniard is a politician' will be represented as ' $S + P$ ' instead of, say, ' $S5 + P5$ '.

' $P1 + L12 + H2$ ' is a triad. Two ways of writing it dyadically are:

- (1) $(P1 + L12) + H2$ someone Paris loves is Helen
- (2) $P1 + (H2 + L12)$ Paris is someone Helen is loved by.

It would appear to be Leibniz's view that 'Paris is a lover' and 'Helen is a loved one' are entailed by 'Paris loves Helen'. And indeed ' $P1 + L12$ ' and ' $H2 + L12$ ' follow by simplification from (1) and (2), respectively. But two interpretations of ' $P1 + L12$ ' are possible. On one reading we construe it as an open *relational* sentence 'Paris is a lover of . . .'. On a second reading it is a closed *monadic* sentence 'Paris is a lover' or 'Paris loves'. Leibniz clearly favors the monadic reading when he says that 'Paris loves Helen' is equivalent to 'Paris loves and *eo ipso* Helen is loved'. We could explicate the '*eo ipso*' in Leibniz's formula by understanding his argument to be based on the equivalence of 'Paris loves Helen' to

the truth functional conjunction of (1) and (2), from which the closed sentences ‘Paris loves’ and ‘Helen is loved’ follow by simplification. In holding to a closed sentence interpretation, the terminist logician maintains that the degree of an expression is determined contextually. In the triad ‘P1 + L12 + H2’ the expression which transcribes ‘loves’ is of degree two, since it pairs simultaneously with ‘P1’ and ‘H2’. But in a dyad both terms are read monadically. The particular meaning of a term within its dyad may then be determined by the term it pairs with. Thus the common index of the term pair in ‘P1 + L12’ is ‘1’; this determines the interpretation of ‘L12’ as ‘lover’, the second index being idle and unengaged. On the other hand, in ‘H2 + L12’ the common index is ‘2’, engaging the passive face of ‘L12’ and giving ‘Helen is loved’ as the reading.

[Being of no fixed degree, the term letters of a primitive vocabulary for a term functor language are not superscripted. Some terms (sortals, abstract nouns, and proper names) do not admit of relational use but others can occur in environments that give them different degrees. Thus ‘W’ in ‘S1 + W12 + C2’ (‘snow is whiter than cream’) is a two-place term; the same term is monadic in ‘S1 + W1’ (or ‘S1 + W12’ where this is got from ‘S1 + W12 + C2’ by simplification). Similarly, in ‘S1 + F1’ (‘snow is falling’) ‘F’ is monadic; in ‘S1 + F12 + H2’ (‘snow is falling on a house’) it is dyadic. When a term occurs “relationally” it is often accompanied by a preposition like ‘of’, ‘on’, ‘by’ or an expression of comparison such as ‘more . . . than’. The terminist does not construe these as part of the term but as mere indices directing us to a term with which the indexed term is being paired. For example, ‘3’ represents ‘from’ in ‘T1 + B123 + C2 + H3’ (‘Tom bought a Cadillac from Harriet’).]

3.1 In general, then, the terminist approach to the syntactic analysis of a sentence is to make explicit its dyadic structure by pair indexing its codenoting terms. Consider a more complex sentence that involves two- and three-place relations and a relational product:

(A) A robber of an owner of a farm sold a truck to a neighbor.

This may be transcribed as

$$(R12 + (023 + F3)) + (S145 + T4 + N5).$$

Some of the term pairs that figure in (A) are

R12, 023	robbed, owner
F3, 023	farm, owned
R12, S145	robber, seller
T4, S145	truck, sold
N5, S145	neighbor, buyer
T4, (S145,N5)4	truck, sold to neighbor
(R12,023)1, (S145,N5)1	robber of owner, seller to neighbor.

In terminist analysis any sentence is a term/functor dyad each of whose nonelementary terms must itself be dyadic in structure. A nonelementary term is either compound (as in ‘some philosopher is a *farmer and a gentleman and a scholar*’), or relational (as in ‘a man is *selling a truck to a neighbor*’). We noted

earlier that when a compound term is not overtly dyadic in structure, it can be given dyadic form by *grouping* as in ‘some philosopher is a ⟨farmer and ⟨gentleman and scholar⟩⟩’. A similar parsing may be given for a sentence containing a relational term of degree greater than two.

A dyadic grouping for sentences with relational terms of degree higher than two is made by a dyadic bracketing that places each subject term to the left of its own predicate term. For example,

(B) A man is selling a truck to a neighbor

is first freely transcribed as

$$M1 + (S123 + T2 + N3).$$

This transcription, which follows the English vernacular in a straightforwardly linear way, we may call the English Normal Form (ENF). The ENF of a sentence will not normally have the canonical dyadic structure in which every nonelementary term is a dyad (that may embed other dyads). However, any ENF has such a canonical paraphrase. Thus we may *regroup* the terms of B’s ENF in a new transcription

$$M1 + (T2 + (N3 + S123))$$

which may be read as ‘a man a truck to a neighbor is selling’. In this paraphrase, which we call the Dyadic Normal Form (DNF), each subject term is to the left of its own predicate term and each term/functor dyad is a sentence or subsentence. Thus, ‘T2 + (N3 + S123)’ is the subsentence ‘a truck is to a neighbor being sold’ and ‘N3 + S123’ is the subsentence ‘a neighbor is buying’.

3.2 A sentence in DNF is in strict formal correspondence to a formula of modern predicate logic that is its standard translation. One rule for translating a DNF formula to the corresponding formula of MPL is the bridging rule:

$$\mathbf{R1} \quad Sn + Pn = (\exists x)(Sx \& Px).$$

Applying R1 in translating (B) we proceed as follows

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|--|--------|
| 1. M1 + (S123 + T2 + N3) | ENF |
| 2. M1 + (T2 + (N3 + S123)) | 1, DNF |
| 3. $(\exists x)(Mx \& (T2 + (N3 + Sx23)))$ | 2, R1 |
| 4. $(\exists x)(Mx \& (\exists y)(Ty \& (N3 + Sxy3)))$ | 3, R1 |
| 5. $(\exists x)(Mx \& (\exists y)(Ty \& (\exists z)(Nz \& Sxyz)))$ | 4, R1. |

These steps may be reduced to three by initially assigning x , y , and z to the numerical indices 1, 2, and 3. (Note that R1 is applied from outside in.)

In translating sentences containing conjunctions we treat ‘⟨S + P⟩n’ as ‘Sn + Pn’. Consider

(C) No owner of a handgun votes for a liberal politician.

Its ENF is

$$-((012 + H2) + (V13 + \langle L + P \rangle 3)).$$

Its DNF is

$$-((H2 + 012) + ((L + P)3 + V13)).$$

By successive applications of R1 this translates as

$$-((\exists x)((\exists y)(Hy \& Oxy)) \& (\exists z)((Lz \& Pz) \& \forall xz)).$$

(For rules of translation between TFL and MPL see [4], [10], and [20].)

4 The predicative functor The similarities of TFL and MPL are best studied by comparing DNF forms to the corresponding forms in MPL. But our present concern is with how TFL and MPL *differ* in logical syntax and with how the differences affect their respective inference powers. Briefly, MPL comes off the worse because its singular sentences lack the formal properties of a mediating predicative sign tying together the two parties of predication, while TFL has the plus sign which behaves as ‘and’ behaves. We now take a closer look at the formal properties of the binary functor as they are manifested in its purely predicative occurrences.

The commutivity of the predicative functor is manifested in the converse equivalences of singular and particular predications:

$$A + B = B + A$$

$$\text{(some) } A \text{ is (a) } B = \text{(some) } B \text{ is (an) } A$$

for example, in the equivalent pairs of statements:

- (1) Some Spaniard is a painter; some painter is a Spaniard.
- (2) Twain is Clemens; Clemens is Twain.

Associativity is manifested in equivalences of form:

$$X1 + (R12 + Y2) = (X1 + R12) + Y2$$

$$\text{(some) } X \text{ is } R \text{ to some } Y = \text{something (an) } X \text{ is } R \text{ to is (a) } Y$$

for example in the equivalence:

$$P1 + (L12 + H2) = (P1 + L12) + H2$$

$$\text{Paris loves Helen} = \text{Someone Paris loves is Helen.}$$

The laws governing the plus sign apply only to well-formed dyads. For example, ‘P1 + H2’ is not well-formed, so we cannot use commutation and association on ‘P1 + (L12 + H2)’ to derive ‘(P1 + H2) + L12’.

Finally, just as ‘+’ in its conjunctive meaning obeys the law of simplification that allows us to detach ‘(p + q)’ from ‘(p + q) + r’, so too are we permitted to detach ‘X1 + R12’ from ‘(X1 + R12) + Y2’. For example, given ‘Paris loves Helen’ we derived ‘Paris loves’, since from P1 + (L12 + H2)’ we get to ‘(P1 + L12) + H2’ by association, and from ‘(P1 + L12) + H2’ to ‘P1 + L12’ by simplification. Similarly, we derive ‘Helen is loved’ by commutation—P1 + (H2 + L12)—and simplification: H2 + L12. (And, again, since ‘P1’ is not a well-formed sentence it makes no sense to derive it from ‘P1 + (H2 + L12)’.)

Having seen that predicative expressions of the form ' $X + Y$ ' obey the law of commutation and that expressions of the form ' $X1 + (R12 + Y2)$ ' obey the laws of association and simplification, we now look at some types of inferences in which these formal properties of the predicative functor come into play.

4.1 Simplification Consider the following dialogue:

Fred. Does June ever smile?

Leah. As it happens she is smiling at someone this very moment.

Fred. So she does smile!

The little argument implicit in this exchange is:

(A1) June smiles at someone \therefore June smiles.

Letting P stand for 'person' the premise of (A1) is ' $J1 + (S12 + P2)$ '. The conclusion of (A1) then follows by association and simplification:

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|----------------------|-------------------|
| 1. $J1 + (S12 + P2)$ | premise |
| 2. $(J1 + S12) + P2$ | 1, association |
| 3. $J1 + S12$ | 2, simplification |

Consider the following very similar argument:

(A2) June smiles at Tom \therefore June smiles.

Here too ' $J1 + S12$ ' follows from ' $J1 + (S12 + T2)$ ' by association and simplification.

Turning to MPL's treatment of (A1), we find that it cannot be expressed in it. The premise of (A1) is represented as ' $(\exists x)(Sjx)$ '. But MPL cannot provide a conclusion that differs from the premise. (A2) fares a bit better. The premise is ' Sjt ' from which ' $(\exists x)(Sjx)$ ' follows by existential generalization. Note, though, that the price of this way of accounting for (A2) is that although 'June smiles at Tom' is a singular ("atomic") statement, the conclusion 'June smiles' must be read as a general statement. It would in any case seem that a correct account of (A1) and (A2) should treat them in the same way. The moral of these examples and those that follow is that the lack of a predicative functor puts MPL at a singular disadvantage.⁴

4.2 Associative shift Consider the following inferences:

(A3) Someone Wittgenstein did respect was Augustine.

(So) Wittgenstein respected Augustine.

(A4) Someone Wittgenstein did respect was an uncle of Augustine.

(So) Wittgenstein respected an uncle of Augustine.

TFL treats (A3) and (A4) in a uniform manner:

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|----------|------------------------------|----------------|
| (A3) TFL | 1. $(W1 + R12) + A2$ | premise |
| | 2. $W1 + (R12 + A2)$ | 1, association |
| (A4) TFL | 1. $(W1 + R12) + (U23 + A3)$ | premise |
| | 2. $W1 + (R12 + (U23 + A3))$ | 1, association |

Both (A3) and (A4) are instances of an associative shift to the right.

To handle (A3) and (A4), MPL needs identity.

- (A3) (MPL) 1. $(\exists x)(Rwx \ \& \ x = a)$ premise
 2. Rwa 1, Leibniz's Law

A more complicated rendering, employing the identity sign, may be used for representing the premise and conclusion of (A4).

4.3 The passive transformation The trivial inference ‘some boy loves every girl, hence every girl is loved by some boy (or other)’ is represented in MPL as

$$(\exists x)(Bx \ \& \ (y)(Gy \rightarrow Lxy)) \ \therefore \ (y)(Gy \rightarrow (\exists x)(Bx \ \& \ Lxy)).$$

But though we may read L as meaning ‘is loved by’ the formula does not contain the passive form, and the conclusion really says something like ‘every girl is such that there is a boy who loves her’. In any case, MPL is unable to express, let alone account for, inferences like the following one:

(A5) Paris loves Helen, hence Helen is loved by Paris.

Either (A5) is represented as ‘Lph \therefore Lph’, which is unacceptably insensitive to the difference between premise and conclusion, or (letting ‘L*’ stand for ‘is loved by’) (A5) is represented as ‘Lph \therefore L*hp’, in which case we have no way to prove its validity without appealing to a passive transformation. Of course we may explicitly acknowledge this appeal by introducing ‘*’ as a new primitive predicate conversion functor, perhaps in the form of a definition like ‘L*yx =_{def} Lxy’.

By contrast, passive transformations are easily and naturally derivable in TFL for both singular and general propositions. The derivation of ‘Helen is loved by Paris’ is:

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| 1. P1 + (L12 + H2)1 | premise | Paris is lover of Helen. |
| 2. (P1 + L12)2 + H2 | 1, association | Someone Paris loves is Helen. |
| 3. H2 + (P1 + L12)2 | 2, commutation | Helen is someone that Paris loves. |
| 4. H2 + (L12 + P1)2 | 3, commutation | Helen is loved by Paris. |

The same method of proof applies when the statements are general, moving, for example, from ‘some Pole loves a Hungarian’ to ‘some Hungarian is loved by a Pole’.

4.4 Frege's dodge These days the move from active to passive is not given a logical account. Frege explicitly rules out passive transformations as of “no concern to logic”, treating the difference between ‘a gives b to c’ and ‘b is given to c by a’ as merely stylistic and on a par with the difference between ‘this dog howled’ and ‘this cur howled’. His argument is that in either case one and the same thought is expressed since there can be no difference of truth value (see [7], p. 141). If one took this argument seriously one would be led to say that the inference from ‘some A is B and C’ to ‘some C is B and A’ should be of no concern to logic, since here too both have the same truth conditions. But it is

clear that what Frege counts as a logical transformation must be expressible as such in his concept-script and the passive transformation is not. His position is dictated by the consideration that his logical language provides no way of expressing or justifying a passive transformation for a sentence like ‘Paris loves Helen’, while it does have a way of expressing the difference between ‘some A is B and C ’ and ‘some C is B and A ’ and of showing how one follows from the other. Sentences that cannot be distinguished in the canonical language and that have the same truth conditions are mere paraphrases of no concern to logic. One may agree with this. But if one’s “concept-script” is defective, it is short of being canonical and some logical transformations will have been ruled out by default. One may put this point another way: of two concept-scripts, the superior is the one that comprehends more transformations *within* the “concern” of logic.

4.5 Dative movement Dative movement is a kind of inference involving relations of more than two places that is structurally related to the moves between active and passive forms of two-place relational terms. An example is:

(A6) Dave sold a truck to June \therefore Dave sold June a truck.

MPL must leave (A6) to the linguists. But TFL can give a straightforward derivation:

- | | |
|------------------------------|----------------|
| 1. $D1 + (S123 + T2 + J3)$ | premise in ENF |
| 2. $D1 + (T2 + (J3 + S123))$ | 1, DNF |
| 3. $D1 + ((J3 + S123) + T2)$ | 2, commutation |
| 4. $D1 + (J3 + (S123 + T2))$ | 3, association |
| 5. $D1 + (J3 + (T2 + S123))$ | 4, commutation |
| 6. $D1 + (S123 + J3 + T2)$ | 5, ENF |

The penultimate line is the DNF paraphrase of the conclusion of (A6).

5 The negative copula MPL and TFL are to a large extent intertranslatable but they are not mere “notational variants”, and not only due to the differences in expressive power we have been discussing. The more fundamental difference pertains to the very nature of the predicative relation and to the nature of the categorematic expressions being predicatively tied together. Does predication relate predicate to name or term to term? Does it have formal properties of the kind that some relations have, such as symmetry or transitivity? The relation tying name to predicate in MPL is by now the subject of a vast literature, but for present purposes all that needs to be noted is that it has no formal properties of any kind, since such properties as symmetry, transitivity, and reflexivity are precluded by the syntactical rules that prohibit the interchangeability of the parties to a predicative tie. By contrast, the I-functor, which joins interchangeable terms, is symmetrical though not transitive. Moreover, we can use the symmetry of the predicative I-functor and the distributive properties of the unary negative functor to define a new binary term connective, the so-called A-functor, which is transitive and reflexive but not symmetrical, and which, as we are now about to see, is opposed to the I-functor as a negative copula is opposed to a positive copula.

Henceforth we write ‘ $P + S$ ’ for ‘ P some S ’, ‘ PiS ’, or ‘ P belongs-to-some S ’ (the Aristotelian form of ‘some S is P ’). In effect we are adopting the Aristotelian convention of putting the predicate term first. We now define an algebraic representation for ‘ PaS ’ in which ‘every’ (the A-functor) plays the role of a binary predicative connective. We use as definiens the sentence ‘not((non- P)i S)’ or ‘ $-((-P) + S)$ ’, and proceed by obversion to distribute the minus sign inwards, changing the predicate term from negative to positive and changing the binary connection from ‘belongs to some’ to ‘belongs to every’. Algebraically this transformation is represented by the first of the following three variants of the definitional equivalences:

$$\begin{aligned} P - S &=_{def} -((-P) + S) \\ PaS &=_{def} -((-P)iS) \\ P \text{ every } S &=_{def} \text{not (non-}P \text{ some } S). \end{aligned}$$

Unlike the unary minus signs representing the negative particles in the definiens on the right, the minus sign representing ‘every’ or ‘A’ in ‘ $P - S$ ’ is a binary functor. Thus ‘I’ is to ‘A’ as addition is to subtraction. Term/functor logic can get by without the A-functor (just as MPL, using only ‘not’, ‘ $\exists x$ ’, and ‘and’ can get by without introducing the universal quantifier, and just as arithmetic can get by without the use of a subtraction operator), but once we introduce it its nature is clear and much circumlocution is avoided.

5.1 We now have *two* predicative forms, ‘ $P + S$ ’ and ‘ $P - S$ ’, in which ‘some’ and ‘every’ are the logical copulas or predicative functors. A sentence whose copula is positive will be called *positive in valence* or *particular in quantity*. A sentence whose copula is negative will be called *negative in valence* or *universal in quantity*. Sentences of positive valence claim existence. Sentences of negative valence claim nonexistence. For example, ‘ $P - S$ ’ claims the nonexistence of things that are S but non- P . Two sentences of the same valence are called *convalent*. Negation reverses valence. Thus ‘ $-(P - S)$ ’ is positive in valence and ‘ $-(P + S)$ ’ is negative in valence. Convalence is a necessary condition for equivalence; divalent sentences cannot be equivalent since the existence of something cannot be the same state as the nonexistence of something. It can be shown that the following necessary and sufficient condition for equivalence is entailed by the commutivity of the binary plus functor and the distributive properties of the unary minus functor:

PEQ Two sentences are equivalent if and only if they are convalent and algebraically equal.

We call this the Principle of Equivalence (PEQ). According to PEQ ‘some S isn’t P ’ and ‘some P isn’t S ’ are not equivalent since they are algebraically unequal: $(-P) + S \neq (-S) + P$. Moreover, ‘every S is P ’ and ‘some non- S is P ’ are not equivalent since (although $P - S = P + (-S)$) the two sides of the equation are not convalent. On the other hand ‘every non- S is P ’ and ‘every non- P is S ’ are convalent as well as equal, so they are equivalent.

5.2 Natural transcription Algebraically a sentence like ‘some boy loves every girl’ would be transcribed as ‘(L12 – G2) + B1’ – ‘loves every girl some boy’, whose order is not “natural”. Note however that ‘loves every girl’ does have an Aristotelian order in English. More generally, a relational term of the form ‘*R* some/every *S*’ in a language like English has a naturally Aristotelian form: there is no grammatical copula and the predicate term appears before the subject term connected to it by ‘some’ or ‘every’. It is obviously convenient to have a mode of algebraic transcriptions from the vernacular that is linear and natural for the *whole* sentence. So instead of transcribing ‘some *S* is *P*’ as ‘*P* + *S*’ we shall transcribe it as ‘+*S* + *P*’, reading the first sign as ‘some’ and introducing a syn-categorematic plus sign for the grammatical copula ‘is’. Similarly ‘–*S* + *P*’ now transcribes ‘every *S* is *P*’. In this mode of transcription the binary predicative functors are represented by *ordered pairs* of signs: ‘+,+’ for the I-functor and ‘–,+’ for the A-functor, wherever the grammatical copula is part of the vernacular expression for the predicative relation. However, in relational phrases such as ‘taller than every Swede’, there is *no* grammatical copula and we *retain* the Aristotelian form transcribing such phrases with a single binary sign (e.g., as ‘T12 – S2’). In natural transcription, therefore, we are in effect representing ‘some’ and ‘is’ as plus signs, and ‘every’ as a minus sign.

The natural transcription of ‘some boy loves every girl’ is ‘+B1 + (L12 – G2)’. And, naturally transcribed, ‘every boy envies some owner of a dog’ is ‘–B1 + (E12 + (023 + D3))’.⁵ Note also that obversion, in which the distribution of an external sign of negation into a sentence changes its quantity and the quality of its predicate term, works for relational clauses in just the way it works for whole sentences. Thus ‘some boy is not a lover of a girl’, which transcribes as ‘+B1 + (–(L12 + G2))’, is equivalent to ‘some boy fails-to-love every girl’, or ‘+B1 + ((–L12) – G2)’. This too is what we should expect from the sentential nature of relational clauses in the logic of terms.

5.3 General syllogistic Syllogistic inference is governed by a rule expressing the Aristotelian principle that what is true of every *M* is true of any *M* (*M* being the middle term of the syllogism). For example, since ‘is mortal’ is true of every animal it is true of Socrates who is an animal. Let ‘*E*(*M*)’ be a statement containing a positive occurrence of *M* in the environment *E*. And let ‘*E**(*–M*)’ be a statement in which ‘–*M*’ means ‘every *M*’ and *M* occurs negatively in the environment *E**. The general rule of syllogistic inference is

$E^*(-M)$	premise 1 “Donor premise”
RS $E(M)$	premise 2 “Host premise”
$E(E^*)$	conclusion.

In effect, the conclusion is got by adding the premises in a manner that cancels the positive middle term in *E*(*M*), replacing that middle term by *E**.

Suppose our premises are ‘no *M* is *P*’ and ‘some *M* is not *S*’. The first of these is universal but it has no expression of the form ‘every *M*’. However, we can apply the Principal of Equivalence which shows that ‘–(*+M* + *P*)’ is equivalent to ‘–*M* + (–*P*)’. We now have

$E^*(-M)$	$-M + (-P)$	every M is non- P
$E(M)$	$+M + (-S)$	some M is non- S
$E(E^*)$	$+(-P) + (-S)$	some non- P is non- S .

Any argument makes a claim of validity. Call the conjunction of its premises and the denial of its conclusion its Counterclaim. A standard syllogism or sorites consists of n statements and n recurrent terms. It can be shown (by RS and PEQ) that any such argument is valid if and only if its Counterclaim satisfies the following two conditions:

- (1) it contains exactly one particular statement
- (2) it sums to zero.

Such a Counterclaim is inconsistent. It is easy to decide the validity of any standard syllogistic argument: simply form its Counterclaim and examine it for inconsistency.

Relational syllogisms Arguments involving relations are nonstandard. An example of how RS applies to relational arguments is

(A.1)	some atheist scoffs at every prayer	$+A1 + (S12 - P2)$
	every congregant recited a prayer	$-C3 + (R32 + P2)$
(hence)	every congregant recited something some atheist scoffs at.	$-C3 + (R32 + (+A1 + S12))$

The first premise of (A.1) is of the form $E^*(-M)$ with E^* as the expression '+A1 + S12'. The second is of the form $E(M)$ in which E is the expression '-C3 + (R32 + ...)'. So the conclusion, $E(E^*)$, is

$$-C3 + (R23 + (+A1 + S12))$$

which, by commutation of '(+A1 + S12)', is equivalent to '-C3 + (R32 + (S12 + A1))', or 'every congregant recited something scoffed at by an atheist'.

Note that RS does *not* permit us to draw conclusions of the form $E^*(E)$. Thus it does not allow us to move the Host into the Donor premise, yielding

$$+A1 + S12 + (-C3 + R32) \text{ some atheist scoffs at something every congregant recited}$$

even though this conclusion is algebraically equal to the previous one.

Sometimes the Donor premise must be teased out. Consider the premises 'every owner of a dog is enviable' and 'every boy loves some dog'

- 1. $-(O12 + D2) + E1$
- 2. $-B3 + L32 + D2$.

To apply RS we need to get an equivalent to (1) that has the phrase 'every dog' in it. The following transformations achieve this:

- 3. $-(-E1) - (O12 + D2)$ every unenviable person fails to own a dog
- 4. $-(-E1) + ((-O12) - D2)$ every unenviable person is a nonowner of every dog.

Applying RS to (4) and (2) we get

$-B3 + L32 + (-(-E1) + (-O12))$ every boy loves something every unenviable person fails-to-own

which, by obversion, is equivalent to

$-B3 + L32 + (-(+(-E1) + O12))$ every boy loves something no unenviable person owns.

Representing binary connectives by pairs of signs, ‘both p and q ’ is transcribed algebraically as ‘ $+p + q$ ’, and ‘if p then q ’ is defined by distributing the minus sign of ‘ $- (+p + (-q))$ ’, thereby giving its algebraic transcription as ‘ $-p + q$ ’ (see Chapter 9 of [20]).

Now, in ‘ $E^*(-p)$ ’, ‘ $-p$ ’ means ‘if p ’. Modus ponens illustrates how RS applies to sentential arguments:

$$\begin{array}{ll} E^*(-p) & -p + q \\ E(p) & p \\ E(E^*) & q. \end{array}$$

Here the environment of ‘ p ’ in the second premise is null.

RS allows penetration into embedded contexts. From the premises ‘if what Sally is driving is not a Plymouth then Tom isn’t with her’ and ‘every Plymouth is manufactured by Chrysler’ we may derive the conclusion, ‘If what Sally is driving is not manufactured by Chrysler then Tom isn’t with her’.

$$\begin{array}{ll} \text{(A.8)} & E^*(-P) \quad -P2 + (M23 + C3) \\ & E(P) \quad -[+(+S1 + D12) + (-P2)] + [+T4 - (W41 + S1)] \\ & E(E^*) \quad -[+(+S1 + D12) + (-M23 + C3)] + [+T4 - (W41 + S1)]. \end{array}$$

The embedded occurrence of P in the second premise of (A.8) is algebraically positive, so RS applies.

To show that ‘some girl is loved by every boy’ is incompatible with the denial of ‘every boy loves some girl (or other)’ we consider their conjunction and derive a contradiction. ‘ $-(-B1 + (L12 + G2))$ ’ and ‘ $+B1 - (L12 + G2)$ ’ are equivalent by PEQ. Taking ‘B1’ as middle term we syllogistically add ‘ $+G2 + (L12 - B1)$ ’ to ‘ $+B1 - (L12 + G2)$ ’ to get ‘ $+(+G2 + L12) - (L12 + G2)$ ’ or ‘ $+(L12 + G2) - (L12 + G2)$ ’, ‘some lover of a girl isn’t a lover of a girl’.

[Every well-formed TFL sentence has two terms. But an English sentence such as ‘every boy loves some girl’ is not overtly bracketed. Most “hear” it as ‘every boy loves some girl or other’, but some say it can also be heard as the claim that someone every boy loves is a girl. The difference is in whether one implicitly brackets ‘ $-B1 + L12 + G2$ ’ as ‘ $-B1 + (L12 + G2)$ ’ or as ‘ $+(-B1 + L12) + G2$ ’. Bracketed the second way, its MPL translation is ‘ $(\exists y)((x)(Bx \rightarrow Lxy) \& Gy)$ ’ (see Note 5).]

6 For a final illustration of the logistic price being paid for the functorless character of predications in modern logic we compare MPL and TFL with respect to the logic of identity. In TFL an identity sentence is simply a sentence

both of whose terms are uniquely denotative. Identities are nonrelational: instead of finding an 'is' of identity in 'twice two is four', TFL finds an 'equals' of predication in 'twice two equals four', reading it as 'twice two is four'. Both sentences are of the form '+ $X + Y$ ' and like all singular sentences they entail their universalizations. For example, if 'Twain is Clemens' is true, so is 'every Twain is Clemens'. This gives us the liberty of treating 'Twain is Clemens' as either '+ $C + T$ ' or '- $C + T$ '. Since identities are indifferently particular or universal they exhibit the transitivity and reflexivity of universal sentences as well as the symmetry of particular sentences. Given 'Twain is Clemens', 'Clemens is Twain' follows by the symmetry of the I-functor. Given 'Twain is Clemens and Clemens is Sam', 'Twain is Sam' follows by the transitivity of the A-functor. Similarly, the law of indiscernibility, which some systems of MPL introduce as an axiom governing the relation of identity, is just another syllogism: given '(some) a is b and (every) a is P ', the conclusion '(some) b is P ' follows by RS.

Modern logic treats the functor deficiency in the identity sentences of MPL by adding '=' as a supplementary logical particle and supplying axioms of identity to yield the needed properties of symmetry, transitivity, and reflexivity, properties that occur in their natural state in the nonrelational identities of MPL. For example, in MPL 'Twain \neq Twain' is ruled out as a violation of the law that identity is a reflexive relation. But in TFL '(some) Twain is not Twain' is simply a contradiction of the form 'some X is not X '.⁶

7 The discussion so far has focused on syntax and more particularly on the syntax of singular sentences. Another area of comparison concerns semantics. I shall touch briefly on only one aspect of this topic: the semantics of mass terms.

Mass terms pose difficulties for the current theory of predication which do not arise for term logic. To my knowledge this was first noted by van Heijenoort in [9]. van Heijenoort contrasts Frege's subject-predicate theory of predication, which he calls a landmark of Western philosophy, with Aristotle's two-term analysis where both terms are "of the same level and where, consequently, the sentences are not of the subject-predicate form" ([9], p. 256). Emphasizing the centrality of the subject-predicate form in standard analysis, van Heijenoort says:

The subject-predicate form has, in modern logic, taken a primordial role in two closely connected ways. First, all atomic sentences are of subject-predicate form with subject places possibly occupied by variables that are subsequently captured by quantifiers. Second, the semantics of that logic requires a domain of individuals. ([9], p. 258)

Being semantically tied to a domain of individuals renders modern logic inadequate when it comes to expressing "stuff talk". It is, for example, incorrect to represent a sentence such as 'Gold is present' as ' $(\exists x)(Gx)$ '. "The reason is that when we have ' $\exists x(Gx)$ ' we are able to say there are one, two or more x 's such that Gx . We can count . . ." ([9], p. 262).

Quantifiers range over individuals, and the problem will not be solved by efforts to parse sentences with mass terms into the quantificational idiom:

We cannot squarely fit mass terms into the subject-predicate form, with its companion ontology of individuals. It seems more natural instead to recognize, beneath our language, a stuff ontology coexisting with but different from the ontology of individuals. ([9], p. 263)

Nor is Quine's idea that mass terms represent an archaic survival of a preparticular level of thought acceptable:

One can even claim that stuff talk corresponds to a higher level of thought. Stuff lends itself to magnitude . . . hence to the mathematical view of the world achieved by modern physics. ([9], p. 265)

van Heijenoort argues that "stuff talk" constitutes a major challenge for modern logic, comparable to the challenge faced and successfully met by Frege in dealing with relations:

Modern logic was able to unbury relation talk beneath property talk and thus to create quantification theory, in which a one place predicate is recognized as a special case of a many place predicate. No such unburying has taken place for stuff talk. Quite the contrary. In today's logic, quantification theory, the ontology of individuals has become exclusive: first in the subject-predicate form of its prime sentences, and second in its semantic, based on an ontology of individuals, to which are attached properties and relations. Thus quantification brings out in its pure form the main ontology of the natural language, to the exclusion of every other. ([9], pp. 265-266)

The article ends with the reminder that the "systematized language of Aristotelian syllogistic . . . escapes that ontology".

There is, for van Heijenoort, no question that MPL is superior to the older term logic. Indeed he says that such a semantic advantage as may accrue to the older analysis of predication is due entirely to its being restricted to monadic terms, a limitation that allows it to do its logic without quantifiers binding individual variables, but which renders it incapable of handling relational arguments. "It is because Aristotle could dispense with quantifiers that he could also dispense with the subject-predicate form of sentences" ([9], p. 257). Thus the semantic advantage is gained at a prohibitive cost in inference power. On this point van Heijenoort simply repeats the received view. We have seen, however, that a quantifier-free term/functor logic can be extended to relational arguments and we may now view the semantic advantage that van Heijenoort takes note of without thinking of it as hollow.

In giving the truth conditions of a sentence of the form ' $P + S$ ' one does not necessarily identify an individual referred to by a subject and then attend to what is said of that individual by the predicate. Instead one attends to the question whether the world is characterized by the existence of anything codenoted by the terms. The semantics of TFL is a semantics of presence (existence) and absence (nonexistence): ' $P + S$ ' is true just in case the presence of (an) S that is P characterizes the world; ' $-(P + S)$ ' is true just in case the absence of (any) S that is P characterizes the world. To characterize the domain of interpretation is to "obtain" or to be a fact. For example, the presence of white swans is a fact; the presence of red swans is not a fact. The domain of natural numbers is characterized by such facts as the presence of an even prime and the

absence of a greatest prime. These facts render ‘some prime is even’ and ‘no prime is greater than every other prime’ true.

The world is characterized by the presence of many things that are non-individuals. Thus the presence-of-snow-that-is-melting-in-Boston is the fact that makes ‘snow is melting in Boston’ true, and similarly for many other true sentences about stuff. What is important for our topic is that a statement of the form ‘ $P + S$ ’ is not necessarily a statement whose subject refers to an individual or whose quantifier ranges over individuals. The plus functor is essentially a copula connecting the terms of a sentence claiming presence. But how we read the copula as a term connective depends entirely on what sort of terms it ties together. Where ‘ S ’ is a general term denoting distinct individuals (e.g., ‘children’ in ‘children were shouting’) there, indeed, we interpret the copula as a binary quantifier. But where ‘ S ’ denotes stuff, a species of individual or a generic attribute like salinity, such an interpretation of the copula will often be inappropriate. A similar point was made earlier about singular sentences with uniquely denoting proper name subjects where we do not give the predicative functor a quantifier reading.

The formal properties of the predicative functor are unaffected by differences in the kinds of terms it joins. The semantics of TFL neither dictates nor excludes reference to any sort of thing. It is tolerant of individuals, stuff, species, abstract objects, and other kinds of things whose presence or absence may be said to characterize the world. TFL is in this sense a logic without an ontology. It is, in any case, not bound to a particular conception of the domain of interpretation as a domain of individuals of the kind that are eligible to serve as referents for the subject expressions of the atomic formulas of a standard language of MPL.⁷

8 Conclusion It is generally believed that the elementary sentences of a logically correct language have the logical syntax of predications in MPL. This assumption underlies a great deal of what has been written concerning the nature of names and predicates and the way a predicate is tied to a name. Much of it (by Frege, by Russell, by Strawson, by Quine, by Kripke, by Dummett) cannot be properly evaluated until we fix on the right answer to the question that divides MPL and TFL concerning the nature of the predicative tie in elementary singular sentences. This article has presented reasons to favor a syntax that construes predication as tying term to term in a term/functor language for logic.

Historically, term logic was superseded as inferentially weak due to its alleged inability to give a logical account of relational inference. The current poor repute of term logic is in part due to the efforts of many clearheaded but wrongheaded philosophers who repeated the claim that the traditional “two term” logic is essentially incapable of dealing with inference involving multiple generality. But this claim of superiority for MPL turns out to have been mistaken. Indeed, as has here been argued, MPL’s incompetence in the area of singular inference gives TFL a slight but significant advantage in expressive and inferential capability. Moreover, it remains true that this sort of advantage is what one looks for in judging the claim to canonical status of a logical syntax. On these grounds, then, the logic of terms deserves some of the technical, schol-

arly, and philosophical attention that has been given to modern predicate logic in this century.

NOTES

1. Functor logic has ancient roots but it has just begun to be studied with care. The seminal papers are [14]–[18]. Quine’s predicate functor logic depronominizes MPL, still taking predicates or verbs as the material expressions but introducing adverbs or predicate functors as formatives doing the work of bound variables or pronouns. The result is a variable-free first-order predicate logic. Sommers’ term/functor system [19] and [20] is algebraic, using plus and minus signs for the formative expressions. Most recently Bacon [1] has proved the completeness of a predicate functor logic, and he notes (pp. 923–926) that the proof carries over to Sommers’ system as extended by Lockwood [10].
2. Strawson speaks of the tie between subject and predicate as “nonrelational”. Considered as a predicative relation between terms, the I-functor is symmetrical and the A-functor is transitive. The MPL view that the elements in a predicative tie are semantically distinct (Frege’s “object word” and “concept word”) and syntactically noninterchangeable precludes thinking of a relational tie with such formal properties as symmetry or transitivity. The view squares with current linguistic theory, which takes NP/VP as the basic way of parsing sentences. The linguist’s noun/verb parsing is thus acceptable to practitioners of MPL at the level of atomic sentences. On the other hand, it appears to have little to recommend itself as a general method of parsing sentences. Thus Geach rebukes Chomsky for applying NP/VP in parsing general sentences ([8], pp. 116–117). But see Strawson’s ecumenical attempt in [22] to reconcile the syntax of MPL with the NP/VP style of parsing general sentences. For the terminist view that linguists as well as logicians should seriously attend to the term/functor grammar of the Port Royalists, see Sommers [21].
3. The assertion that ‘and’ and ‘some (is a)’ are formally similar is subject to rules of well-formedness. Thus ‘ p follows from ‘ $p + (q + r)$ ’ by simplification but ‘ P ’ is not a sentence and so does not follow from ‘ $P + \langle Q + R \rangle$ ’. On the other hand ‘ $Q + R$ ’ (‘some Q is R ’) is well-formed and it does follow from ‘ $P + \langle Q + R \rangle$ ’ by simplification.
 In Sommers [20], the commutative functors ‘and’ and ‘some’ are transcribed as ‘+’ and the unary functor ‘not’ as ‘–’. ‘If’ and ‘every’ are then algebraically defined in a way that introduces a binary use of ‘–’ for them via the equivalences ‘ q if $p = \text{not} (\text{not } q \text{ and } p)$ ’ and ‘ P every $S = \text{not} (\text{not } P \text{ some } S)$ ’. Algebraically: $q - p =_{\text{def}} -((-q) + p)$; $P - S =_{\text{def}} -((-P) + S)$. Arguments are reckoned algebraically. See below, Section 5.3.
4. Some of the symptoms of functor deficiency have been ingeniously treated by Donald Davidson, who showed that we could derive, say, ‘Caesar was killed’ from ‘Brutus killed Caesar with a dagger’ if we construe the premise as being about a killing of which it is being said that it was by Brutus, of Caesar, and with a dagger, and the conclusion as ‘there was a killing of Caesar’. The construals are implausible. The premise in TFL is ‘ $B1 + K123 + C2 + D3$ ’ whose DNF is ‘ $B1 + (C2 + (D3 + K123))$ ’. From this (by commutation, association, and simplification) we derive ‘ $C2 + K123$ ’ – ‘Caesar was killed’.

5. For formulas of the form ‘ $-An + Bn$ ’ the bridging rule to MPL is

$$\mathbf{R2} \quad -An + Bn = (x)(Ax \rightarrow Bx).$$

Applying R1 and then R2 to ‘ $+B1 + (-G2 + L12)$ ’, the DNF of ‘some boy loves every girl’, we get ‘ $(\exists x)(Bx \ \& \ (y)(Gy \rightarrow Lxy))$ ’. The DNF of ‘every boy envies an owner of a dog’ is ‘ $-B1 + (+(+D3 + O23) + E12)$ ’. Applying the two bridging rules to this gives ‘ $(x)(Bx \rightarrow (\exists y)((\exists z)(Dz \ \& \ Oyz) \ \& \ Exy))$ ’.

6. Identity as a relation a thing bears to itself may be mysterious but *difference* hardly seems mysterious as a relation between two things. Be that as it may, TFL has no more logical need of ‘ \neq ’ than of ‘ $=$ ’. The sign of relational inequality figures in the MPL version of statements containing exceptive words like ‘else’ or ‘other’; for example, in (i) ‘Cicero envies everyone else’ or (ii) ‘There is exactly one divine being’ (i.e., ‘some being is divine and no other is’). In TFL, inequality is monadic and is expressed by using the contrary of a proper name or pronominal term. Thus (i) transcribes as ‘ $+C1 + (E12 - (-C)2)$ ’ and (ii) transcribes as ‘ $+ [+B^i + D] + [-(+(-B^i) + D)]$ ’, with ‘ B^i ’ a “proterm”. For pronominalization in TFL, see [20], [10], and [11].

Relational identity and its laws are not the only supplement used by MPL to make up the functor deficiency. The law of existential generalization, which MPL uses to get from ‘Jim is rich and Jim is famous’ to ‘someone rich is famous’, plays no part in TFL, which derives ‘ $+R + F$ ’ syllogistically from ‘ $+J + R$ ’ and ‘ $-J + F$ ’. Nor does TFL have need of a primitive semantic law equating ‘ a is P and a is Q ’ and ‘ a is P and Q ’. To derive ‘Jim is rich and Jim is famous’ from ‘Jim is rich and famous’ we move by association and simplification from ‘ $+J + \langle +R + F \rangle$ ’ to ‘ $+J + R$ ’ and similarly (by the above and commutation) to ‘ $+J + F$ ’. To derive ‘ $+J + \langle +R + F \rangle$ ’ from ‘ $+J + R$ ’ and ‘ $+J + F$ ’, we apply iteration, association, and commutation to ‘ $+J + R$ ’ to get ‘ $+J + \langle +R + J \rangle$ ’. To this we add ‘ $-J + F$ ’ syllogistically to get ‘ $+J + \langle +R + F \rangle$ ’.

7. For more on the semantics of Term Functor Logic, see Lockwood [10], and Sommers’ “Truth and existence,” pp. 299–304 in [6].

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