Notre Dame Journal of Formal Logic
Volume 37, Number 2, Spring 1996

Editors' Introduction

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The idea of combining logics, structures, and theories has recently been attracting interest in areas as diverse as constraint logic programming, theorem proving, verification, computational linguistics, artificial intelligence—and indeed, various branches of logic itself.[1] It would be an exaggeration to claim that these (scattered, and by-and-large independent) investigations have crystallized into an enterprise meriting the title "combined methods"; nonetheless, a number of interesting themes are emerging. This introduction notes some prominent ones and relates them to the papers in this special issue.

The attraction of combined methods is probably most clearly visible in real world applications of logic. Modularity—and, more generally, the need to reason about the flow of information in a structured way—is important if complex systems are to be properly designed and maintained. As any interesting real world system is a complex composite entity, it seems natural to describe them using "combined languages," that is, languages made up of a number of sublanguages, each of which is tailored to the descriptive requirements of a particular subsystem. This approach decomposes tasks such as specifying or verifying complex systems into the interaction of simpler, more restricted specifications and verifications, and thus offers a plausible way of tackling complex modeling tasks.

This "divide and conquer" strategy is certainly natural, but a number of important decisions have to be made and a number of problems solved. For example, it may be natural to describe the information state of an assembly-line robot over a period of time using some combination of a temporal logic $\mathbf{L}^T$ and an epistemic logic $\mathbf{L}^E$. But what sort of combination? Intuitively, this very much depends on the degree of interaction between the *belief level* and the *temporal level*. If the "belief level" is simply a collection of databases, one for each time point, containing only nontemporal information, it may suffice to use very simple modes of combination. To give an extreme example, perhaps epistemic formulas need only ever be used as "complex atomic formulas" of temporal expressions (that is, perhaps no epistemic operator ever needs to have a temporal operator in its scope). This is a strikingly simple (and as it turns out, logically well behaved) mode of combination. On the other hand, if the belief and temporal components interact in complex ways, we may have to adopt more complex modeling strategies. Indeed, we may need to add new relations to mirror the effects of the various interactions, and this in turn may lead to syntactic extension: new operators or predicate symbols may need to be defined over this additional structure. How well behaved is the result? That is, given that $\mathbf{L}^T$ and $\mathbf{L}^E$ have pleasant logical properties, do these survive in the combined language? Such *transfer problems*, for various methods of combining logics, are the focus of much current interest.

It is worth emphasizing that such issues are not new. For example, arguably they have been at least implicitly present throughout most of the history of philosophical logic, and on occasions they have stepped into the limelight. A particularly nice example of this is Thomason's [11] overview of combinations of tense and modality. Both the structural combinations required to model this problem domain adequately (in particular, should one simply form the product $W \times T$ of possible worlds and times, or work with branching treelike models?) and the need for new logical methods to lift completeness results to the combined systems is acknowledged (this article was one of the earliest sources to note the utility of Gabbay-style irreflexivity rules in rich modal languages; cf. Gabbay [6]). Nonetheless, by and large, it is the newer applications of logic in computer science, artificial intelligence, and so on, that have driven the idea of logical combination. The sheer variety of these new applications, and the obvious need to obtain systematic solutions, has tended to encourage innovative approaches to logical modeling.

Similar themes involving logical combination—or its converse, logical decomposition—have arisen in various branches of logic itself. One recent example of logical decomposition is the analysis Meyer and Mares [9] give of the notion of entailment in terms of component logics for necessity and implication. Intriguingly, their analysis leads fairly directly to combinations of models reminiscent of semantic structures proposed in more applied work. Modal logic, on the other hand, is a rich source of problems concerning logical combination—and for a somewhat curious reason. The impressive technical advances made in modal logic in the 1970s and 1980s were by and large confined to uni-modal languages. It is natural to ask which of these results do (or do not) generalize to richer modal systems, such as multi-modal logic, temporal logic, and propositional dynamic logic. In short, for purely historical reasons, modal logic has generated a host of unanswered transfer questions, and a systematic exploration of them has only just begun. Papers initiating this line of inquiry include Kracht and Wolter [8], Fine and Schurz [4], and Wolter [12].

The papers in this issue can be grouped (somewhat roughly) into the following categories: Classical Questions from Logic, Real World Applications, Combining Structures, and Towards a Mathematical Framework. We hope this classification helps highlight the main contributions of the various papers.

Questions from logic Wolter, in his contribution to this issue, studies the problem of transferring properties from a modal logic $\mathbf{L}$ to its *minimal tense extension*. Suppose we are working with a modal logic $\mathbf{L}$, and for every operator in the language of $\mathbf{L}$ we add a corresponding backward-looking operator. Now consider the smallest logic $\mathbf{L}'$ (in the enriched language) that contains $\mathbf{L}$; what properties does this minimal tensed extension $\mathbf{L}'$ inherit from $\mathbf{L}$? Wolter shows that, in general, neither completeness nor the finite model property transfer. In fact, he constructs a normal extension of $\mathbf{K4}$ with the finite model property whose minimal tense extension is frameincomplete.

In a similar vein, Hemaspaandra investigates the effect on the satisfiability problem of enriching modal languages with the universal modality, and the reflexive, transitive closure modality. (The need for such extensions arises naturally in a wide variety of settings, ranging from analyzing program behavior to reasoning about the common knowledge possessed by interacting agents.) As Hemaspaandra demonstrates, the increase in the complexity of the satisfiability problem can be dramatic: in fact, one can move from PSPACE-complete to highly undecidable. Moreover, Hemaspaandra shows that, with the exception of a number of special cases, adding the universal or reflexive, transitive closure modality to a multi-modal logic with independent modalities causes EXPTIME-hardness of the satisfiability problem.

The negative transfer results obtained by Wolter and Hemaspaandra should be contrasted with the positive transfer results due to Kracht and Wolter [8] and Fine and Schurz [4]. The conclusion to draw is that transfer generally succeeds in the absence of interaction of the component logics, but can easily fail when the component logics are allowed to interact.

Real world applications In real world applications—such as temporal databases, expert systems, or specification tools—more exotic combinations of logics may be needed than those just described. A number of papers in this issue explore such applications and the combinations they give rise to.

Finger and Gabbay consider four methods of combining a logic with a temporal logic to arrive at two-dimensional temporal logics; the various methods are motivated by applications in temporal databases. The main focus of this paper is on transfer results; the properties considered include soundness, completeness, and decidability, and the authors show that there is a clear trade-off between transfer of properties and expressive power of the combined logic.

Engelfriet focuses on dynamic aspects of nonmonotonic reasoning in artificial intelligence systems. He starts by considering a temporalized modal logic in the sense of Finger and Gabbay [5], and then introduces a notion of *minimal information* by imposing a preference relation on models. This enables him to use the combined language to describe the behavior of an agent performing default reasoning. Engelfriet exhibits a decision procedure and proves a number of complexity results for the resulting system.

The logics considered by Montanari and Policriti deal with combinations of temporal domains of different granularity. Motivated by applications in real time specification, the authors show how the decision problem for their *layered logic* can be reduced to the decision problem for one of the layers, namely the finest one; and the latter is shown to be decidable using an embedding into **S1S**.

Combining structures Although the papers discussed so far combine structures in various ways, the theme of structural combination has largely been implicit. However, the theme is explicit in much recent work in computer science and natural language semantics, especially when notions such as communication and dynamics enter the picture. What sort of ontologies provide good models of these concepts—and can these ontologies be viewed in composite terms?

In computer science, such questions are perhaps to be expected. After all, issues such as modularity and concurrency can be naturally couched in terms of composite structures. Rensink's contribution belongs to this tradition. Rensink focuses on the algebraic theory of a particular class of complex structures used in concurrency theory, namely order-deterministic pomsets. These pomsets are equipped with a sound and complete equational theory. Rensink shows how this framework can be extended to deal with *refinement*, the operation of replacing the elements of a pomset by entire pomsets.

However, a similar trend can be discerned in current *dynamic* theories of natural language semantics. The basic idea of dynamic semantics is to explain the meaning of expressions in terms of their potential to change the information state of participants in a discourse. In their paper, Visser and Vermeulen make systematic use of composite structures to provide interpretations for arbitrary chunks of natural language text. By repeatedly applying the Grothendieck construction the authors build up elaborate meaning objects that form a monoid. Different levels of structure are used to model content and to model the (changing) context. The heart of the analysis is an account of how change on one level of structure induces change on an other level, and how to keep track of what information belongs together.

Towards a mathematical framework As should be clear from the papers discussed so far, while there are many ways of combining logics or systems, there seems to be a number of ideas common to most of the approaches. The paper by Jánossy, Kurucz, and Eiben attempts to isolate this common mathematical core by taking an algebraic point of view. They use the general methodology of algebraizing logics and represent combinations of logics as the co-limit of the component logics in the category of algebraizable logics. The authors show that this category is co-complete and that it is isomorphic to a category of certain first-order theories.

To conclude this introduction, let us briefly consider where the idea of logical combination could—and perhaps should—be leading. For a start, we expect interest in logical combination to increase; quite apart from anything else, the demands of real world problems make the basic idea too attractive to ignore, and this alone is likely to give rise to considerable activity. Moreover, the idea of logical combination (and decomposition) can be a natural way of formulating purely logical questions, and this is another possible source of interest. But—in our view—perhaps the most pressing issue in the area is a quite fundamental one: finding a suitable mathematical framework for conducting combined investigations.

We have already mentioned the algebraic approach adopted by Jánossy, Kurucz, and Eiben in this issue. This seems an eminently sensible approach, and it is to be hoped that their investigations can be extended. But algebraic logic is not the only plausible path towards generality. For a start, it is natural to enquire whether categorical methods could be helpful. Moreover, a number of reasonably general approaches have been proposed in the literature on logical combination; how far can they be developed?

The best known of these proposals is undoubtedly Gabbay's technique of combining logics through *fibering* (see [7]). This is a general method for combining any two logics, whether they are given semantically (in terms of their model theory) or syntactically (in terms of some deductive calculus). The method has a preprocessing stage in which logics are equipped with a Kripke-style semantics; the actual combination step then proceeds by weaving these semantic entities together. The method is both suggestive and intuitive; clarifying exactly how widely it can be applied, and to what effect, would be a fundamental contribution to the development of combined methods.

There are a number of other suggestions which invite further exploration. Seligman in an unpublished manuscript, for example, aims to achieve a high level of generality through his syntactic methods of combining arbitrary logics. In another proposal, due to Blackburn and de Rijke [3], so-called *trios* consisting of a pair of (classes of) models and a collection of relations between those models are introduced. The models are assumed to come equipped with their own logic and language, and the relations model the interaction between the models. (The difference between this approach and most of the approaches represented in the papers in this special issue lies precisely in the fact that the interactions between the combined models are made explicit, with special syntactic items referring to them.) Again, further work is needed to develop this idea. Finally, there is a large body of closely related work on combining decision procedures. Early work in this area is due to Nelson and Oppen [10], and Baader and Schulz [1] give a fairly extensive overview. A comparison of the methods used in this tradition with the approaches used in this issue is called for.

Acknowledgments Most of the papers in this special issue are based on presentations at a workshop held during the 1994 European Summer School in Logic, Language and Information, which took place in Copenhagen, Denmark, in August 1994. The workshop was made possible by generous donations from the Center for Computer Science in Organization and Management (CCSOM), Amsterdam; the Centre for Mathematics and Computer Science (CWI), Amsterdam; the European Association for Logic, Language and Information (FoLLI); the Institute for Logic, Language and Computation (ILLC), Amsterdam; the Netherlands Organization for Scientific Research (NWO), Project NF 102/62-356; the Dutch PhD Network for Logic, Language and Information (TLI); and the Department of Computer Science at Utrecht University. Their support is gratefully acknowledged. We also want to thank Michael Detlefsen, Martha Kummerer and Larry Moss for their valuable help in bringing this issue about.

NOTE

1. For example, papers on combined systems drawn from most of these fields were presented at the *Frontiers of Combining Systems* (FroCoS) workshop in München, Germany, March 1996. FroCoS seems to have been the first open workshop devoted solely to logical combination; the proceedings [2], which contain further pointers to relevant literature, may be of interest to readers of this special issue.

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