Situations in Which Disjunctive Syllogism
Can Lead from True Premises
to a False Conclusion

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Abstract   Disjunctive Syllogism, that is, the inference from 'not-A or B' and 'A', to 'B' can lead from true premises to a false conclusion if each of the sentences 'A' and 'not-A' is a statement of a *partial* truth such that affirming one of them amounts to denying the other, without each being the *contradictory* of the other. Such sentences inevitably occur whenever a situation which for its proper precise description needs the use of expressions such as 'most probably true' and so forth, is described (less precisely) by sentences not containing such expressions.

1 The question raised by relevant logicians—Can disjunctive syllogism, under certain circumstances, lead from true premises to a false conclusion?   Relevant logicians believe that in classical logic sufficient attention has not been paid to the "relevance" of the premises of an argument to its conclusion, and on this ground consider some inference forms of classical logic to be not valid. In particular they hold the rule

> From a disjunction 'A or B', and the negation of a disjunct, 'not-A', one may infer the remaining disjunct 'B'

is not acceptable, if the disjunction is interpreted as truth functional; it is acceptable if the disjunction is interpreted as intentional (Anderson and Belnap [1], pp. 176–7). This has led Haack to remark that the relevant logicians "don't deny that if 'P $\vee$ Q' (where '$\vee$' is truth functional) is true, and '$\neg$P' is true, then necessarily 'Q' is true" ([7], p. 201) but that the relevant logicians would not call such an inference valid 'in their sense'. Geach obviously considers the relevant logician's claim to be that disjunctive syllogism is not valid, even in the classical logician's sense of 'valid'; and has remarked that not only had no counterexamples to the validity of disjunctive syllogism been offered, but that it was clear none could be forthcoming: "It would be difficult to describe even a possible set up in which . . . 'A $\vee$ B' should be true, but both

its disjuncts false" ([6], p. 495). About disjunctive syllogism with a truth functional disjunction, Read states "In general this inference fails. It is an interesting question much studied in relevant logic, under what conditions it holds" ([14], p. 30). Clearly relevant logicians have raised the question: Can disjunctive syllogism, under certain circumstances, lead from true premises to a false conclusion?

To meet Geach's challenge, some philosophers have wondered if 'This sentence is false' could be cited as an example of a sentence which is both 'true' and 'false'. This would certainly not do; for the whole problem with 'This sentence is false' is that whether we assume it to be true, or assume it to be false, we seem forced to the conclusion that the assumption is wrong (Bhave [2], p. 465). There have been several other suggestions. Under the American plan the possibility that some propositions may be 'true', 'both', 'neither', or 'false' is considered. Under the Australian plan, to each proposition a there is a corresponding dual proposition a^*, the truth values of a and a^* being related in a specified way with the truth values of a under the American plan. Thus the American plan "gives up bivalence, whereas the Australian plan keeps it at the cost of introducing a twist into the homomorphic truth conditions on $\sim$" (Meyer and Martin, [9], p. 308).

All the different suggestions mentioned above have one common insight, namely, that in a situation for the proper description of which truth values (or expressions) other than 'true' and 'false' become necessary, counterexamples to the validity of disjunctive syllogism might arise. In the succeeding sections of this paper we shall show that this is indeed the case.

2 Conditions under which disjunctive syllogism could fail   If it is known that

'A or B'

(where 'A' and 'B' are sentences which can only be either true or false) is true, and that the sentence

'not-A'

(that is, the sentence 'It is not the case that A') is also true, then disjunctive syllogism in the form

'A or B', 'not-A'; therefore B

gives the conclusion that 'B' is a true sentence. So the only situation in which disjunctive syllogism could fail to yield a true conclusion from true premises is one in which 'A' and 'not-A' are both true, even though 'not-A' is the *negation* of 'A', so that the inference

'not-A' is true; therefore 'A is false'

fails. So the only situation in which disjunctive syllogism can fail is when though 'not-A' is the *negation* of 'A', it is not the *contradictory* of 'A'.

Logicians dealing with sentences which can only be either true or false, (and with situations which can be adequately and precisely described by such sentences) have considered the *negation* of a sentence the same as its *contradictory*. Thus Quine [12], p. 9, has stated

> To deny a statement is to affirm another statement known as the *negation* or *contradictory* of the first . . .
>
> The commonest method of forming the negation of statements in ordinary language is by attaching 'not' (or 'does not' etc.) to the main verb . . . . If the statement is compound, and thus has no main verb, its negation has to be phrased more elaborately; e.g., 'It is not the case that . . . and that . . . '.

But philosophers have pointed out that "A proposition and the same proposition preceded by 'It is not the case' need not express contradictory propositions" (Wolfrom [16], p. 177). In subsequent sections we shall show that when the situation is such that for its proper description sentences with truth values (or expressions) other than 'true' and 'false' are necessary, sentences can occur such that, although they and their negations can only be either true or false, their negations are *not* their contradictories. If such a sentence and its negation both express different partial truths regards the actual situation, both have the truth value 'true'; and disjunctive syllogism can fail if such a sentence and its negation are used to infer the truth of a third sentence.

3 Sentences which are, in certain contexts or situations, neither 'true' nor 'false', but only 'most probably true', and so forth   Consider the sentence

'There will be a sea battle tomorrow'.

Many philosophers from the time of Aristotle until today have felt that this sentence is neither true nor false. Some philosophers consider such a sentence to have some third truth value, other than 'true' and 'false'; some consider it to have *no* truth value, that is, they consider it to be occupying a *truth value gap*.

Whatever be the correct position regarding the truth value of the sentence 'There will be a sea battle tomorrow', there is not much doubt that the sentence can be described as being 'most probably true', or 'most probably false', and so forth, depending on things like the state of the strained relations between the two countries, the distance between the two navies, the orders issued by their respective naval headquarters, and so on. Furthermore sentences like

' 'There will be a sea battle tomorrow' is most probably true'

can be judged to be either true or false, on the basis of the situation today; and the war offices of the concerned countries are often anxiously considering truth values of such sentences; on a proper ascription of truth value (either 'true' or 'false') to such sentences much war strategy often depends.

Even a sentence about a past event (for instance, 'There was a sea battle near Crete twenty-five hundred years ago') can be properly described as 'most probably true', or 'most probably false', and so forth, in the context of historical research. In this paper we shall call such expressions truth values of sentences, just like 'true' and 'false', following the practice of those who have developed many valued logics. But the argument of this paper remains unaffected whether they are called truth values or expressions.

We note that in the natural languages truth values like 'most probably true' are in use; and in the natural sciences (for instance, in quantum theory) expressions such as 'with probability q' (where q is a real number in the range 0 to 1) are in use and

are in fact unavoidable. Here $q = 0$ corresponds to 'is false'; $q = 1$ corresponds to 'is true' and the degree of probability x is as near 'true' as $(1 - x)$ is near 'false'.

We note that in the natural languages a particular degree of probability is expressed in different equivalent ways. Thus the following sentences have the same meaning.

'There will most probably be a sea battle tomorrow'

' 'There will be a sea battle tomorrow' is most probably true'

Also, to say 'p is x-probably-true' where x is a particular degree of probability is the same as saying 'p is $(1 - x)$-probably-false', where $(1 - x)$ is a degree of probability as near 'false' as x is near 'true'.

4 Sentences about a sentence p, when p can have a truth value other than 'true' and 'false'   Consider the sentence p, when p can have, depending on the situation, any one of the truth values 'true','most probably true', . . . ,'less probably true', . . . 'false'. If the actual situation is such that the sea battle the next day is 'most probable', then p ('There will be a sea battle tomorrow') is a false sentence, and has the truth value 'false'. Its negation 'not-p' ('There will not be a sea battle tomorrow') is also a false sentence. Though p and 'not-p' are negations of each other, they are not contradictories of each other. The sentence

(a)   'p is most probably true'

and its negation

(not-(a))   'p is not most probably true'

are, however, sentences which are the contradictories of each other. Similarly the sentence

(b)   'p is most probably false'

and its negation

(not-(b))   'p is not most probably false'

are sentences which are contradictories of each other. This is because 'not most probably true' means the disjunction of all possible truth values of p excepting 'most probably true'; and the words 'not most probably false' means the disjunction of all possible truth values of p with the exception of 'most probably false'. Consider the sentence

(c)   ' 'p is true' is false'

and its negation

(not-(c))   ' 'p is true' is not false'

The sentence 'p is true' ascribes a particular truth value to the sentence p ; so 'p is true' can be true if it ascribes the correct truth value to p; it can be false if it ascribes a wrong truth value to p. So the sentence 'p is true' can only be either true or false. The words 'not-false' in not-(c) can, in view of this situation, only mean 'true'. So if (c) and not-(c) are considered as sentences about the sentence 'p is true', then they

are not merely the negations of each other, but also the contradictories of each other. But (c) and not-(c) say something about the sentence p also. It is shown in the next section that, considered as sentences about p, (c) and not-(c) are negations of each other, but are not the contradictories of each other.

5 *The sentences (c) and not-(c) considered as sentences about the sentence* p   Unlike the sentence 'p is true', the sentence p can have (depending on the actual situation) any one of the truth values 'true', or 'most probably true', or . . . , or 'false', that is, any one of the entire range of truth values from 'true' to 'false'. Any actual stuation can be correctly described by putting in the place marked by $*$ in either

(1)   'p is $*$'

or

(2)   ' 'p is true' is $*$'

a suitable truth value from among all the possible truth values of p. Keeping this in mind, we see that

(c)   ' 'p is true' is false'

says about the object language sentence p that to call p true is false; that is, p has a truth value other than 'true'. (c) means that p is either most probably true or . . . or less probably true, or . . . or false. In other words (c) means that (1) with some truth value other than 'true' in place of '$*$' is a true sentence.

What not-(c) says about p is that to call p 'true' is not false; that is, to call p 'true' is either true, or most probably true or . . . (a disjunction of all truth values other than 'false'). In other words not-(c) means that (2) with some truth value other than 'false' in place of '$*$' is a true sentence.

Thus, considered as sentences about the sentence p , (c) and not-(c) are negations of each other but if the actual situation is that the naval battle the next day is either most probable, or probable to some degree (other than 0 and 1), the sentences (c) and not-(c) are both true sentences and are not contradictories of each other.

6 *Disjunctive syllogism in the form (C), 'not-(C) or B'; therefore B*   Suppose B is a sentence which can only be either true or false. Then clearly, if B is false, disjunctive syllogism in the form

(C), 'not-(C) or B'; therefore B

would lead from true premises to a false conclusion.

However, suppose 'or' in '(C) or B' is interpreted as intensional, so that '(C) or B' means 'if not-(C), then B'. Then clearly, disjunctive syllogism in the form

'not-(C)', '(C) or B'; therefore B

is valid in the classical logician's sense, and cannot lead from true premises to a false conclusion. For instance, suppose that in a situation in which p can have, in addition to 'true' and 'false', other truth values such as 'most probably true', and so forth, the naval headquarters of one of the countries concerned has decided that if there is any probability of a naval battle the next day, (that is, any probability other than 0), fighter aircraft in support of the naval units shall be sent to the area, and if there is *no*

probability of such a naval battle, fighter aircraft shall not be sent to the area. Let B be the sentence

'Fighter aircraft in support of the naval units shall be sent to the area'.

Disjunctive syllogism in the form

'not-(C)', '(C) or B'; therefore B

is now valid in the classical logician's sense.

7 Conclusion   We have shown a case of disjunctive syllogism being invalid in the classical logician's sense of 'invalid', unless 'or' in the disjunction is interpretable as intensional. To this extent the intuitions of the relevant logicians appear to be correct. However, the claim that disjunctive syllogism is valid in the classical logician's sense of 'valid' only so long as the 'or' in the disjunction can be interpreted as intentional certainly goes too far. To such a claim philosophers have responded with instances of valid disjunctive syllogism that appear to employ only the truth functional 'or'. Sanford [15], pp. 131–32, has given one such example. It would appear logicians must carefully consider what restrictions need to be placed on the sentences used in disjunctive syllogism to ensure its validity in the classical logician's sense of valid.

It would be useful to connect the findings in this paper with the points raised in earlier papers on disjunctive syllogism published in this journal in the past. In Burgess [3] and [4] the question is raised whether in view of examples of valid (in the sense of truth preserving) extensional disjunctive syllogism (EDS), the relevant logician's insistence on 'relevance' should be considered a fallacy. In the subsequent debate in which Read [13], Mortensen [10] and [11], and Lavers [8] participated, the main questions and positions that emerged were the following.

1. Does intensional disjunctive syllogism (IDS) exist, separately from extensional disjunctive syllogism (EDS)?
2. Is EDS (always) valid (Burgess)?
3. Is EDS (never) valid, but when it appears valid this is because of extra conditions ensuring truth preservation (Mortensen)?
4. Is EDS (never) valid, but when it appears valid this is because of the fact that the premise for IDS is also true (Read, Anderson, and Belnap)?

As for (1), we note that the EDS

'not-(C)', '(C) or B'; therefore B

is essentially different from the IDS

'not-(C)', '(C) or$_1$ B'; therefore B

(where 'or$_1$' is a connective different from 'or' and such that '(C) or$_1$ B' is true only if the subjunctive conditional corresponding to '(C) or B' is true). So we can only expect that IDS should sometimes exist separately from EDS. An instance of this happening is the one shown in Section 6 of this paper where the IDS

'not-(C)', '(C) or$_1$ B'; therefore B

is valid (i.e., truth preserving), and exists separately from the corresponding EDS

'not-(C)', '(C) or B'; therefore B

which is not valid (that is, not truth preserving).

As for (2), in view of the instance shown in Section 6 of this paper, it is clear that EDS is not always valid (in the sense of truth preserving).

As for (3), EDS is not always valid; it is valid only if certain conditions are satisfied. These are that each one of the sentences A, not-A, and B is such as can only be either true or false, and that A and not-A should have opposite truth values.

As for (4), it is not true that EDS is never valid except when the premise for the IDS is also true. Sanford [15] has given an instance of a valid EDS when the disjunctive premise, if interpreted intensionally, is false. An instance of a valid EDS in a situation in which the corresponding IDS does not exist is the EDS

'not-A', 'A or B'; therefore B

when not-A, A, and B are all statements about natural numbers, and B is a true statement. The intensional disjunction

'A or$_1$ B'

would be true if the corresponding subjunctive conditional

'Were B to be false, A would be true'

is true. But when B is a true statement about natural numbers, we cannot imagine what would be the situation if B were to be false, and whether in such a situation A would be true. It is difficult to understand the meaning of the subjunctive conditional, and whether it should be considered to be true or false. In such cases, clearly there is no IDS corresponding to the valid EDS. Friedman and Meyer's [5] result that some theorems of classical Peano Arithmetic (that is, Peano postulates added to classical logic which includes disjunctive syllogism as a valid inference form) cannot be proved in relevant Peano arithmetic (that is, Peano postulates added to relevant logic which does not include disjunctive syllogism as a valid inference form) is a consequence of the fact EDS can be valid, when the corresponding IDS does not exist.

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