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Book Review

Yael Cohen. *Semantic Truth Theories*. Translated by Mark Steiner and Naomi Goldblum. The Magnus Press, Jerusalem, 1994.

Yael Cohen addresses several topics: presupposition, the Raven Paradox, negation, and truth. There is a unifying theme: Cohen's goal is to develop an interpreted formal language in which it can be claimed that certain sentences are truth valueless; she also hoped to deepen our understanding of connections between truth, presupposition, and relevance. On the formal side, Cohen begins with Kripke's idea that the truth predicate need not be completely defined—the model allows that some sentences can be neither true nor false. She then seeks to improve on Kripke's model by adding an "exclusive" negation which is used to provide (some of the expressibility) that "neither true nor false" provides. As she puts it:

> An interpreted language in Kripke's sense which contains its own truth predicate is not the same as a natural language including its truth predicate, if only because: (a) the fixed point is defined in a set theoretic metalanguage rather than the language itself; and (b) there are assertions about the object-language that cannot be interpreted in the interpreted object-language. Thus, for example, a sentence such as (1) :
>
> (1) is false.
>
> is not true in the object-language, in the sense that there is no fixed point with respect to which it is true, yet the interpretation given to the truth predicate and to the negation operator prevents us from saying this in the interpreted language.
>
> As Kripke says, "The ghost of the Tarski hierarchy is still with us." (p. 44)

Hence Cohen's goal is to provide *within* the object language a way of saying that sentences are neither true nor false. Cases she has in mind are not just paradoxical and related sentences that are self-referential and contain 'true' and/or 'false'; but also sentences that have been said to be neither true nor false because they have—in a given context—a false presupposition (e.g., 'All of John's children are asleep' when said in a context where the person referred to by 'John' has no children). To this end Cohen includes in her formal system two negations.

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> I have found it necessary to distinguish two negation operators partly because our intuition distinguishes between sentences like "The king of France is not bald" and "It is not the case that the king of France is bald"...but mainly because of formal considerations of functional completeness. (p. 108)

Following Kripke [3], Cohen uses three values (T, F, I) to define the propositional connectives, but whereas Kripke uses Strong Kleene truth tables, Cohen uses the Weak Kleene tables. The truth table for choice negation '$\sim$', the usual negation, is: $\langle \mathrm{T}, \mathrm{F}\rangle, \langle \mathrm{F}, \mathrm{T}\rangle, \langle \mathrm{I}, \mathrm{I}\rangle$; whereas the truth table for exclusive negation '$\neg$', is: $\langle \mathrm{T}, \mathrm{F}\rangle, \langle \mathrm{F}, \mathrm{T}\rangle, \langle \mathrm{I}, \mathrm{T}\rangle$, (p. 108). The introduction of exclusive negation rules out any kind of wholesale adoption of Kripke's minimal fixed point (the point at which grounded sentences are determined to be either true or false). For Kripke builds up the extensions of 'true' and 'false' in stages, with the subformulas of a formula being included in the extension of either 'true' or 'false' before a determination is made regarding the formula itself. This means that sentences that at one stage are neither true nor false may at a later stage be placed in one or other of the extension of 'true' or 'false'. Cohen describes the problem on page 192: "If at a certain stage a given sentence is neither valuated as true nor as false, we cannot, of course, require that the exclusive negation of the sentence be true at the next stage." For the sentence itself—without the exclusive negation—may turn out true at the next stage.

So Cohen develops a more complicated model that employs a slew of new concepts and nonstandard truth conditions for the quantifiers. Some of the key concepts include: *cluster*, a consistent set of ordered pairs $\langle A, \delta\rangle$, where A is a formula and either T or F, and satisfies certain (partially defining) truth-condition-like conditions; *conflict sequence of A from cluster* α, a "computation sequence" from α containing $\langle A, \mathrm{T}\rangle$ and $\langle A, F\rangle$; *conflict-generator*; *maximal proper cluster*; and finally *supercluster*. As for the quantifiers, $\forall x \varphi x$ is true if φx is true for some assignment of an element of the domain to x while there is no such assignment which makes φx false; $\forall x \varphi x$ is false if φx is false for some assignment of an element of the domain to x; otherwise it is undefined (i.e., has the value I).

In developing the concept of a supercluster, Cohen sought a model in which (as many as possible of) the following adequacy conditions would be satisfied.

1. No sentence is both true and false (in a given world). (p. 157)
2. Any sentence not containing 'T' or 'F' is bivalent. (p. 157)
3. Every universal (existential) sentence all of whose substitution instances are true (false) is true (false). (p. 158)
4. Universal (existential) sentences in which 'Tx' or 'Fx' appear may also be true (false). (p. 158)
5. The truth-value of a compound sentence is determined "after" or "on the basis of" the truth-values of its components. (p. 159)
6. As many sentences as possible should be true or false. (p. 161)
7. For a given domain D and valuation s, in every cluster, and for every paradoxical sentence A (relative to the cluster) with name k, Tk, preceded by sufficiently many exclusive negations '$\neg$', is true; and so for Fk. (p. 194)

Cohen has some success in meeting her goals, for superclusters satisfy all except adequacy conditions (3) and (7); and (3) and (7) are partially satisfied. So in this respect, she has made progress in pursuing the improvements on Kripke's model.

Furthermore, by page 197, Cohen argues that some violations of condition (3) are "natural." She provides a motivating example, the generalization (G), "For every Cretan and for every sentence that is true or false, if the Cretan utters the sentence, then the sentence is false." This can be neither true nor false. But now consider the case where there are just two Cretans, Alpha and Beta, and their only utterances are (G) and "$2+2=5$"; then the two substitution instances "Every true or false sentence uttered by Alpha is false" and "Every true or false sentence uttered by Beta is false" are both true (on the supposition (G) is neither true nor false). We have a case here of the substitution instances being true and the generalization neither true nor false.

The reference in adequacy condition (7) to "sufficiently many exclusive negations" needs explanation. Though for any sentence 'p' that is not bivalent one might want to have '$\neg p$' come out true, things cannot be so simple. Consider Cohen's self-referential

$$(3)\ \neg\mathrm{T}(3)$$

on page 192. (3) is neither true nor false—the fate for any formula in Cohen's system that would be both true and false if either true or false. But clearly, if (3) is not valuated as true, this just means that '$\neg\mathrm{T}(3)$' is not valuated as true. What Cohen does have valuated as true is '$\neg\neg\mathrm{T}(3)$'. But because this cannot be equivalent to '$\mathrm{T}(3)$', double exclusive negations are not eliminable. Cohen's formula for representing the claim that (3) has neither truth value employs two exclusive negations: it is '$\neg\neg\mathrm{T}(3) \wedge \neg\mathrm{F}(3)$'. Indeed, Cohen argues that 'it is not the case that p' is ambiguous; on some occasions it is appropriately represented by just one exclusive negation, and on other occasions by longer finite strings of exclusive negations.

Cohen provides a counterexample to the satisfaction in superclusters of adequacy condition (7).

(i) $\forall x(Px \rightarrow \mathrm{T}x)$;
(ii) $\forall x(Qx \rightarrow \mathrm{T}x)$;
(iii) $\neg(\mathrm{T}(i) \leftrightarrow \mathrm{T}(ii))$. (p. 200–1)

There is no supercluster in which either (i) or (ii) is bivalent, and yet for no finite string of exclusive negations is either '$\neg\ldots\neg\mathrm{T}(i)$' or '$\neg\ldots\neg\mathrm{T}(ii)$' true. It is '$\neg(\mathrm{T}(i) \wedge \mathrm{T}(ii))$' that takes the value T. For the purpose of developing a theory of presupposition, Cohen defines a Δ-operator. It is first introduced with the truth table: $\langle\mathrm{T},\mathrm{T}\rangle$, $\langle\mathrm{F},\mathrm{I}\rangle$, $\langle\mathrm{I},\mathrm{I}\rangle$. 'All John's children are asleep' is represented by '$\forall x(\Delta(x$ is a child of John's$) \rightarrow x$ is asleep$)$' which has a truth value T, or F, if and only if John has children. Once her formalism is developed, both the Δ-operator and the presupposition relation are defined. The definition of the operator involves the truth predicate and self-reference.

I found Cohen's discussion of the Raven Paradox interesting. Using the Δ-operator she is able to represent 'All ravens are black' by '$\forall x(\Delta Rx \rightarrow Bx)$' and 'All non-black things are non-ravens' by '$\forall x(\Delta{\sim}Bx \rightarrow {\sim}Rx)$'. While these have the same truth conditions when the "presuppositions" are true (i.e., when there are ravens and there are non-black things), she argues that these call for different testing procedures. Though her arguments (one argument involving probabilities and another that appeals to scientific properties) are interesting, I am not convinced that the formalism —employment of the Δ-operator—is on target. For while in conversation there certainly may be conventions regarding presuppositions, I am not convinced that the case of scientific generalizations is similar.

Though fairly technical, much of the material covered in this book will be accessible to readers with just a smattering of logic. Cohen gives a careful presentation and she is meticulous in presenting the intuitions that motivated the development of each piece of her formalization. She also has a dogged honesty that readers will find both helpful and a refreshing novelty. A primary shortcoming of the book is that there are many loose ends. The early chapters on truth treat their topic too sparsely to be of much value, and the diverse topics on which Cohen makes substantial contributions (truth definitions and presupposition) are only loosely tied together. Though Cohen has clearly put much careful thought into this research, the book comes across as still somewhat preliminary; it is a work that awaits a more coherent presentation, both philosophically and formally. (From various comments in the book, I get the impression that Cohen herself had such a concern.) This does not mean that the book should be disregarded. (Isn't most of what philosophers do preliminary?) The book is full of lots of challenging examples—and interesting suggestions; for those working on the logic of "neither true nor false," or presupposition, there is much to explore further. (I am not as skeptical as Cohen is (p. 190), for example, that the formal connections she has demonstrated to exist between presupposition, self-reference, and the truth predicate, may not cast light on our understanding of assertion and presupposition.)

Readers should note that Cohen's untimely death in 1992 has meant that some of the translation and editing of the book was not only carried out by others (Steiner and Goldblum) but much of that work was done posthumously. Note also that since the early 1980's there has been an upsurge in publications on truth—Gupta and Belnap [1], Horwich [2], and McGee [4], to mention just a few. Yet Cohen's bibliography contains no publication dated later than 1979. Readers need to think of this work as written around 1980. The absence of an index proved an inconvenience because there are many definitions and symbols of which one must keep track; there are also many typographical errors.

REFERENCES

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[4] McGee, V., *Truth, Vagueness, and Paradox: An Essay on the Logic of Truth*, Hackett, Indianapolis, 1991. Zbl 0734.03001 MR 92k:03004 1

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