Notre Dame Journal of Formal Logic
Volume 39, Number 4, Fall 1998

AE (Aristotle-Euler) Diagrams: an Alternative Complete Method for the Categorical Syllogism

MARIO SAVIO
1942–1996

Mario Savio is widely known as the first spokesman for the Free Speech Movement. Having spent the summer of 1964 as a civil rights worker in segregationist Mississippi, Savio returned to the University of California at a time when students throughout the country were beginning to mobilize in support of racial justice and against the deepening American involvement in Vietnam.

His moral clairty, his eloquence, and his democratic style of leadership inspired thousands of fellow Berkeley students to protest university regulations that had severely limited political speech and activity on campus. The nonviolent campaign culminated in the largest mass arrest in American history, drew widespread faculty support, and resulted in a revision of university rules to permit political speech and organizing. This significant advance for student freedom rapidly spread to countless other colleges and universities across the country. Mario Savio went on to become a college teacher of physics, logic, and philosophy, to speak and organize in favor of immigrant rights and affirmative action and against U.S. intervention in Central America. He died on November 6, 1996, in the middle of a struggle against California State University fee hikes that hurt working-class students.

Savio had submitted this article to the *Notre Dame Journal of Formal Logic* before he died. Final revisions were made by Philip Clayton with the assistance of Mario's colleagues at Sonoma State University. As reader for the *Journal*, George Englebretsen not only provided an extensive commentary on the article—much of which has been incorporated here—but also assisted in the difficult task of making revisions without changing the substance of Mario's style or thought.

It is fitting that this, Savio's final publication, would be pedagogical in orientation. For him, moral considerations were no less pertinent in logic than in philosophy's less abstract fields. The usual student confusion with Venn diagrams led him to develop the new pictorial device presented in the following pages, which he believed was more sensitive to user psychology. It is hard to miss the political overtones in Savio's closing worry that in Venn diagrams "information of real significance may occasionally appear hidden and distorted."

Received August 5, 1996; revised April 8, 1998

The decision by the *Notre Dame Journal of Formal Logic* to publish this piece posthumously is a testimony that logic, no less than other fields of philosophy, can be a tool of free speech and political change—as powerful in its way as the rhetorical brilliance of that young man standing on top of a police car who launched a worldwide movement with the words, “There is a time when the operation of the machine becomes so odious, makes you so sick at heart, you can’t take part.”

1 Introduction The categorical syllogism seems to spark a perennial fascination. On the theoretical level this is perhaps confirmed by recent work on intermediate quantity or by the publication of Englebretsen’s *The New Syllogistic* [7].[1] The present paper, however, despite its also adopting a general viewpoint in the final section, is mainly offered as a contribution to the lively field of practical syllogistic. Indeed, virtually all current textbooks of introductory logic, or of informal logic or critical thinking, include some exposition of the syllogism. Often, however, either the treatment of valid moods is limited to a few very obvious ones, or else Venn diagrams are adopted as the only pictorial test of validity. In the first case, we must question whether teachers would choose to belabor those few obvious forms if a *simple but complete* diagrammatic method were available; in the second, we wonder why introductory courses especially in *informal* logic must emphasize a device which, for all its admitted elegance, has never yet found any application in real forensic practice.

In this essay we develop AE (Aristotle-Euler) diagrams, a quick and complete alternative to Venn diagrams both for testing syllogisms and for drawing conclusions given the premises only. Sensitive both to user psychology[2] and to the actual structure of syllogistic reasoning, this new approach renders two-thirds of the valid moods (including all moods of the first and second figures) immediately obvious and is only a little less powerful in all other cases. The essence of this technique is to picture not the three syllogistic terms on equal footing, as Venn diagrams do, but rather to represent the two premise judgments and then to foreground the unique role of the middle term. In this way, all valid categorical syllogisms are seen to embody just a single basic mode of reasoning encountered in only two significantly distinct varieties, an important simplifying observation originating with Aristotle. Categorical syllogisms are very simple things; their analysis should therefore render them transparent. This is precisely what the method of AE diagrams is designed to do.

2 Motivation It may first help to review precisely what it is about Venn diagrams that has suggested to many teachers of elementary logic that an alternative might be desirable. Consider first the universal negative. When one thinks *No A are B*, one has in mind some notion of *separation*. To put it crudely, one imagines the *A*s (whatever they might be) in one corner of the room and the *B*s (whatever they might be) in another. Or perhaps one sees two spectral, nonintersecting corrals, one fencing in *A*s, the other *B*s. Such is the intuition of the universal negative before the mind’s eye. Perhaps the easiest externalization (usually credited to Euler), and the one which almost always comes to be used in elementary discussions, is that of two nonoverlapping circles. Nonetheless, the generality of the Venn diagram approach requires that we begin instead with *overlapping* circles. We then mark out the cell where no individuals are to be found. We begin, that is, by ignoring the possible locations for *A*s and *B*s, concentrating our attention entirely on a location that is forbidden to both.

The argument thus begins with an operation, however meaningful, which simply sets aside an almost universal intuitive response.

Turning now to the universal affirmative, our intuition in this case suggests inclusion. Again, however, we start with overlapping circles, and before doing anything else we direct our attention to a cell which is *excluded*. Focusing on the obverse proposition, we mark as empty the intersection of A and non-B. As for the particular propositions: *Some A are B* also suggests inclusion and *Some A are not B* suggests separation. An intuitively appealing approach should therefore start, if possible, with "existential" proposition representations strongly similar to the corresponding universal ones. Alas, the Venn diagrams for A and I categoricals are constructed by marking *different* cells. The same is true for E and O. Most logic instructors would probably agree that the several abrogations of intuition required to mark Venn diagrams accurately contribute to student confusion and ultimately to the disuse of this method beyond the course in logic. It is no help, of course, that mathematicians and logicians often mark the circles differently! The method of AE diagrams is designed to remedy all the foregoing defects. It does so, first of all, by adopting the familiar and suggestive Euler universal judgment diagrams [10]. There has been a recent upsurge of interest in Euler diagrams. Articles by Armstrong and Howe [1] and by Bennett and Nolt [3] describing Euler tests for given syllogisms are especially noteworthy. Cognitive psychologists have also used Euler-type diagrams to model natural syllogistic reasoning.[3] The author believes, however, that the present paper describes the first comprehensive system based upon carrying out the "uncompleted project" of adapting the distinctive Euler universal diagrams to represent the particular judgments also.

If we next consider the combination of premises in an asserted syllogism, we find that the Venn conclusion appears as a sort of artifact. The student must inventory marks on the subject-predicate side of the Venn tableau to see whether one of several possible inferences just happens to be indicated. Although the order of premises never has any bearing on validity, if one premise happens to be particular, then the order in which the premises are *entered* is critically important. Otherwise, an X may fall *avoidably* on an arc! Finally, in the event that no conclusion appears, a further check may still be needed to tell whether an additional mark will enable a "conditional" conclusion. Most instructors have experienced how slow and confusing these inventories and precautions can be for beginners. By contrast, AE conclusions always arise from a unique role that the middle term plays in connecting subject and predicate. This simple relationship, known at the outset, is readily recognized when it occurs; and it can easily be sought by a single, definite operation in that minority of valid moods in which the conclusion is latent. The judgment that some conclusion may be inferred is thus cleanly isolated from the further issue as to what the conclusion might be. As a result, AE diagrams yield their inferences both more rapidly and with greater insight than Venn diagrams generally seem to do. This unique role of the middle term derives directly, as we shall see, from Aristotle's assertion that all valid moods are reducible to two varieties: Barbara/Darii (dictum de omni) and Celarent/Ferio (dictum de nullo).[4] "Aristotle-Euler" is thus not merely honorific, but an accurately descriptive designation.

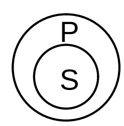
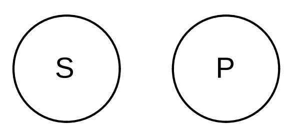
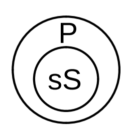
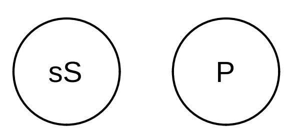
We note, finally, that Venn diagrams make existence versus nonexistence the fundamental distinction of categorical logic. On this basis the universal judgments are sharply delimited from the particular. By selecting inclusion versus separation as primary, AE diagrams maintain an almost complete existential neutrality. Thus the realization that nonexistential particulars are perfectly comprehensible and frequently met with offers yet another argument that the usual Venn techniques are somewhat at odds with intuition. For consider the following two judgments.

Some gryphons are gentle. Some unicorns are not shy.

The sort of "existence" needed to justify such propositions implies no more than that the subject concepts are sufficiently vivid and rich to permit distinguishing among gryphons or among unicorns and that there is no *evident* contradiction in doing so. Of course, use of the AE *method*, as described below, does not require the adoption of any particular opinion on the question of existential import.

3 The four basic AE diagrams A categorical syllogism is a two-premise argument each of whose three categorical judgments can be expressed in one of the four standard, subject-predicate forms (traditionally designated A, E, I, O) which appear below. Capital letters will be taken as abbreviations for categorical *terms* (nouns or adjectives, in a broad sense); and, in accordance with one common usage, we will assume that a categorical syllogism, *by definition*, has exactly three terms.

Table 1: The Four Basic AE Diagrams

	Standard-Form Categorical	AE Diagram
A	All S are P (Every S is a P)	
E	No S are P (No S is a P)	
I	Some S are P (Some S is a P, i.e., at least one S is a P)	
O	Some S are not P (Some S is not a P, i.e., at least one S is not a P)	

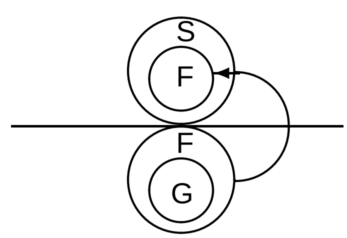
It is well worth reiterating that the author makes no specific "existential" assumptions in merely drawing these basic diagrams. The universal "corrals" might have or be deemed to have occupants; the particular "corrals" might well be empty except conceptually. However, the reader is completely at liberty to regard the particulars as always referring to "real existents." Note also that the order in which negative diagram circles are drawn relative to one another (left/right, up/down, etc.) has no real impact on usefulness. The only essential is that the two circles be nonoverlapping. Indeed, in some cases it may actually be necessary to read such representations contrary to the expected order. But one of the desirable features of AE diagrams is their evident naturalness. Whenever possible, therefore, students should be encouraged to follow the "normal" order for negatives as given by the formulation of the categoricals in language.

For later use we present here two examples of (valid) categorical syllogisms, along with their corresponding forms.

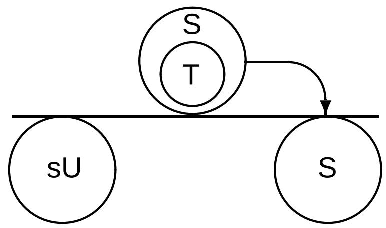
All fierce creatures are scary.	All F are S
All gryphons are fierce.	All G are F
All gryphons are scary.	All G are S
All timid creatures are shy.	All T are S
Some unicorns are not shy.	Some U are not S
Some unicorns are not timid.	Some U are not T.

4 The two varieties of syllogistic reasoning Many traditional logicians recognized only two notably distinct syllogistic varieties (or principles of syllogistic validity), referred to as *dictum de omni* and *dictum de nullo*. In the former, the middle term establishes an *inclusive* linkage, so that part or all of the subject of the conclusion is thereby *included* in the predicate. In the latter, the middle term effects a *separation* whereby part or all of the subject of the conclusion is *excluded* from the predicate. Yet in both, as we shall see, the middle term actually plays a very similar role. The two sample syllogisms given above would typically be assigned to these two varieties respectively.[5] In the AE method, the two premise diagrams are drawn and inspected. If a conclusion can be validly inferred, it will be readily apparent that one or the other of the above two modes of inference is involved. Drawn below are the AE diagrams for the two valid syllogisms given above. The curved arrows are helpful but not essential; they are intended to represent, as it were, the movement of the "mind's eye."

All F are S
All G are F
All G are S



All T are S
<u>Some U are not S</u>
Some U are not T



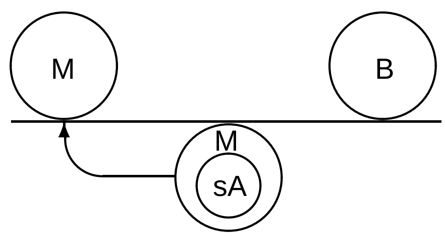
As should be quite evident, the method consists in finding the *complete* middle term "external" in one premise diagram and "internal" in the other. Each categorical diagram consists of a pair of circles providing exact visual representations of these two relationships. Only a circle forming the overall boundary of a diagram is to be regarded as "external." For a *complete* term circle "internal" corresponds precisely to the usual notion of *distribution*. Whenever the easily recognized "external"/"internal" configuration of the *complete* middle term is present, a valid inference may be drawn. Drawing the inference consists in imagining the "external" middle term along with its "contents" to be lifted as a sort of "pancake sandwich" or a pair of coins and superposed upon the "internal" middle term. By contrast with the relatively static Venn procedures, AE operations thus typically involve a degree of activity and dynamism explicitly suggesting the movement of the mind in "judgment." Note finally that the conclusion the user draws will be that of a normal categorical syllogism whenever the resulting relationship between the subject and predicate of the conclusion corresponds to one of the four basic AE diagrams for a standard categorical. *To summarize: the subject is "internal" in the diagrams for A and E categoricals; the predicate is "internal" in the diagrams for E and O; and in the A and I diagrams the predicate is "external."*

5 Multiple conclusions by simple conversion or subalternation The AE method, of course, makes no distinction between order of premises, since this order has no bearing whatever on validity. Therefore, as is true also of Venn diagrams, there will sometimes be more than one conclusion which may be validly drawn from given premises. Thus, if the premises support a particular affirmative conclusion (*Some A are B*), they also support the converse conclusion (*Some B are A*). Similarly, if the premises support a universal negative conclusion (*No A are B*), they support the converse conclusion in this case as well. Moreover, and once again this is equally true of the Venn technique, the inference of a particular ("existential") conclusion from a pair of universal premises requires an ancillary presupposition. Thus, if a pair of AE diagrams warrant a universal conclusion, they may also permit drawing particular conclusions. The conclusion *All A are B* may therefore also entail *Some A are B*, and if so, then *Some B are A* must automatically be allowed. Similarly, the conclusion *No A are B* may entail *Some A are not B* (provided the subject class is deemed nonempty) and may also entail *Some B are not A* (provided the predicate class is deemed nonempty), or possibly both (in case both presuppositions obtain).

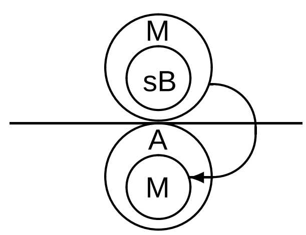
In the first syllogism presented above, for example, the given conclusion is mandatory. If, in addition, we are justified in asserting that *Some gryphons are scary* (because gryphons are, in an appropriate sense, deemed to exist), then, of course, we are also committed to asserting that *Some scary creatures are gryphons*. On the other hand, if some syllogism warrants the conclusion *No gryphons are timid*, then *No timid creatures are gryphons* also obtains. *Some gryphons are not timid* will require the existence of gryphons; *Some timid creatures are not gryphons* will require the existence of timid creatures. It ought to be clear that we could have either one existing, or neither, or both. The sort of "existing" required here has already been discussed. It should be noted that AE and Venn diagrams have precisely the same status with regard to multiple conclusions.

We conclude this section with three additional valid syllogisms and their verifying AE diagrams. The first illustrates dictum de nullo, the second and third, dictum de omni. Since premise order has no importance for the AE method, the fact that these and other examples in this essay are arranged in standard form should not be accorded any special significance. Note that in the first example there exists only a single standard categorical conclusion. In the second example the stated conclusion is actually the converse of the one immediately warranted by the diagrams. In the third example both subalternation and conversion are involved.

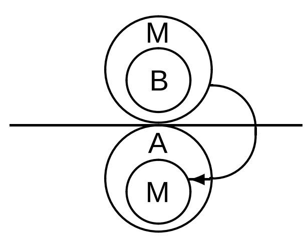
No M are B
Some A are M
Some A are not B



Some B are M
All M are A
Some A are B

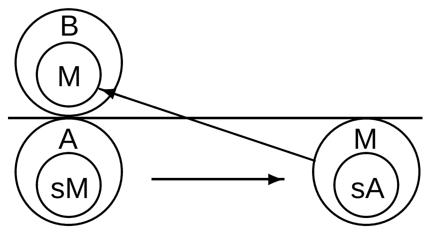


All B are M
All M are A
Some A are B



6 Completing the system: the four converses One third of the valid syllogistic moods (including all moods of the third figure) are neither discoverable nor verifiable by the simple system so far developed. In these cases, however, only a single premise replacement will prove necessary to render validity self-evident. An example will make clear the need for such replacement. Consider the following valid form and its corresponding AE diagrams and note how the indicated conversion of one premise diagram makes the inference obvious.

All M are B
Some M are A
Some A are B

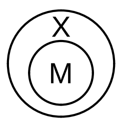
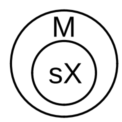
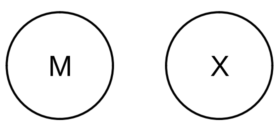
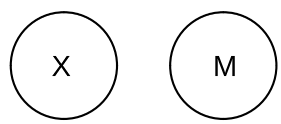
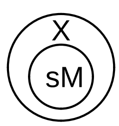
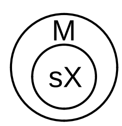
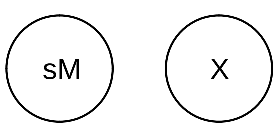
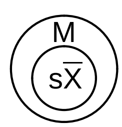


In every instance where such premise replacement is necessary the difficulty to be overcome is precisely the same. We do not have a given situation in which the complete middle term is “external” in one premise diagram, “internal” in the other; hence the two middle-term circles cannot be superposed. The remedy is also the same in every such valid instance: replacement of a *single* premise diagram by its “converse” will enable an inference in the usual way, by superposition of an “external” middle upon an “internal” one. Of course, since “internality” corresponds exactly to *distribution*, the complete middle term which is (or which may be taken as) “correctly” configured is “internal” in every valid case. Thus, the purpose of premise replacement will always be to supply a premise diagram in which the middle term is “external.” Replacement will often consist of substituting for a particular affirmative premise diagram its (simple) converse, as in the example above. Sometimes, as will be shown, we require a replacement “converse” for a universal affirmative or a particular negative diagram. It will never prove useful to replace a universal negative since subject and predicate are both “internal” in the universal negative and in its (universal negative) converse. These two converse diagrams, which could differ at most by relative placement of their two term circles, are equivalent so far as the AE method is concerned.

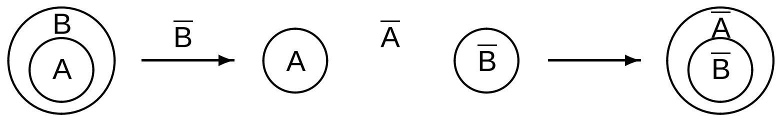
Now it may strike the reader as odd to speak of the “converses” of the universal affirmative or particular negative categoricals. In modern textbooks such “converses” are rarely encountered. Only the universal negative and particular affirmative have converses in the simple, usual sense in which conversion means no more than interchange of subject and predicate. Once again, however, the practice of traditional logicians is instructive. The older tradition recognized “converses” of sorts for both the A and O categoricals—in both cases these are the simple converses of closely related particular affirmatives.[6] In the case of the universal affirmative, the process was referred to as *conversion per accidens* (or as “conversion by limitation” in more modern treatments). The “converse” so obtained is actually the simple converse of the particular affirmative subalternate to the given judgment (and therefore the term *conversion by subalternation* might be more informative).[7] From the point of view of the traditional logic, such “conversion” was always justified since all terms were assumed nonempty. For our purposes, we shall allow such "conversion" only for subject terms *known* (or *deemed*) to be nonempty. In the case of the particular negative, the traditional logicians had recourse to yet another sort of "conversion," *conversion by contraposition*. The contrapositive of *Some S are not P* is *Some non-P are not non-S*. These two judgments are precisely equivalent on all presuppositions. Then by obversion we obtain *Some non-P are S*. An alternative route would be to recognize that *Some S are not P* is obviously equivalent to the obverse *Some S are non-P*; then by simple conversion we again obtain *Some non-P are S*. Of course, the student need only observe that a sign of negation must move with the predicate term.

From this point on we shall drop quotation marks and simply adopt this generalized sense of converse. These results are summarized below, employing the usual overbar notation for class complement. Recall that we shall be using these converses in premise replacement in order to "externalize" a complete middle term; hence the choice of *M* in the following table.

Table 2: The Four Converses

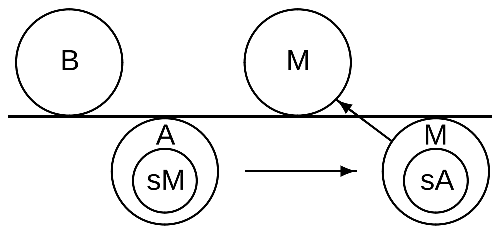
Given Categorical	Generalized Converse
All M are X	Some X are M
	
No M are X	No X are M
	
Some M are X	Some X are M
	
Some M are not X	Some non-X are M
	

The reader will have noted the free use just made of immediate inferences. In a classroom development, AE diagrams can help establish these results also. Very frequently such inferences will turn on the trick of recognizing that in the basic AE diagrams a complementary significance can be assigned to the region *surrounding* the circles. Observe once again the active, dynamic style of AE inference in this single example of contraposition.

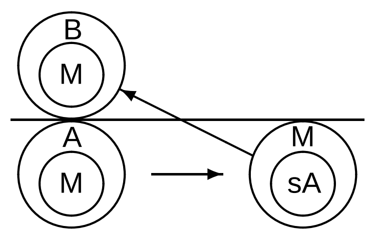


We conclude this section with three additional syllogisms illustrating the process of premise replacement by generalized conversion.

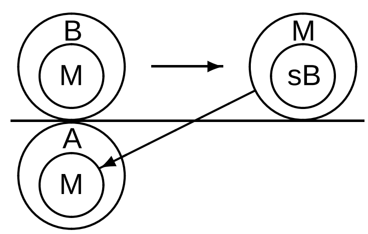
No B are M
Some M are A
Some A are not B



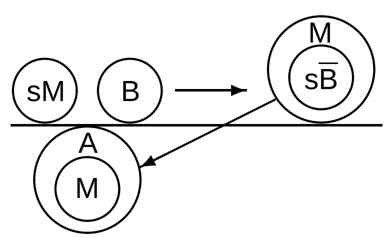
All M are B
All M are A
Some A are B



or



Some M are not B
All M are A
Some A are not B



The third example warrants particular attention, since the conclusion initially obtained *must* be converted in order to affirm a standard categorical of the given terms. This, however, is the only mood (OAO-3) *requiring* conversion of the conclusion. This mood is also unique, on the present analysis, in proceeding by dictum de omni to a negative conclusion. The controversy regarding the meaningfulness of the dicta, which we took note of above (see again footnote 5), is now unavoidable. On the one hand, with all the author's students (and Aristotle) concurring, Barbara and Celarent really must be, psychologically speaking, self-evident. Reduction to one of the dicta properly exploits this self-evidence. The difficulty appears to arise from insisting that any given valid mood *belongs* to a given dictum. Strictly speaking, none do. By obversion of the major premises, Barbara can be cast in Celarent and Celarent in Barbara. More to the point, perhaps, Bocardo can easily be placed "correctly" in dictum de nullo—for example, by converting the major premise and obverting the minor. But the AE method has nothing whatever to do with re-engaging a musty controversy. Rather, the main claim of the AE diagram method is that every valid mood can be assigned to one dictum or the other by one single generalized conversion at most. This claim requires proof, which we now provide.

7 *Proof of completeness and summary instructions*

Proof: Anyone who has read thus far will recognize the *sufficiency* of the AE diagram method. In other words, this method quite plainly yields only valid inferences. In the final section of this essay we will argue further that AE diagrams conveniently embody the fundamental necessary and sufficient conditions for syllogistic validity. From the latter point of view, the basic AE method of superposing "external" and "internal" middle terms will also appear to be "obviously" necessary. For now, however, we simply show that the method of AE diagrams is *complete* in the sense that it yields its valid results in all instances in which any correct system would.

We have already alluded to the fact that requiring every acceptable premise diagram pair to include one diagram with internal middle term is merely another way of stating that in a valid syllogism the middle term (designated below by 'M') must be distributed at least once. We make this the basis for a complete division of all possible initially given premise combinations.

Case 1: Neither given premise diagram has 'M internal'. In this case no valid syllogism could be determined by any correct method, since the fundamental rule of syllogism requires that the middle term be distributed at least once. Otherwise, the two premises could have reference to different "parts" of the middle term and there would then fail to be any definite link between the extremes.

Case 2: One given premise diagram has 'M internal', the other has 'M external'. In this case AE diagrams yield valid results in every instance.

Case 3: One given premise diagram has 'M internal', the other has neither 'M internal' nor 'M external'. In this case there are only two possibilities for the premise diagram in which we do not have 'M internal'. These correspond to the categoricals *Some M are X* and *Some M are not X*. Since these categoricals both have diagrams convertible so as to "externalize M," this case also includes no valid moods inaccessible to AE diagrams.

Case 4: Both given premise diagrams have 'M internal'. In this case we need examine only those pairs of premises in which both diagrams have 'M internal' and in which neither diagram is convertible so as to "externalize M." Then both premises will be negative, and if one is (or both are) particular, the middle term is in predicate position. Such premise pairs may be illustrated using three circles only, one labeled

'M'. The requirement that the M circle intersect neither of the others obviously imposes no restriction whatsoever on the mutual relationship of the subject and predicate circles. It is thus evident that no such pair of premises can establish any definite link between the extremes. □

In view of the foregoing proof by cases, we can reduce the method of AE diagrams to the following summary instructions. In order to determine what, if any, standard-form categorical conclusion follows from a given pair of categorical premises, first note whether the premises contain just three terms with a common "middle" (designated below by 'M'). If so, then, choosing three suitable capital letters for the terms, draw a pair of premise diagrams, one above the other (in either order), separated by a line.

7.1 Immediate decision

1a. If neither premise diagram has 'M internal' there is no valid result.
1b. Does one diagram have 'M internal', the other 'M external'? If 'yes' and if the resulting conclusion is (or is equivalent to) a standard-form categorical, then the premises are acceptable for a categorical syllogism. Read out the conclusion(s).

7.2 Conversion required

2a. One premise diagram has 'M internal', the other has neither 'M internal' nor 'M external'. Convert the diagram that does not have 'M internal' so as to "externalize M" and go to (1b) above.
2b. Both premise diagrams have 'M internal'. If possible, convert either premise diagram so as to "externalize M" and go to (1b) above. If neither premise diagram can be so converted there is no valid result.

It is perhaps worth reiterating that "conditionally valid" syllogisms, those requiring an "existential" assumption, reveal themselves in one of two ways when AE diagrams are employed: in explicit subalternation of an initially universal conclusion or in conversion by subalternation of an A premise.

7.3 Tabular summary On first reading, the above summary instructions will perhaps seem to make a very simple system appear complex. This is probably due to the fact that most persons will actually perform a very rapid but largely unconscious "parallel process" classification of any given premise pair. Even so, many readers will prefer the following diagrammatic approach.

7.4 Discussion of the preceding two tables In Table 3 of this section we present the basic result (derived from Tables 1 and 2 on pp. 4 and 9) that there are just three ways each for the middle term to occur either "externally" or "internally." The column headings *Generalized Minor* and *Generalized Major* are explained in the final section of the paper. The selection of one diagram from each column, in every way, yields nine valid inferences. Six of these have standard categorical conclusions. These six are presented in Table 4. The latter sets forth the only possible syllogistic segments

Table 3: All Possible Premise Diagrams for a Valid Inference
(following generalized conversion of a single premise if necessary)

Generalized Minor | Generalized Major

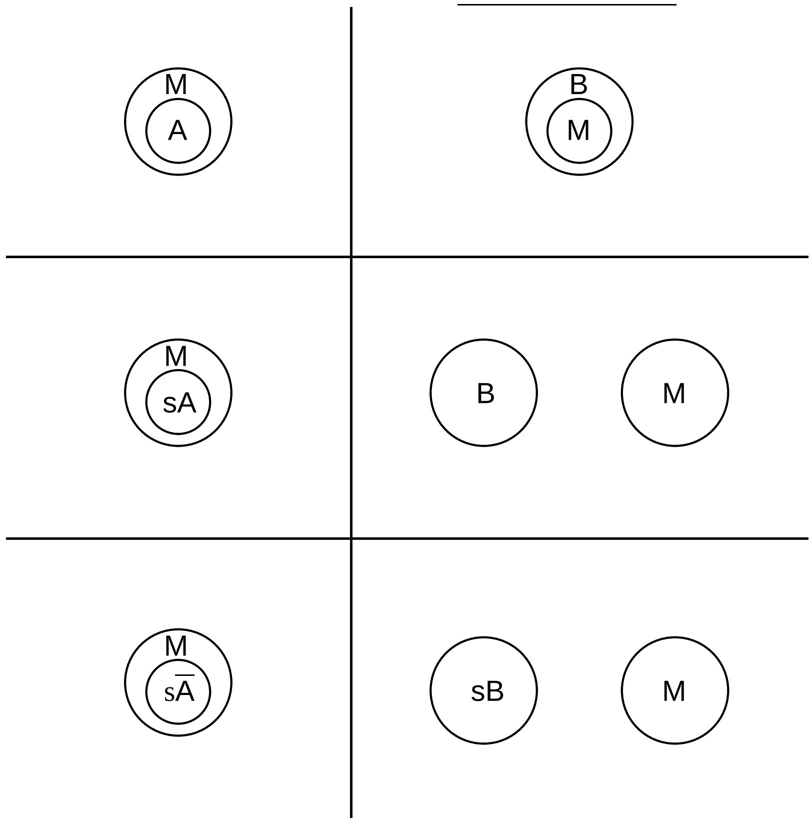
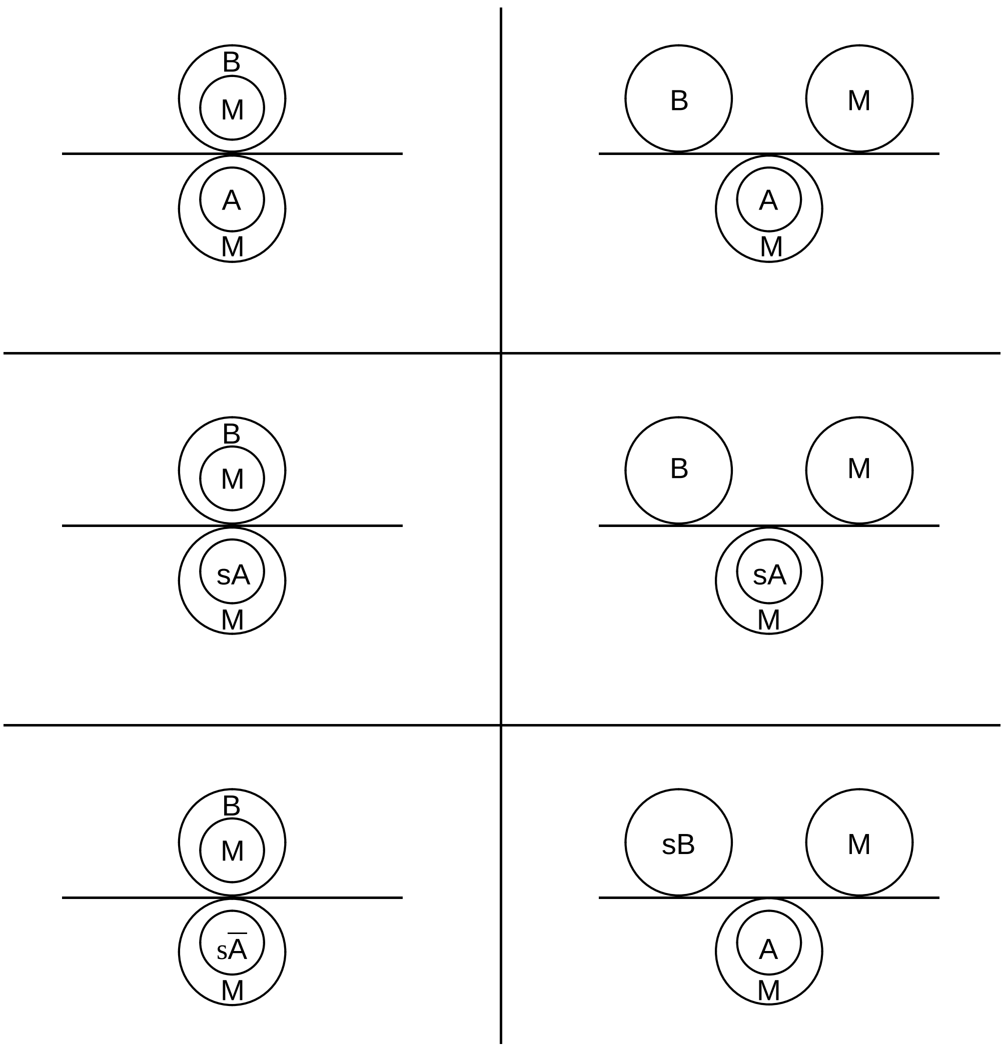


Table 4: The Six Basic Syllogistic Patterns

Dictum de omni | Dictum de nullo



of AE "paths"; that is, syllogistic reasonings may begin with the illustrated premises given. Alternatively, however, these premises may be reached following a single conversion. Similarly at the output end, a syllogism may terminate on the conclusion immediately warranted. Alternatively, simple conversion or subalternation may lead to a further conclusion.

This second table of basic syllogism patterns, which students should be encouraged to examine carefully, has many uses. Indeed, it renders syllogistic reasoning transparent. For example, we see here that neither two negative nor two particular premises are allowed. Moreover, a negative conclusion is permissible if and only if there is a negative premise; a particular conclusion, if and only if there is a particular premise. Or suppose that we require premises to support some predetermined conclusion. They can, with experience, be read directly off this table. Thus there is only one way each to conclude in A, E, or I. By contrast, there are three distinctive paths to a conclusion in O.

Finally, it should again be emphasized that this table of basic patterns achieves its simplifying usefulness partly by isolating the purely syllogistic part of a given reasoning from any ancillary immediate inference. This further illustrates the clarifying AE focus upon the essential role of the middle term.

8 The AE criterion of validity, full comparison with Venn diagrams, and conclusion

We have shown above that the AE system is complete in the sense that it yields valid conclusions in all cases in which any correct system would. A stronger assertion is possible, however: the AE diagram method embodies a criterion for validity of categorical syllogisms which is both necessary and sufficient. This is due to the fact that AE diagrams represent a convenient pictorial model of the class inclusion inference structure which is at the heart of syllogistic. For if the (possibly hypothetical) entities referred to by one conclusion term are to be included among those referred to by the other—and if that inclusion is to be effected by the middle term—then the underlying form of inference can only be that of repeated class inclusion. This is brought out most clearly if we exploit our earlier observation that every valid syllogism can be assigned to either dictum. We apply that observation to the diagrams in Table 3 of the previous section. Specifically, while retaining the given middle term, we cast all moods suggested in that table in dictum de omni. If we then express this result employing the familiar symbol for the transitive relation of class inclusion, we obtain the following very compact description of all the basic valid moods from the AE point of view. Note that we have taken the minor liberty of employing term symbols to represent classes also.

$$\left.\begin{array}{l}\mathrm{A}\\ \mathrm{s}\mathrm{A}\\ \mathrm{s}\overline{\mathrm{A}}\end{array}\right\} \subseteq \mathrm{M} \subseteq \left\{\begin{array}{l}\mathrm{B}\\ \overline{\mathrm{B}}\\ \overline{\mathrm{aB}}\end{array}\right.$$

By analogy with Barbara and Celarent, the three possible premises of the form $\mathrm{M} \subseteq \mathrm{Y}$ may be regarded as *generalized major premises* (premises in which the middle term is "distributed relative to" a "generalized predicate"); those of the form $\mathrm{X} \subseteq \mathrm{M}$ would then be *generalized minors* (in which a "generalized subject" is "distributed relative to" the middle term itself). Now since every valid syllogism must have a premise with distributed middle, the other premise could only usefully serve to identify "part" of the middle term with the "generalized subject." Thus we may take as the single essential rule of syllogism that *every valid categorical syllogism must have both a generalized major premise and a generalized minor premise*. If the middle term is distributed in one given premise only, that premise determines the generalized major; in the few valid moods having two given premises with distributed middle terms the generalized premises are not uniquely determined and must be assigned by some process involving subalternation (for example, by conversion of an A premise). In any case, the single "rule of syllogism" stated above is simply another way of understanding the AE requirement that one premise diagram have 'M internal' and the other be (or entail) a form with 'M external'. In other words, the AE diagram method depends upon, and could have been derived from, the class inclusion criterion for syllogistic validity. The AE method is thus neither more nor less obviously "correct" than, say, the Venn diagram method; it simply represents a different approach, one which, although perhaps more abstract when stated, may strike many students as a more intuitive form of representing arguments. Further, the "motion" implied in AE diagrams—the student can imagine picking up the diagram of one premise and placing it in the corresponding place within the other—makes it a particularly valuable pedagogical tool.

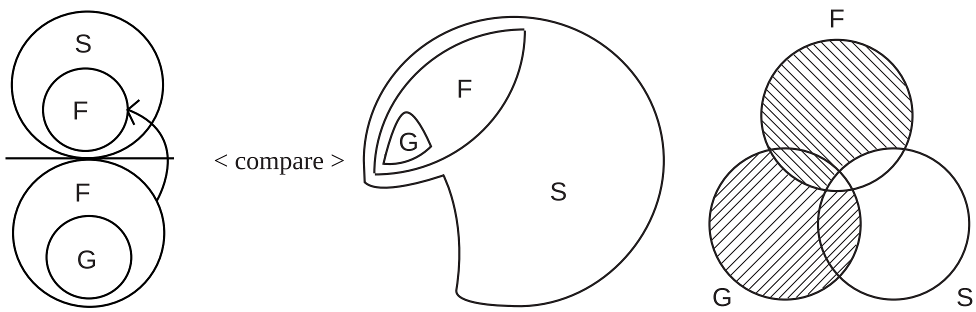
For completeness, we will now also show directly that Venn diagrams (hence any "obviously correct" approach) give precisely the same results that AE diagrams do. For consider the six valid syllogism classes indicated above, and restrict attention, for the present, to those cases in which the generalized minor premises have either been given as such or else have been arrived at by conversion of equivalent particular categoricals. For definiteness, let us focus our attention on the following specific example.

All A are M
Some B are not M
———————
Some B are not A $\qquad$ $A \subseteq M \subseteq \overline{sB}$

Suppose that we imagine diagramming these two premises on a Venn tableau. Since Venn diagrams are based on the formal analogy of class inclusion to planar region inclusion, it follows that when we diagram *All A are M* ($A \subseteq M$), what remains of the A region automatically comes to fall entirely within the M region. And when we diagram *Some B are not M* ($M \subseteq \overline{sB}$), the M region automatically comes to appear within the complementary region outside the cell where we place our mark corresponding to sB. Now a main point of the analogy of class inclusion to planar region inclusion is that these are both *transitive* relations. Hence, automatically, the AE diagram conclusion, *Some B are not A* ($A \subseteq \overline{sB}$), will also appear on the tableau. But Venn conclusions are unique up to subalternation and simple conversion (just as AE conclusions are). Hence we conclude that Venn diagrams and AE diagrams give exactly the same results in the six syllogism classes we have been considering. Finally, consider the cases where AE diagrams require the *nonequivalent* conversion of an A categorical. In these few cases the choice of generalized premises is not unique since both initially given premises have a distributed middle. It is easily verified, however, that in these "conditionally valid" cases also (including AAI-3, EAO-3 and 4) AE and Venn diagrams give identical results. Therefore, the two systems are effectively equivalent.

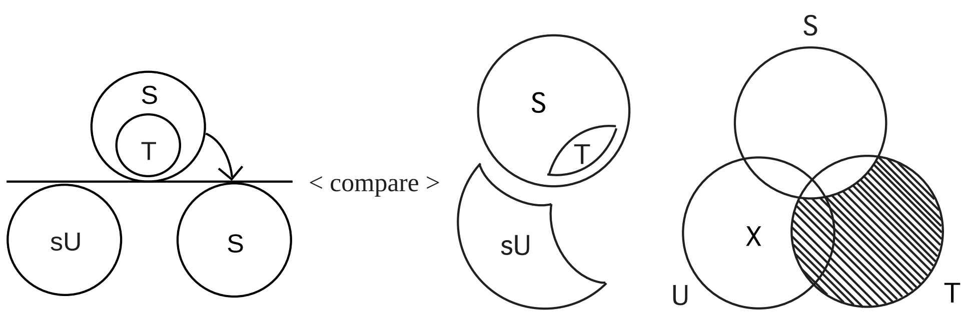
But whereas the systems are indeed equivalent from a logical point of view, they are by no means equivalently easy to use. We conclude this paper by returning to the two examples with which we began.

All fierce creatures are scary.
All gryphons are fierce.
———
All gryphons are scary.



Notice how on the Venn tableau the G region does indeed lie wholly within the F, and the F region wholly within the S! Just as we know they're supposed to do!

All timid creatures are shy.
Some unicorns are not shy.
———
Some unicorns are not timid.



And notice in this case how the T region lies entirely within the S, but how the region here labeled 'sU' for ease of comparison lies wholly outside S! Again, just as we know they're supposed to do!

There are a variety of pictorial devices currently in use, some of which share the completeness demonstrated above for AE diagrams. To establish the superiority of AE diagrams over other options—such as Englebretsen line diagrams, for example[8] —is a difficult task that cannot be undertaken here. Nonetheless, this paper has shown that categorical syllogisms are analyzed with particular rapidity and insight by the method of AE diagrams. This is indeed particularly apparent when given premises fail to yield a valid conclusion. It is then evident at a glance exactly what is wrong with the premises and the way to amend them becomes equally clear. In view of the two comparisons between Venn and AE results given immediately above, however, the author will perhaps be forgiven if he makes a stronger claim. As depicted on Venn diagrams, information of real significance may occasionally appear hidden and distorted, almost as in certain puzzles where we are challenged to find a cleverly concealed face! Is this a worthwhile sort of occupation for our students? Isn't it time for a change?

NOTES

1. See, for example, Peterson [15]. See also [7] and especially Englebretsen [8].
2. Recent years have seen important new work on the diagrammatical representation of mental reckoning. Especially important are Barwise and Etchemendy [2], Glasgow, Narayanan, and Candrasekaran [11], Johnson-Laird [13], Rips [16], Shin [18], and Sowa [19].
3. See, e.g., Erickson [9], and Guyote and Sternberg [12].
4. See Rose [17], Chapter V and Appendix III. Aristotle is the initiator of validation by reduction to the first figure and preferred reduction by conversion.
5. The meaningfulness of the "dicta" remains controversial. For a generally negative view and summary see Dopp [5], pp. 164–6. For a more favorable assessment, expressed with characteristic clarity, see Stebbing [20], pp. 86–7. The author's view is stated in the text at the end of Section 6.
6. For the traditional doctrine of conversion see Dopp [5], pp. 133–8.
7. For recent texts which include conversion by limitation of the universal affirmative (what we are here referring to as "conversion by subalternation") see Copi and Cohen [4], p. 173 and also Kahane and Tidman [14], pp. 276–7.
8. See Englebretsen [6].

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