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# Reconciling Aristotle and Frege

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**Abstract** An account of Aristotle's syllogistic (including a full square of opposition and allowing for empty nouns) as an integral part of first-order predicate logic is lacking. Some say it is not possible. It is not found in the tradition stemming from Łukasiewicz's attempt nor in less formal approaches such as Strawson's. The Łukasiewicz tradition leaves Aristotle's syllogistic as an autonomous axiomatized system. In this paper Aristotle's syllogistic is presented within first-order predicate logic with special restricted quantifiers. The theory is not motivated primarily by historical considerations but as an accurate account of categorical sentences along lines suggested by recent work on natural language quantifiers and themes from supposition theory. It provides logical forms which conform to grammatical ones and is intended as a rival to accounts of quantifiers in natural language that appeal to binary quantifiers, for example, Wiggins or to restricted quantifiers, for example, Neale.

***1 Introduction*** Łukasiewicz's pioneering study, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, was an attempt to relate "The Philosopher's" work to the modern formal logic Frege initiated. While the tradition growing out of Łukasiewicz's work does relate Aristotle's syllogistic to modern formal logic, it fails to rescue the syllogistic from its anomalous status. By "anomalous" I mean that the syllogistic is not actually reduced to first-order predicate logic. As the title of his study reveals, Łukasiewicz presents Aristotle's syllogistic *from* the standpoint of modern formal logic. It is not an account of the syllogistic *in* modern formal logic, first-order predicate logic. The syllogistic is presented via syntactical rules of well-formedness and axioms outside of predicate logic which are the basis of a formal system. This is similar to other extra-predicate logic systems such as mereology and set theory.

Łukasiewicz's study was in part motivated by historical considerations. In his initial Chapters I–III he demonstrates how fine a formal logician Aristotle is. The syllogistic is reconstructed along the lines Aristotle laid out, while indicating how the addition of accepted principles of first-order predicate logic justify Aristotle's systematization. Chapter IV provides a rational reconstruction rather than a historical

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account. In Chapters I–III and in Chapter IV, Aristotle's syllogistic appears as an autonomous system supplementing first-order predicate logic. Thus, in Chapter IV the basic well-formed formulas, the **A**, **E**, **I**, **O** form categorical sentences, are taken as syntactically and semantically basic.[1] They are not characterized merely in terms of the predicate logic notation of atomic sentences plus truth functional and quantificational compositions. Łukasiewicz has primitive *a*, *e*, *i*, and *o* functors that go between noun/term expressions to form the **A**, **E**, **I**, **O** well-formed formulas. Four axioms are provided which govern inferences involving these formulas, that is, immediate inferences and syllogisms ([11], p. 91). As axioms they are neither derived from nor suggested by ordinary predicate logic. The first two axioms correspond to Barbara and Datisi. The last two are laws of identity: All $A$ are $A$, Some $A$ are $A$. They are involved in deriving as theorems laws of subordination for the affirmative and for negative premises, that is, that an **A**-form sentence implies an **I**-form one and that an **E**-form sentence implies an **O**-form one. Thus, that **A** implies **I**, is demonstrated by substituting in the "axiom" Datisi; that is, 'All $A$ are $B$', 'Some $A$ are $C$', so 'Some $C$ are $B$'. On substituting $A$ for $C$ we obtain 'All $A$ are $B$', 'Some $A$ are $A$', so 'Some $A$ are $B$', where the second premise is dispensable since it is an instance of the second law of identity.

By and large the accepted view in modern first-order logic as found in countless texts is that the full set of Aristotelian theses including principles such as a full square of opposition (with an **A**-form sentence implying an **I**-form one) while allowing for empty nouns is not justifiable within predicate logic. It was Frege who initiated the currently entrenched practice of representing **A**-form sentences as universal generalizations of conditionals: $(x)(Ax \to Bx)$, and **I**-form ones as existential generalizations of conjunctions: $(Ex)(Ax \,\&\, Bx)$. Notable attempts have been made to follow Łukasiewicz in showing how Aristotle's theses can be justified outside of predicate logic. While there is disagreement as to how accurate Łukasiewicz's treatment was to the historical Aristotle (for example, whether Aristotle stated his theses as axioms or as rules of inference), the various sides on such disputes agree in holding that the solution to justifying the theses should be formal and has to be given outside first-order logic.

Some logicians attempt to give an informal and in some sense "pragmatic" or "context-based" solution. Strawson's work is a prominent attempt in this direction. It is not formal in that no special logical form is assigned to categorical sentences. It is pragmatic or context dependent in that unless a sentence grammatically of **A**, **I**, **E**, or **O** form is used in a context where an existential presupposition is fulfilled, no truth vehicle (no statement) exists and so no relation of implication obtains. An **A**-form truth vehicle does in a sense "imply" an **I**-form one, but it is not a matter of formal considerations.

It is the goal of this paper to change the status of Aristotle's syllogistic by placing it within modern logic, by reducing it to predicate logic plus identity with restricted quantifiers of a special sort. The treatment of these restricted quantifiers is suggested by an Aristotelian tradition, that of Terminist logicians such as Ockham [15] and Buridan [5]. Utilizing certain ideas derived from this tradition's theory of supposition, a formal account of restricted quantifiers can be constructed which falls squarely within first-order predicate logic. On this basis Aristotle's theses concerning categorical sentences are accounted for within predicate logic. The logical forms assigned categorical sentences are not primitive as on the Łukasiewicz view nor is the quantification unrestricted. A full syllogistic with a full square of opposition is given in terms of a notation for first-order predicate logic and identity with restricted quantifiers and rules of inference for those quantifiers suggested by those of first-order predicate logic.

Unlike the line taken by Łukasiewicz and others, I am not trying to account for Aristotle's syllogistic along the lines Aristotle pursued. I am trying to show that his results, Aristotle's syllogistic—its theses—are derivable in predicate logic using techniques and concepts he may have had no idea of, for example, quantifiers ([11], p. 83). Łukasiewicz described his rational reconstruction of Aristotle's syllogistic as follows:

> This chapter [IV] does not belong to the history of logic. Its purpose is to set out the system of non-modal syllogisms according to the requirements of modern formal logic, but in close connection with the ideas set forth by Aristotle himself. ([11], p. 77)

In saving the theses of Aristotelian logic within modern predicate logic several bonuses accrue. We can offer an exact restatement and explanation of why the traditional rules of quality and quantity (though not part of Aristotle's own approach) work when suitably construed in terms of modern predicate logic. The problem of empty nouns so scantily dealt with in Aristotle himself, his 'goat-stag' example, though a source of controversy after him, is of a piece with and goes the same way as the problem of empty names and free logic in predicate logic. Some results from current investigations of Aristotle's modal syllogistic can be expressed and justified in terms of the present approach. A theory of quantified noun phrases emerges which rivals accounts involving binary quantifiers and other views of restricted quantifers.

***2 Assertoric syllogistic*** Let us start afresh by focusing on regimenting English so as to formulate canonical statements in English of categorical sentences. Let us adopt 'Every $A$ is a $B$' as our regimentation of an **A**-form sentence and not the familiar plural form 'All $A$ are $B$' which requires more than one $A$. Parallel canonical regimentations of **I**-, **E**-, and **O**-form sentences are 'At least one $A$ is a $B$', 'No $A$ is a $B$', that is, '(Not one $A$) is a $B$' and 'At least one $A$ is not a $B$'. To begin with we focus on

Every $A$ is a $B$

parsed as

(Every $A$) is (a $B$).

'(Every $A$)' is represented as the restricted quantifier

$$(x, Ax).$$

'(a $B$)' is represented as the restricted existential quantifier

$$(Ey, By).$$

Since the copula 'is' will go between two singular terms, the variables '$x$' and '$y$', the obvious choice to represent it is '$=$'. An **A**-form sentence is formally represented as

$$(x, Ax)[(Ey, By)x = y].$$

As convincing and natural as this symbolic representation is, if we treat the restricted quantification as it is usually done in most logic texts, we will not arrive at a satisfactory account of Aristotle's views. On the usual treatment the restricted quantification in the **A** form would appear with unrestricted quantifiers as a universal generalization of a conditional:

$$(x)[Ax \to (Ey)(By \,\&\, x = y)].$$

On this representation we fail to do justice to Aristotle's view that 'Every $A$ is a $B$' logically implies 'At least one $A$ is a $B$'. According to the usual account of the relation of restricted to unrestricted quantification the **A**-form sentence does not imply the corresponding **I**-form one,

$$(Ex, Ax)[(Ey, By)x = y], \text{ i.e., } (Ex)[Ax \,\&\, (Ey)(By \,\&\, x = y)].$$

To deal with this problem we put aside the usual translation of restricted to unrestricted quantifications. Instead we take our cue from the following observation about quantified noun phrases: '(Every $A$)', '(Every $B$)' and corresponding demonstrative noun phrases '(That $A$)', '(That $B$)'. There is some sort of formally specifiable relation between '(Every $A$)' and '(That $A$)' and between '(Every $B$)' and '(That $B$)'. No such relation obtains between '(Every $A$)' and '(That $B$)'. The presence of the same noun, the same restriction on the quantifier and on the demonstrative, is what we want to capture. The problem then before us is of developing a predicate logic notation, semantics, and rules of inference that will do justice to this insight about surface grammar.

Let us take as a further guide the account of logical truth found in Bolzano, Ajdukiewicz, and Quine. It can be construed as enforcing a close connection between logical truth and surface grammar. In Quine's version, a sentence is logically true if, and only if, it is true and replacement of the nonlogical parts *salva congruitate* yields only truths. I propose that conditionals with components containing quantified and demonstrative noun phrases having the forms

(Every $A$) is a $B \to$ (This/That $A$) is a $B$

and

(This/That) $A$ is a $B \to$ (Some $A$) is a $B$

are logical truths. Both intuitively and given the semantics provided in a later section, these qualify as logical truths. Thus, take any true sentence of the form '(Every $A$) is a $B \to$ (This $A$) is a $B$' and replace the predicates/nouns $A$, $B$ uniformly with any predicate/noun and you get another true sentence. Of course, the status of this conditional as a logical truth justifies the associated inference from '(Every $A$) is a $B$' to '(This $A$) is a $B$'. Similar remarks apply to the second conditional: '(This/That) $A$ is a $B \to$ (Some $A$) is a $B$'. By hypothetical syllogism applied to these two conditionals we get '(Every $A$) is a $B \to$ (Some $A$) is a $B$'. This is the logically true conditional that corresponds to an **A**-form proposition implying an **I**-form one. The latter is at the heart of the traditional full square of opposition.[2] The usual Frege-inspired textbook treatment of such categorical sentences as unrestricted universal generalizations over conditionals and existential generalizations over conjunctions violates conservatism when it prohibits such inferences.[3] By contrast, earlier logicians regarded such inferences as valid.

It is a purely formal logical relation between sentences containing quantified noun phrases and demonstrative noun phrases that we want to capture. Note that we are not considering an enthymeme. The question is not: Does the **A** plus other suppressed premises imply the **I**? The claim is that it is intuitive and correct to say that every human is a mammal by itself purely logically implies that at least one human is a mammal. Furthermore, the account is offered as a purely formal one and not as involving pragmatic elements.

***2.1 A full square of opposition in predicate logic*** An account of categorical sentences can be given within predicate logic notation by mimicking in predicate logic notation the way the quantified noun phrases correspond to the demonstrative noun phrases in the above conditionals. In this section we turn to the square of opposition, in the next to the remainder of the syllogistic, and in Section 4 to additional applications. The theory is offered as a logic of quantified and demonstrative noun phrases. Instead of 'All $A$---' having the logical form $(x)(Ax \rightarrow [\text{---}x\text{---}])$, it has the form $(x, A)[\text{---}x\text{---}]$ as though the English sentence were parsed '(All $A$)---'. 'Some $A$ are $B$' does not have the form $(Ex)(Ax\ \&\ \text{---})$, but rather the form $(Ex, A)[\text{---}x\text{---}]$ which one can associate with an English sentence parsed as '(Some $A$)---'. So doing we start by abiding, at least syntactically, by the constraint that the only difference between the logical forms assigned is in the quantifiers. As suggested by the logical truths,

(Every $A$) is a $B \rightarrow$ (This/That $A$) is a $B$,

and

(This/That) $A$ is a $B \rightarrow$ (At least one $A$) is a $B$,

it is a logic of restricted quantifiers in terms of such quantified and demonstrative noun phrases. The syntax for the formal notation will be described and then rules of inference for these restricted quantifiers will be given. The rules of inference are for semantic tableaux (trees). They provide a refutation procedure as well as a proof procedure.

Assume the syntax of first-order predicate logic and the method of tree rules (semantic tableaux). Similar rules can be given for your favorite system of natural deduction. The expressions "substituend," "instance," "canonical substituend," and "canonical instance" are used as follows. In first-order predicate logic a substituend can be characterized as an expression used to replace a variable. Thus in going from $(x)(Fx \rightarrow Gx)$ to $Fx \rightarrow Gx$ and then to $Fa \rightarrow Ga$, the expression substituted, $a$, is the substituend for the variable, $x$. The resulting formula, $Fa \rightarrow Ga$, is the instance. Contexts such as $Fa$ are basic instances. We will need parallel notions of a canonical substituend, a canonical instance, and a canonical basic instance for restricted quantifiers.

Add to the language of predicate logic restricted quantifiers such as $(x, Ax)$, $(Ex, Bx)$. These quantifiers are the representations in predicate logic form of English quantified noun phrases: 'Every $A$', 'At least one $B$'. Placing these in front of appropriate open sentences yields well-formed formulas. We need symbolic counterparts of the English demonstrative noun phrases: 'This $A$', 'That $B$', which will serve as the canonical substituends of restricted quantifiers and occur in the sentences serving as canonical instances of such generalizations. Since these canonical substituends are singular terms, use lowercase letters with superscripts such as $a^1$, $b^2$, in a special way. Just as the English 'this $A$' somewhat formally indicates by the presence of the same noun that it is an appropriate substituend for the restricted quantifier 'Every $A$', use a lowercase letter (with a superscript), $a^1$, $a^2$, that is, the lowercase version of the capital letter occurring in the quantifier phrase: $(x, Ax)$. In this notation, $Ba^1$ would be a correct or canonical substituend for the formula $(x, Ax)Bx$, but $Bb^1$ would not. Using the same letter of the alphabet in a lower case as the restriction on the quantifier mimics in our notation the relation in English of the restriction on the natural language quantifier to its canonical demonstrative noun phrase. The superscript on the singular term serves to distinguish the substituends: $a^2$, $b^1$, for restricted quantifiers from the ordinary singular terms: $a$, $b$, $c$, $x$, $y$ which serve as substituends for variables of unrestricted quantifiers.

As rules of inference to be added to tree rules for unrestricted quantifiers we add four rules which apply solely to restricted quantifiers.[4]

*Restricted Universal Instantiation*

$$\frac{(x, Ax)\Phi x}{\Phi a^1}$$

(as individual constants use the lowercase letter of the restriction on the quantifier with superscripts to distinguish these canonical instances for restricted quantification from instances associated with unrestricted quantification).

*Restricted Existential Instantiation*

$$\frac{(Ex, Ax)\Phi x}{\Phi a^1}$$

where $a^1$ is new to the tree.

*Quantifier Interchange (Duality)*

$$\frac{-(x, Ax)\Phi x}{(Ex, Ax) - (\Phi x)}$$

$$\frac{-(Ex, Ax)\Phi x}{(x, Ax) - (\Phi x)}$$

Each of the relations in the full square of opposition can be easily tested as well as others, that **A** forms imply **I** forms but **O** forms don't imply **I** forms.

Duality principles in several areas of logic are not always recognized as such: the duality principles in sentence logic (their alias there is "De Morgan's laws"); duality relations between quantifiers (unrestricted ones) in predicate logic (often referred to as "quantifier interchange"); the interchange of necessity and not possibly not in modal logic; and in Aristotelian logic the full "square of opposition." Throughout these areas the same relations of duality are found. This suggests (and ordinary intuitions demand) that a classical treatment of these subjects maintain these analogies (and/or, all/some, and necessity/possibility are all analogous). In sentence logic '$p$ & $q$' implies '$p \vee q$' and '$p$ & $q$' is equivalent to '$-(-p \vee -q)$'. In full predicate logic with unrestricted quantifiers '$(x)(\ldots x \ldots)$' implies '$(Ex)(\ldots x \ldots)$' and '$(x)(\ldots x \ldots)$' is equivalent to '$-(Ex) - (\ldots x \ldots)$'. Similar remarks apply to necessity and possibility. In other words, there are full squares of opposition in sentence logic (De Morgan's laws), in predicate logic (Quantifier Interchange), in modal, and in deontological logic. An additional argument for a full square of opposition emerges when one maintains duality for restricted quantifiers construed as quantified noun phrases: '(All $A$). . . .)' implies '(Some $A$)(. . . .)' and '(All $A$)(. . . .)' is equivalent to '$-$(Some $A$)$-$(. . . .)'. The latter is a more generalized form than that found in the special case referred to as the "traditional," "full," or "Aristotelian" square of opposition since the subquantificates might consist of more complex constructions such as conditionals, generalizations, or modal formulas (see Section 4). Here are the above duality principles arranged as squares of opposition.

*Duality (Full Squares of Opposition)*

| | |
|---|---|
| $p$ & $q$ | $-p$ & $-q$ |
| $(x)\Phi x$ | $(x)-\Phi x$ |
| Nec $p$ | Nec $-p$ |
| $(x, A)\Phi x$   (Every $A$)$\Phi B$ | $(x, A)-Bx$   (No $A$)$\Phi B$ |
| This $A$ is a $B$ & That $A$ is a $B$ | $-$This $A$ is a $B$ & $-$ That $A$ is a $B$ |
| $p \vee q$ | $-p \vee -q$ |
| $(x)\Phi x$ | $(Ex)-\Phi x$ |
| Pos $p$ | Pos $-p$ |
| $(Ex, A)\Phi x$   (least one $A$)$\Phi x$ | $(Ex, A)-\Phi x$   (At least one $A$)$-\Phi x$ |
| This $A$ is a $B \vee$ That $A$ is a $B$ | $-$This $A$ is a $B \vee -$That $A$ is a $B$ |

***2.2 Syllogistic in predicate logic*** When we symbolize categorical sentences as we have done so far, we have enough logical structure to explain the full square of opposition but not a full syllogistic. The duality relations involved in the full square pertain to a grosser logical form than that needed for a full syllogistic. Abiding by the constraint that the only difference in logical form between 'Every $A$ is a $B$' and 'At least one $A$ is a $B$' is in the initial quantifier phrases, we need appropriate identical subquantificates, that is, one and the same [ - - - $x$ - - - ] for $(x, Ax)$[ - - - $x$ - - - ] and $(Ex, Ax)$[ - - - $x$ - - - ]. As mentioned earlier the solution consists of treating 'a $B$' as a restricted existential quantifier and 'is' as identity. 'Every A is a $B$' is represented as $(x, Ax)[(Ey, By)x = y]$ and 'At least one $A$ is a $B$' as $(Ex, Ax)[(Ey, By)x = y]$.

This solution as to what to provide as a common subquantificate for the **A** and **I** and for the **E** and **O** forms was initially suggested by an aspect of the descent to singulars doctrine of supposition theory. While Terminist logicians such as Ockham did not provide a special notation for quantifiers, they did have a firm grasp of some crucial features of its theory. The part of supposition theory known as descent to singulars is a special case of providing conjunctive and disjunctive expansions for generalizations. Universal generalizations (**A**- and **E**-form sentences) were correlated with conjunctions and particular generalizations (**I**- and **O**-form sentences) with disjunctions. Relating generalizations to these sentence connectives and then getting down to singular sentences parallels features of what is now referred to as compositionality.

The question arises as to how far the descent to singulars should proceed: exactly what kind of singular sentences are at the bottom of the descent? Which type will account for the grosser logical structure of the square and which for a full syllogistic?

For a full syllogistic the descent for '(Every $A$) is a $B$' does not stop at the singular sentences in the conjunctions:

This $A$ is a $B$ & That $A$ is a $B$ & $\cdots$.

While this aspect or step of descent might do for saving the square of opposition, it does not yield sufficient logical structure to account for a full syllogistic. For example, descent to such singulars will not explain the valid conversion of an **I**-form sentence. So, the descent-expansion of the **I** form,

(At least one $A$) is a $B$

to the expansion

This $A$ is a $B \vee$ That $A$ is a $B \vee \cdots$

will not justify the inference to the **I** form's converse:

(At least one $B$) is an $A$

having as its expansion/descent

This $B$ is an $A \vee$ That $B$ is an $A \vee \cdots$.

The needed additional logical structure is found in the 'is a $B$' component of 'Every/At least one $A$ is a $B$'. The 'is' in 'is a $B$' can be construed (using current predicate logic parlance) as '$=$' . The indefinite article 'a' in 'is a $B$' can be thought of in terms of the "existential"/particular quantifier and the expression 'a $B$' as a restricted particular quantifier, that is, '(At least one $B$)'. On the descent to singulars account of quantifiers ('a $B$', i.e., 'at least one $B$') would, so to speak, involve a descent to

this $B \cdots \vee$ that $B \cdots \vee \cdots$.

Putting together the singular form of the copula/identity and the restricted quantifier, that is, $=$(At least one $B$), we get descent to

$=$ this $B \vee =$ that $B \vee \cdots$.

Finally putting $=$(At least one $B$) in the scope of the overall initial restricted quantifier we have

(At least one $A$)[$=$(At least one $B$)].

(At least one $A$)[$=$(At least one $B$)] is an attempt at a rational reconstruction in contemporary predicate logic of the Terminist conception of the **I** form: 'At least one $A$ is a $B$'. So the descent to singulars Ockham would supply for the **I** form 'At least one $A$ is a $B$' is

(This $A$ is this $B \vee$ This $A$ is that $B \vee \cdots$.)
$\vee$ (That $A$ is this $B \vee$ That $A$ is that $B \vee \cdots$ )
$\vee \cdots$.

This solution, suggested by supposition theory, is given below in a predicate logic notation, accompanied by expansions/descent to singulars, to illustrate the logical relations involved.

**Every $A$ is a $B$ as $(x, Ax)[(Ey, By)x = y]$**

$$\begin{array}{lllllllll} & a^1 = b^1 & \vee & a^1 = b^2 & \vee & a^1 = b^3 & \vee & \cdots \\ \& & a^2 = b^1 & \vee & a^2 = b^2 & \vee & a^2 = b^3 & \vee & \cdots \\ \& & a^3 = b^1 & \vee & a^3 = b^2 & \vee & a^3 = b^3 & \vee & \cdots \end{array}$$

**At least one $A$ is a $B$ as $(Ex, Ax)[(Ey, By)x = y]$**

$$\begin{array}{lllllllll} & a^1 = b^1 & \vee & a^1 = b^2 & \vee & a^1 = b^3 & \vee & \cdots \\ \vee & a^2 = b^1 & \vee & a^2 = b^2 & \vee & a^2 = b^3 & \vee & \cdots \\ \vee & a^3 = b^1 & \vee & a^3 = b^2 & \vee & a^3 = b^3 & \vee & \cdots \end{array}$$

**Every $A$ is not a $B$ /No $A$ is a $B$ as $(x, Ax)[-(Ey, By)x = y]$**

$$\begin{array}{lllllllll} & a^1 \neq b^1 & \& & a^1 \neq b^2 & \& & a^1 \neq b^3 & \& & \cdots \\ \& & a^2 \neq b^1 & \& & a^2 \neq b^2 & \& & a^2 \neq b^3 & \& & \cdots \\ \& & a^3 \neq b^1 & \& & a^3 \neq b^2 & \& & a^3 \neq b^3 & \& & \cdots \end{array}$$

**At least one $A$ is not a $B$ as $(Ex, Ax)[-(Ey, By)x = y]$**

$$\begin{array}{lllllllll} & a^1 \neq b^1 & \& & a^1 \neq b^2 & \& & a^1 \neq b^3 & \& & \cdots \\ \vee & a^2 \neq b^1 & \& & a^2 \neq b^2 & \& & a^2 \neq b^3 & \& & \cdots \\ \vee & a^3 \neq b^1 & \& & a^3 \neq b^2 & \& & a^3 \neq b^3 & \& & \cdots \end{array}$$

One now has a full Aristotelian syllogistic and a counterpart in contemporary logic of aspects of Terminist supposition theory. It is a simple exercise in predicate logic to show the immediate inferences of conversion of the **I** and **E** forms and a full syllogistic as coded in the Barbara, Celarent, Darii, Ferio mnemonic lines. Obversion and inferences such as contraposition based on it are not valid. To illustrate some of these matters, consider the following trees for Darii and for the invalidity of converting an **A**-form sentence.

**Darii** Every $C$ is an $A$, Some $B$ is a $C$, so Some $A$ is a $B$. The tree closes:

| | | |
|---|---|---|
| 1. | $(x, Cx)[(Ey, Ay)x = y]$ | |
| √2. | $(Ex, Bx)[(Ey, Cy)x = y]$ | |
| √3. | $-(Ex, Ax)[(Ey, By)x = y]$ | |
| 4. | $(x, Ax) - [(Ey, By)x = y]$ | 3, Quantifier Interchange |
| √5. | $[(Ey, Cy)b^1 = y]$ | 2, E.I. |
| 6. | $b^1 = c^1$ | 5, E.I. |
| √7. | $[(Ey, Ay)c^1 = y]$ | 1, U.I. |
| 8. | $c^1 = a^1$ | 7, E.I. |
| √9. | $-[(Ey, By)a^1 = y]$ | 4, U.I. |
| 10. | $(y, By) - a^1 = y$ | 9, Quantifier Interchange |
| 11. | $-a^1 = b^1$ | 10, U.I. |
| 12. | $-c^1 = b^1$ | 8, 11, Identity |
| 13. | $c^1 = b^1$ | 6, Identity |
| | X | |

**A-form conversion** Every $A$ is an $B$, so Every $B$ is an $A$. The tree remains open:

| | | |
|---|---|---|
| 1. | $(x, Ax)[(Ey, By)x = y]$ | |
| $\surd$2. | $-(x, Bx)[(Ey, Ay)x = y]$ | |
| $\surd$3. | $(Ex, Bx) - [(Ey, Ay)x = y]$ | 2, Quantifier Interchange |
| $\surd$4. | $-[(Ey, Ay)b^1 = y]$ | 3, E.I. |
| 5. | $(y, Ay) - b^1 = y$ | 4, Quantifier Interchange |
| $\surd$6. | $(Ey, By)a^1 = y$ | 1, U.I. |
| 7. | $a^1 = b^2$ | 6, E.I. |
| 8. | $-b^1 = a^1$ | 5, U.I. |
| 9. | $-b^1 = b^2$ | 7, 8, Identity |

As traditionally stated, obversion consists of changing the quality of a proposition, for example, going from an **A** form to an **E** form; and then negating the predicate. If we take the negation of the predicate as internal negation, then the original and the obverse do not imply each other. Starting with an **A**-form sentence: $(x, Ax)[(Ey, By)x = y]$ and changing its quality by going to a corresponding **E**-form sentence, we obtain: $(x, Ax)[-(Ey, By)x = y]$. However, when we form the internal negation of the predicate, the final obverse appears as: $(x, Ax)[-(Ey, -By)x = y]$. Obversion of this sort is valid in one direction but not the other. $(x, Ax)[-(Ey, -By)x = y]$ does not imply $(x, Ax)[(Ey, By)x = y]$. Given that there are no unicorns, the negative **E** form ‘No unicorns are nonwhite’ can be taken as true and does not imply the false affirmative **A** form ‘All unicorns are white’.

Aristotle considered internal negation in connection with what came to be referred to as “infinite” or “indefinite” terms. In the *Categories* 13b, he discussed the relationship of ‘Socrates is sick’ to ‘Socrates is well’. ‘Socrates is well’ is to be understood as an affirmative proposition involving an internal negation, that is, ‘Socrates is [not-sick]’. Aristotle tells us that

> if Socrates exists one will be true and one false, but if he does not both will be false; neither ‘Socrates is sick’ nor ‘Socrates is well/[not-sick]’ will be true if Socrates himself does not exist at all.

With an expanded tree we can show that such an internal negation, ‘Socrates is [not-sick]’ (represented as $(Ex, -Sx)s = x$) implies its respective external negation ‘It is not the case that Socrates is sick’ (represented as $-(Ex, Sx)s = x$).

| | | |
|---|---|---|
| $\surd$1. | $(Ex, -Sx)s = x$ | The internal negation |
| $\surd$2. | $- - (Ex, Sx)s = x$ | The denial of the external negation |
| $\surd$3. | $(Ex, Sx)s = x$ | 2 |
| $\surd$4. | $(Ex)(-Sx \,\&\, s = x)$ | 1, $(Ex, \varphi x)Fx$ implies $(Ex)(\varphi x \,\&\, Fx)$ |
| $\surd$5. | $-Sa \,\&\, s = a$ | 4, Existential Instantiation |
| $\surd$6. | $(Ex)(Sx \,\&\, s = x)$ | 3, $(Ex, \varphi x)Fx$ implies $(Ex)(\varphi x \,\&\, Fx)$ |
| $\surd$7. | $Sb \,\&\, s = b$ | 6, Existential Instantiation |
| 8. | $-Sa$ | 5 |
| 9. | $s = a$ | 5 |
| 10. | $Sb$ | 7 |

| | | |
|---|---|---|
| 11. | $s = b$ | 7 |
| 12. | $-Ss$ | 8, 9 Identity |
| 13. | $Ss$ | 10, 11 Identity |
| | X | |

With another expanded tree we can show that the external negation does not imply the internal negation.

| | | |
|---|---|---|
| √1. | $-(Ex, Sx)s = x$ | The external negation |
| √2. | $-(Ex, -Sx)s = x$ | The denial of the internal negation |
| √3. | $(x, Sx) - s = x$ | 1 |
| √4. | $(x, -Sx) - s = x$ | 2 |
| 5. | $(x)(Sx \to -s = x)$ | 3, $(x)(x, \varphi x)Fx$ implies $(x)(\varphi x \to Fx)$ and $(Ex)\varphi x$ |
| √6. | $(Ex)Sx$ | 3, $(x)(x, \varphi x)Fx$ implies $(x)(\varphi x \to Fx)$ and $(Ex)\varphi x$ |
| 7. | $(x)(-Sx \to -s = x)$ | 4, $(x)(x, \varphi x)Fx$ implies $(x)(\varphi x \to Fx)$ and $(Ex)\varphi x$ |
| √8. | $(Ex) - Sx$ | 4, $(x)(x, \varphi x)Fx$ implies $(x)(\varphi x \to Fx)$ and $(Ex)\varphi x$ |
| 9. | $Sa$ | 6, Existential Instantiation |
| √10. | $Sa \to -s = a$ | 5, Universal Instantiation |
| 11. | $-Sb$ | 8, Existential Instantiation |
| 12. | $-Sb \to -s = b$ | 7, Universal Instantiation |
| | / \ | |
| 13. | $-Sa$ $\quad$ $-s = a$ | 10 |
| | X $\quad$ / \ | |
| 14. | $--Sb$ $\quad$ $-s = b$ | 12 |
| 15. | X | Open |

***2.3 Semantics—truth conditions*** The truth condition for basic canonical instances such as ‘$Fa^1$’, for example, ‘(This *A*) flies’, in nonvacuous cases is similar to those found in most logic texts for nonvacuous atomic sentences: ‘$Fa^1$’ will be true when there is an $a^1$ and it is an $F$ ($a^1$ is a member of the set ‘$F$’ has as its semantic value). As in full predicate logic the formal logic is neutral on empty/vacuous names. Different accounts of vacuous singular sentences will be considered in Section 3.

The truth conditions for generalizations are an adaptation to restricted quantifiers of the Mates method of beta-variants. Mates [12] provided an account of unrestricted quantifiers which relies on taking a given interpretation of a first-order formula and systematically reinterpreting it (such reinterpretations are called *beta-variants*) so that a given singular term is assigned different objects in different interpretations (beta-variants). A universal generalization is true when it is true for an instance on a given interpretation and remains true for every reinterpretation (every new assignment to that singular term). ‘$(x)Fx$’ is true if and only if an instance of it such as ‘$Fa$’ is true under every interpretation. The singular term, individual constant, ‘$a$’, is assigned different objects. Baldwin [3] pointed out that the substituend, the constant, functions like the demonstratives ‘this’ and ‘that’ of English in being used to refer to different objects. It is as if the instance ‘This is an $F$’ turns out true for any object you use the same singular expression ‘This’ to refer to. Note that it is the semantic (in the sense of truth conditions and the theory of reference) and not the pragmatic or contextual-indexical character of demonstratives that is being appealed to, that is, that ‘this’, the same expression, can be assigned different individuals and nonetheless is of the category of a singular term.

Relying on Mates’s method and Baldwin’s insight, take demonstrative noun phrases such as ‘$a^1$’/‘This $A$’ as playing a similar role with regard to restricted quantifiers as the constant ‘$a$’ does for unrestricted quantifiers but with the following crucial difference: the domain of objects is restricted to the set determined by the noun in the demonstrative noun phrase and the quantified noun phrase. Semantically ‘this man’ functions as a name would in picking out an object from the domain. So ‘(All men) are mammals’, that is, $(x, Ax)[(Ey, By)x = y]$ is true on the beta-variant account. We keep reinterpreting $a^1$, ‘This man’, in the sentence $[(Ey, By)a^1 = y]$ to refer to different objects in the set of men. The canonical instance $[(Ey, By)a^1 = y]$, ‘This man is a mammal’ is true on all such interpretations. ‘(Some man) is a mammal’ is true if and only if at least one beta-variant with respect to the set of men is a mammal.

The usual truth conditions for atomic sentences, for truth functional connectives, and for unrestricted quantifiers are assumed. With identity we run into some complications when we turn to empty nouns.[5]

1. A basic canonical sentence $Fa^1$ is true iff $a^1$ exists and it is a member of the val $F$ (i.e., val$(Ex)(x = a^1 \;\&\; Fx)$ is true).
2. val $(x, Ax)$ is true iff val $Fa^1$ is true under every beta-variant with respect to val $A$.
3. val$(Ex, Ax)$ is true iff val $Fa^1$ is true under at least one beta-variant with respect to val $A$.

In summary, a basic statement with a demonstrative noun will be true if the demonstrative noun has a referent and it is in the set that is the extension of the predicate. Restricted universal (particular) generalizations are true when a canonical instance is true on all (some) of its beta-variant interpretations. Canonical instances of a basic affirmative type, as well as both universal and particular generalizations whose subquantificates are basic predications, are false when the noun is vacuous.

Consider the following examples of how these truth conditions are applied. A sentence such as

Every donkey is a mammal

is true as is its conjunctive analogue,

This donkey is a mammal and that donkey is a mammal and so on,

and so are canonical instances such as

This donkey is a mammal.

Since all the beta-variants of the generalization are true, the generalization is true. In

Section 3 we examine generalizations with vacuous nouns such as Aristotle's goat-stag case or our own 'All purple donkeys are mammals'.

Though there is disagreement about Aristotle's inclusion of singular sentences in his syllogistic, the Aristotelian tradition did later include singular premises and singular conclusions. These can be easily accommodated by treating a sentence such as 'Socrates is a man' as

$$(Ex, x \text{ is a man})\, x = \text{Socrates}.$$

We can even use the above account to justify the treatment of Aristotelian logic found in traditional logic texts which rely on rules of quantity (distribution) and quality (affirmative-negative). The restricted quantifiers in this account of **A**, **E**, **I**, **O**-form sentences capture the notion of a term's being distributed found in such traditional treatments of syllogisms, that is, distributed terms are those nouns/predicates serving as restrictions on restricted universal quantifiers (or those equivalent to restricted universal quantifiers). One might argue that the traditional rules of quantity and quality for testing a syllogism are a decision procedure. The rules of quantity (the middle term must be distributed at least once and a term distributed in the conclusion must be distributed in the premises) summarize quantifier principles (you can't derive a common instance from two existential generalizations and you can't derive a universal generalization from an existential one). The rule of quality: only one premise can be negative and then the conclusion must be negative as well, can be justified by considering the combinatorial possibilities of the identities (affirmative) and inidentities (negative) involved in categorical syllogisms when the categorical sentences are represented as above. That is, if the premises contain identities and the rules of quantity are in force, then the conclusion must involve an identity; and if the premises express an inidentity, it can only be in one premise and then the conclusion must state an inidentity.

***3 Aristotelian free logic*** Given the approach of the previous section it is not difficult to see how Aristotle's syllogistic can serve as a free logic—a logic free of existence assumptions. The matter comes down to the fact that the Aristotelian system is a special case of the more general predicate logic case. In both of these it is a question of accommodating empty nouns and the treatment of singular sentences with empty names. It is customary to classify the different ways of treating such sentences as the negative, positive, or neuter solution. On the negative solution all such vacuous singular sentences are false, on the positive solution some such sentences are true, and on the neuter solution they are neither true nor false. As in the more general predicate logic it is not a purely formal matter which of these solutions is adopted. In other words, the formal apparatus such as the tree rules for the quantifiers are neutral in the sense that it is an open question in choosing between the positive, negative, and neuter solution. Arguments of a more philosophical sort are invoked. Thus, Meinongian arguments might be advanced for treating 'Pegasus is a flying horse' as true or anti-Meinongian arguments for treating that sentence as false or neither true nor false. In considering free Aristotelian logic we have the entire history of philosophy to rummage through. Terminist logicians such as Ockham and Buridan held to the negative solution and there is evidence that Kant did, too. Leibniz held an interesting version of the positive solution while Strawson, object dependent theorists, and Van Fraassen provide the basis for versions of the neuter solution.

It is necessary at this point to interject some remarks on the expression 'free logic'. Originally the phrase was used to refer to logics free of existence assumptions and in particular allow for the presence of empty/vacuous nouns. Most people still use the phrase in this way. However, it has been proposed that the phrase be used in a narrower way ([10], pp. 105, 114–15; [4], pp. 376, 379) for logics that are free of existence assumptions (in that they allow for empty names) and in addition that interpret the $(Ex)$ quantifier as having existential force. Since neither Aristotle himself nor traditional Aristotelian logic treated sentences which can in predicate logic translations be governed by the $(Ex)$ quantifier as having existential force, by definition Aristotelian logic could not be a free logic in this narrow sense. Indeed, if Łukasiewicz is right in saying that Aristotle did not have quantifiers, then the narrow use of 'free logic' seems to rule out in an uninteresting way the question of Aristotle's logic being free of existence assumptions. On this narrow use proponents of substitutional quantification who forgo reading $(Ex)$ as having existential force and those following Lesniewski in according existential force to the copula and not the quantifier (a position that strikes the present author as firmly entrenched in an Aristotelian tradition) are said not to be 'free logics' even though they accommodate empty names. Let us explicitly put aside this narrow use of 'free logic'.

***3.1 The negative solution*** To illustrate the negative solution consider the following generalization with a vacuous noun phrase.

> All purple donkeys are mammals.

It is false on the negative solution as is its conjunction analog,

> This purple donkey is a mammal and that purple donkey is a mammal, etc.,

since a canonical instance such as

> This purple donkey is a mammal

is false. This false substitution instance makes the generalization false.

Parsons presented the following problem. Consider the true sentence, the universal generalization:

> All donkeys are mammals.

Given the following purportedly false "instances" it turns out false:

> (a) Alex's donkey is a mammal.

(a) is false, since it contains a vacuous noun, that is, Alex has no donkey.

(b) Browny is a mammal.

(b) is false. 'Browny' is a name for Alex's donkey, so this vacuous singular sentence is false.

Parsons's argument is that since either of these two instances, (a) or (b), is false, the true universal generalization would on the above account be false as well.

The reply to Parsons is that neither (a) nor (b) are canonical instances of the generalization in question.

(a) does not contain a canonical instance of the generalization in question. Instead it is an instance of the quantified noun generalization

All of Alex's donkeys are mammals

which is false for the same sort of reasons applying to the purple donkeys case.

With regard to (b), names (vacuous and nonvacuous) are not canonical instances of quantified noun generalizations. Names, vacuous or not, can occur as instances of unrestricted generalizations:

Everything is a donkey

is false given the falsity of (b).

If anything is a donkey then it is a mammal

is a true generalization on a conditional. Given the usual truth functional account of conditionals, the false (b) would be involved as a false antecedent of a true conditional serving to make the universal conditional true.

It may be of interest to consider anew from the perspective of the negative solution the familiar argument from

Every horse is an animal: $(x, Hx)[(Ey, Ay)x = y]$

to

Every head of a horse is a head of an animal:

$$(x, Ez(Hxz \,\&\, Hz))[(Ey, Ez(Hyz \,\&\, Az))x = y].^{6}$$

It is frequently cited as an example of a valid argument that cannot be represented as valid in Aristotle's syllogistic. Given the negative solution, figures such as Ockham and Buridan could reply that it is not valid. They might ask us to consider the following analogous reasoning leading from a true premise to false conclusions:

Every horse is an animal;
so every wing of a horse is a wing of an animal (every human sibling of a horse is a human sibling of an animal).

A different but somewhat similar argument from a premise which is an unrestricted quantification with a conditional component:

If anything is a horse, then it is an animal

to the conditional-like conclusion:

If anything is a head of a horse, then it is a head of an animal is valid.

To a great extent the issues concerning free Aristotelian logic can be addressed by focusing on two of Łukasiewicz's axioms, his *laws of identity*:

Every $A$ is an $A$

and

At least one $A$ is an $A$ ([11], p. 86).

Terminist negative solutions deny these laws of identity. Their status is questioned in connection with the sophism: A chimera is a chimera. In his treatise on sophisms, Buridan begins by presenting a case for that sentence being true, then provides a rebuttal, and after that supplies a principled account of the negative solution. The case for the sophism consists of citing Boethius's claim that "no predication is more true than that in which the same thing is predicated of itself" as evidence for the sophism ([5], p. 84). The Boethian maxim is of a piece with Łukasiewicz's two laws of identity. The terminist-negative solution of authors such as Ockham, Buridan, Albert of Saxony, and the Pseudo Scotus denies that all/some chimera are chimera. On the descent to the singular view they would deny as well that this chimera = this chimera. All singular sentences with vacuous terms are false and identity claims with empty singular terms are no exception. From the standpoint of contemporary logic these denials of self-identity are counterexamples to the total reflexivity of identity: $(x)(x = x)$. Denying total reflexivity is not without precedent in contemporary logic. It is denied in Lesniewskian approaches to the copula and identity. In place of total reflexivity we adopt a weaker principle of partial reflexivity of identity. It might be formulated as $(x)[(Ey)(y = x) \to x = x]$ or as $(x)(x \text{ exists} \to x = x)$ or $(x)(Ef)(fx \to x = x)$ where $f$ is schematic for a basic type of predicate.

When we do a tree for Łukasiewicz's laws of identity and do not appeal to the total reflexivity of identity, they are refuted. In the case where $A$ is empty, the tree remains open for the denial of the "laws."

| | | |
|---|---|---|
| 1. | $-(x, Ax)[(Ey, Ay)x = y]$ | |
| 2. | $(Ex, Ax) - [(Ey, Ay)x = y]$ | |
| $\surd$3. | $-[(Ey, Ay)a^1 = y]$ | 2, E.I. |
| $\surd$4. | $(y, Ay) - a^1 = y$ | 3, Q.I. |
| 5. | $-a^1 = a^1$ | 4, U.I. |

The negative solution has quite a history. It has been argued for by invoking principles such as *nihili nullae proprietates sunt* (Nothing [what does not exist] has no properties) ([13], p. 345) and *non entis nulla sunt predicata* "all that is asserted of the [nonexistent] object, whether affirmatively or negatively, is erroneous" ([9], p. 627).[7] These provide the basis for saying that a singular sentence is false when its subject term is vacuous. Since generalizations such as affirmative categorical sentences rely on such singulars they, too, are false. In a well-known passage, Kant appeals to such a principle in rejecting a positive solution for sentences such as those involved in versions of the ontological argument where the predicate has or gives the meaning of the subject term. "If, in an identical proposition, I reject the predicate while retaining the subject, contradiction results . . . . But if we reject subject and predicate alike, there is no contradiction . . . . If its [the subject's] existence is rejected, we reject the thing itself with all its predicates."

One can follow this tradition and provide a free logic along the lines of the negative solution. We adopt a truth condition according to which the semantic value of a singular term is an existent object and the associated singular sentence is true when the predication is true of that object. The singular sentence is false when the object does not exist. It can be argued that this is part and parcel of realist theories of truth. Arguments for holding this view are not matters of formal logic per se. Competing approaches are all feasible solutions from the standpoint of formal logic. The issues in choosing among these alternatives are philosophical or semantic, such as among allowing Meinong-like objects, choice of truth vehicle, or allowing truth-value gaps.

***3.2 Positive solutions*** To achieve the result that Łukasiewicz saw in Aristotle of having the Boethian laws of identity as theses of the system we can change our stance and provide a positive solution to the vacuity cases. On one extreme positive solution all vacuous, as to existence, singular sentences are true. If all such vacuous singular sentences are true, there is no violation of the total reflexivity of identity. We can adopt $(x)(x = x)$ and its tree rule correlate as our tree rule (i.e., any branch containing $-a = a$ closes) and the above tree closes. Besides appearing to require Meinong-like nonexistent objects as semantic values, this seems too extreme in allowing that 'All/Some chimera are chimera' by sanctioning accepting as true both that 'All/Some chimera are black' and that 'All/Some chimera are white'. If we do not want all such vacuous sentences to be true, we need some principled way of distinguishing the true from the false sentences. Leibniz had an interesting suggestion. He maintained that nonexistents still have their essential properties. Accordingly 'This chimera is white' would be false (assuming whiteness is not an essential property) but 'This chimera is this chimera' would be true assuming that self-identity is an essential property. Thus 'All/Some chimera are chimera' would be true. However, this more conservative positive solution and defense of the Boethian laws would also require something like Meinongian objects. These nonexistent objects would be needed to serve as the semantic values/referents of the vacuous terms involved.

***3.3 Neuter solutions*** A non-Meinongian neuter line of investigation was offered in a communication from Strawson. Whereas he approved of the formal way the square of opposition is saved, he preferred his own treatment of cases involving vacuity. It can serve to fit in with Łukasiewicz's, Boethius's, and possibly Aristotle's intentions. For Strawson sentences such as 'All winged horses are white', 'Some winged horses are white', and 'This winged horse is white' are grammatical and meaningful but they do not yield truth vehicles. Recall that for Strawson, sentences with vacuous nouns are meaningful and express propositions. Given the failure of existential presupposition, these sentences are not used to make statements (statements are the truth vehicles—not sentences or propositions). (A parallel but stronger line might be taken by those who follow Evans or Kaplan and hold object dependent views: that vacuous singular sentences are meaningless and do not express propositions.)

There are two complementary ways of bringing a Strawsonian approach to bear. The first treats vacuous singular sentences as not yielding truth vehicles and then argues that strings formed from them by the use of 'and' and 'or' also do not yield truth vehicles, that is, "genuine" conjunctions and disjunctions. Such strings are merely grammatical, combinatorial possibilities without any genuine conjunctive or disjunctive statement force. Thus, the string 'This winged horse is white and that winged horse is white' is not used to make a statement, a conjunctive one. Next, by maintaining the all/some-and/or analogies (a variant of the doctrine of descent to singulars) one can argue that there are no such truth vehicles for such strings as 'All winged horses are white' and 'Some winged horses are white' which merely purport to be genuine generalizations.

The same effect can be achieved without appealing to the expansions or descending to singulars by arguing that genuine generalizations (general statements) require instances which are genuine statements. On the Strawsonian view merely grammatical possibilities such as 'This flying horse is white' do not provide us with statements. Generalizations which involve or presuppose such instances are merely grammatical combinations and do not result in genuine general statements. We thereby save the Boethian principle since neither 'A chimera is a chimera' nor its singular descendants, such as 'This chimera is a chimera', furnish truth vehicles for denying $(x)(x = x)$.

One might also examine this matter by treating vacuous singular sentences in terms of truth-value gaps or many-valued logics. On Van Fraassen's supervaluation view there are, contra-Strawson, truth vehicles, but they do not have a truth-value. (One might even adopt some extra truth-value, as did Kripke on truth, so long as one maintained classical principles of logic and one did not abandon classical theorems of predicate logic.) As far as I can see, on the supervaluation treatment, a vacuous singular identity claim, even of the form $a = a$, would be neither true nor false since on some valuations it might be valued as true and on some as false. Consequently, since there would not be a common truth-value, such identity claims, and hence '$(x)(x = x)$' as well as 'All/some chimera are chimera' would be neither true nor false.

As a final alternative, consider Charles's account in his "Aristotle on names and their signification" [6]. Charles offers an approach motivated, in the main, to give an accurate account of the historical Aristotle's views on names/nouns. In so doing he offers an interesting treatment of vacuous names/nouns. The material relevant to the present essay can be summarized as distinguishing surface grammatical form from logical form. Sentences, both categorical as well as singular sentences containing "improper" nouns, that is, vacuous nouns, do not wear their logical form on their sleeves. Where nouns (singular or common) occur in sentences and do not "refer" to exactly one thing, some other logical form should be assigned to the sentence than that suggested by its surface grammar. Vacuous nouns are improper nouns. The situation is broadly similar to Russell's on improper definite descriptions. It is as though Charles is generalizing Russell's strategy for empty proper nouns to all nouns. Sentences whose surface grammatical form is that of singular or categorical sentences containing vacuous nouns have a different and more complex logical form. Putting aside the question of what the complex logical forms are that will replace the defective surface forms, the upshot for the vacuous cases we are concerned with is clear enough. There will not be vacuous truths of the form: $-a = a$; functioning as counterexamples to the total reflexivity of identity.

***4 Additional applications*** For ease of introduction and reasons bearing on the history of logic the theory was presented for the four sentences of Aristotelian logic. Its scope is broader. It is an account of restricted quantifiers as per quantified and demonstrative noun phrases, and the theory applies to whatever inferences involve them. So, from the fact that this cat likes that man, that is, $Lc^1m^1$, it should follow that at least one cat likes at least one man, that is, $(Ex, Cx)(Ey, My)Lxy$. That at least one cat likes something should follow, too, that is, $(Ex, Cx)(Ey)Lxy$. These as well as other inferences involving both restricted and unrestricted quantifiers should be dealt with. Another topic is that of restricted quantifiers with complex structures, for example, $(Ex, \text{--}x\text{--})$ where $\text{--}x\text{--}$ is not a simple predicate—as in 'Some healthy athletes': $(Ex, Hx \;\&\; Ax)$. Principles are needed to explain valid inferences such as: (Some healthy athletes) eat a hot breakfast, so (some athletes) eat breakfast.

In a like manner modal functors can be components of complex restrictions. Patterson has argued that besides the de re, de dicto, and other modal distinctions needed to explain Aristotle's modal logic, we must take account of two ways of modalizing the copula. By using modal restrictions in the quantifers we can capture Patterson's and Aristotle's thoughts.

The strong form of modalized copula is given in the example

Every triangle (is necessarily) three sided.

The weak form of modalized copula is given in

Every figure drawn on the blackboard (is necessarily) three sided.

In the strong form there is an essential connection between the property of being a triangle and that of being three-sided. In the weak form the connection is not one of an essential connection between being a figure drawn on a blackboard and being three-sided. Let it be given that the only figures drawn on the blackboard are in fact triangles. While qua figures drawn on the blackboard they are only accidentally three-sided, as triangles they are necessarily three-sided. Aristotle's own example of the weak form is that all the white are animals, where the white things are human.

Patterson calls our attention to several different inferences. Among these are conversions involving the strong and weak modalizations ([17], p. 16). Conversions with weak modalizations fail for **A**, **I**, **E**, and **O** forms. Consider conversion of the **I** form with a weak modalized copula:

$$(Ex, Ax)(Ey, \text{Nec } By)x \neq y$$
$$(Ex, Bx)(Ey, \text{Nec } Ay)x \neq y.$$

Its invalidity is a case of going from a contingency in the restricted quantifier in the premise, $Ax$, to a necessity in a restricted quantifier in the conclusion, Nec $Ay$. The strong modalized copula converts for **A** (by limitation), **I**, and **E** forms as in

$$(Ex, Ax)(Ey, By)\text{Nec}\, x = y$$
$$(Ex, Bx)(Ey, Ay)\text{Nec}\, x = y.$$

Another topic the approach applies to is quantifiers in natural language. Several philosophers, logicians, and linguists are not happy with the usual construal of natural language quantifers. When **A-** and **I-**form sentences are represented as unrestricted quantifications over conditionals or conjunctions they exhibit little relation to other quantifiers in English such as 'few $A$' or 'a good many $A$'. Evans [8], Davies [7], and Wiggins [19] offer accounts of quantifiers in terms of binary quantifiers. The logical form assigned to 'All/some/few $A$ are $B$' is $(Qx)(Ax, Bx)$, where $Qx$ is a quantifier and unlike the unrestricted quantifier approach, the quantifier applies to two open sentences. Others such as Bach [2] and Neale [14] advocate restricted* quantifiers. The logical form assigned to 'All/some/few $A$ are $B$' is $(Qx)Bx$. (On the surface such restricted* quantifiers appear to be the same as those appealed to in this paper.) Both the binary and the restricted* quantifier treatments are based on the theory of generalized quantifiers and associated set theoretical concepts. Both also claim two advantages for their approach over the usual unrestricted quantifier treatment: (1) being closer to the surface grammar of natural language quantifiers and (2) affording insight into other natural language quantifiers, that is, plural quantifiers such as 'most men', 'many butterflies', 'few females'.

Binary and restricted* treatments as well as the approach offered in this paper observe a constraint for conforming to surface grammar: that the only difference in logical form assigned to generalizations such as 'All $A$ are $B$', 'Some $A$ are $B$', 'Most $A$ are $B$' should be in the quantifier position. Binary $(Qx)(Ax, Bx)$, restricted* $(Qx)Bx$, and restricted quantifiers $(Qx)Bx$ honor this constraint. The usual Fregean unrestricted account does not abide by this constraint: the subquantificates for an **A-** and an **I-**form sentence differ.

Whereas the binary and restricted* approaches meet this condition, there is not much difference between them and the Fregean unrestricted approach. Given the set theoretical background of generalized quantifiers both the binary and the restricted quantifier* approaches treat 'All $A$ are $B$' as $A = A$ intersection $B$. Understood in this way, an **A** form says that the intersection of the set $A$ and the complement of the set $B$ is empty. This is just another way of saying that $A$ is included in $B$. Such inclusion is rendered on the usual Fregean account as the familiar universal generalization over a conditional: $(x)(Ax \rightarrow Bx)$. Parallel remarks apply to the treatment of **I-**form sentences, treated as: $A$ intersection $B \neq$ the null set, which turns out to be $(Ex)(Ax \,\&\, Bx)$ when put in predicate logic clothing. Binary and restricted quantifiers* differ on categorical sentences from the usual treatment of restricted quantifiers only in a digression through generalized quantifiers.

It is also claimed by some proponents of the binary quantifier and unrestricted/ generalized quantifier approaches that they are superior to the unrestricted approach in giving insight as to the logical form of plural quantifiers such as 'most women', 'several butterflies', 'few men'.[8] How much of an advantage is this over the approach offered in this paper?

Before answering this question, it is necessary to indicate some confusion in an argument which binary and restricted* theorists offer. They argue that in using unrestricted quantifiers one cannot provide a connective that will do the job of plural quantifiers. That is, if the conditional connective is appropriate for universal sentences such as the **A-**form one and the conjunction connective for the **I-**form one, then we are at a loss to say which connective to use for sentences like 'Few men are brave'. This objection is confused, since the goal of staying closest to surface grammar indicates that it is not a question of an appropriate connective but of the quantifier phrase. If we are interested in staying as close as possible to the surface grammar of the English sentences the key observation should be honored: that the only differences in logical form among

‘All *A* are *B*’,
‘Some *A* are *B*’,
‘Few *A* are *B*’,
‘A good portion of *A* are *B*’, and so on

should be in the quantifier phrases and not in any connectives.

The differences among ‘All *A*’, ‘Some *A*’, and ‘Few *A*’, as in other quantifiers and logical constants generally, lies in appealing only to what is essential to those constants when giving their truth conditions (see [16], pp. 102–3). This can be, and is frequently, achieved by using the same logical constant in the analysans-explicans side of the truth condition as is being explicated as in:

‘*p* and *q*’ is true iff ‘*p*’ is true and ‘*q*’ is true.

With the method of beta-variants we can achieve the desired effects for plural quantifiers. Why not say that ‘Few *A*’s are *B*’ is true if and only if few beta-variants with respect to *A* are *B*. The two goals of binary quantifiers and of restricted quantifiers (being closer to surface grammar and giving a uniform treatment of plural and non-plural quantifiers) are salutary and they are achievable along the lines proposed in the body of this essay.[9]

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## NOTES

1. Westerstahl [18], (pp. 6, 8) makes the interesting suggestion that Aristotle’s account of these sentences constitutes an anticipation of relational/binary quantifiers. Unfortunately the account Westerstahl offers has categorical sentences functioning much the same as they do on the Fregean approach where ‘All *A* are *B*’ is true if and only if *A* is included in *B*.

2. The logical constants in the above conditionals are the quantifiers and the demonstratives. That these quantifiers are logical constants should be no surprise (they are continuous with unrestricted quantifiers and maintain duality). Demonstratives are similar to other logical constants in that they are topic neutral. ‘This/that’ are ways of forming a singular term (demonstrative noun phrase) from any predicate. They also feature as natural language counterparts of the Beta-Variant technique that can be used to explain other logical constants.

3. Many studying predicate logic have noted that ‘All *A* are *B*’ does not quantify over everything unrestrictedly; nonetheless, this bit of linguistic intuition/insight is put aside. Some texts contain incredibly poor arguments for the unrestricted conditional, for instance, citing a sign warning that all trespassers are (or will be) prosecuted. However, the

warning has some sort of prescriptive force-deontic status which is irrelevant to making a conditional claim. Moreover, there are numerous cases where the warning is violated, the trespassers are not prosecuted, and we don't say the remark 'All trespassers are (or will be) prosecuted' is false or somehow refuted, but merely in some sense violated.

4. We can supplement the instantiation rules for the unrestricted universal quantifier. The supplement consists of allowing in addition to the usual instances, $a, b, c$, restricted instances: $a^1, b^2$. So an unrestricted universal generalization can have as an instance a restricted instance: $(x)Fx$ yields $Fa$, $Fb^2, \ldots$. A parallel rule for unrestricted existential instantiation is not forthcoming, since 'Something is a $B$' does not imply 'Some $A$ is a $B$'. An unrestricted particular *existential* generalization can in systems containing a rule of unrestricted existential generalization be derived from an instance containing a restricted instance such as: $Fa^1$ therefore $(Ex)Fx$. Other relations between the nonsyllogistic part of predicate logic and the syllogistic part should be considered.

5. For nonvacuous singular terms the usual condition for identity: $\mathrm{val}\, a = b$ is true iff $\mathrm{val}\, a = \mathrm{val}\, b$, obtains. Identity claims containing vacuous singular terms can be treated as false, for example, 'This flying horse = this flying horse', 'Pegasus = that flying horse'. For these cases, in place of the total reflexivity of identity: $(x)(x = x)$, we adopt partial reflexivity: $(x)((Ey)y = x \rightarrow x = x)$ and a tree rule to go with it, namely, a branch closes which contains $(Ey)y = x$ (or $Fx$ where $Fx$ is a basic context) and $-x = x$ as lines. See Section 3. Leibniz's law, $a = b$, $Fa$ therefore $Fb$, is fine as is.

6. Kirwan offered another way of symbolizing this sentence which exhibits more logical structure. It has restricted quantifiers within restricted quantifiers:

$$(x, (Ez, Hz)Hxz)[(Ey, (Ez, Az)Hyz))x = y].$$

7. What Kant means by "asserted negatively" is that the negation is internal negation. The sentence in question is a singular sentence with an internal negation and is not an external negation of a singular sentence. The Russellian treatment of empty names has the same negative effect. Such a sentence containing an empty name involving an internal negation is treated on Russell's theory as false. Scope distinctions for negation achieve similar results as having an internal/external negation distinction for singular sentences.

8. Wiggins and Neale are careful in pointing out that plural quantifiers could be dealt with as variations of standard Tarskian accounts of quantifiers. I am indebted to Stephen Neale for calling this point to my attention (see [14], p. 47 and n. 49).

9. Segal has remarked that claims made in this paper, such as for a full square of opposition, etc., could be duplicated with generalized quantifiers. If this is correct, then that is good news. However, why appeal to generalized quantifiers and set theory when the same results can be accomplished on a more modest predicate logic basis with its established track record of being shown consistent and complete? In addition, I have seen no proof or refutation procedures (such as those provided in this paper) given for the generalized quantifier approaches.

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