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MID-CENTURY IN SEATTLE

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1. Introduction. This is a view of algebraic activity centering around abelian group theory, particularly the theory of torsion-free groups, at the University of Washington roughly during the period from the 1950's to the mid 1960's. Hence, the title. Any such account naturally focuses on two people, R.A. Beaumont and R.S. Pierce, each of whom as it happens has been the subject of recent review (cf. [**13**], [**15**]). To keep within suggested space limits, we take the liberty of referring generously to these accounts, which therefore might be consulted in conjunction with our remarks here. We emphasize that this is in the nature of a shapshot – that large areas, e.g., work on p-groups, are not addressed. Complete bibliographic references may be found in [**13**] and [**15**].

We also adopt a somewhat personal point of view and our comments will reflect to a large extent our own experience. However, we hope the report is broad enough to give a reasonable idea of what was going on, who the major players were and what some of the accomplishments were. We hope also to give some indication of the influence of that work on those of us who were there at the time.

2. People (faculty). The algebraists on the faculty at the time were Ross Beaumont, James Patrick Jans, Ronald Nunke, Richard S. Pierce and John H. Walter.

Of these, Beaumont, Pierce and Nunke are perhaps the ones we know best in the abelian group community.

Beaumont was the senior algebraist, having come to the University in 1940. He received his degree working under Reinhold Baer at the University of Illinois and was Baer's first American student after his, Baer's, arrival from Germany. He was, whether we were his thesis students or not, a central figure in our experience. Friendly, easy-going, encouraging and with an engaging sense of humor, he was a wonderful

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asset to the mathematical community at the time.

Jans worked in the representation theory of rings, having written a thesis (1955) under R.M. Thrall at the University of Michigan. He was particularly interested, as I remember, in the indecomposable representations of algebras. We recall his popular monograph, *Rings and Homology*, and its characterization of semi-simple rings ("everybody splits").

Nunke came to the University of Washington from a post-doctoral position at Yale, after receiving his Ph.D. (1955) at the University of Chicago under Saunders MacLane. He brought a breath of homological algebra to the department which was, I recall, an exciting development to me. He was always willing to share his deep insight and knowledge.

Walter worked on "real groups", that is, nonabelian finite groups, and made a major contribution to the classification of the simple groups. I remember fondly his inimitable teaching style which I experienced in the introductory algebra course as a beginning graduate student. However, he left Seattle around 1960 to accept a position at the University of Illinois.

Rounding out the group was R.S. Pierce who came to Seattle in 1955 from Harvard where he had been a Jewett Fellow working with Birkhoff. Between his Ph.D. at Cal Tech under Dilworth in 1952 and the Harvard appointment, Pierce spent a post-doctoral year (1952–1953) at Yale on an ONR research appointment, attracted there I believe because of Ore. (Just ten years later it was my pleasure to spend a year at Yale under the same auspices.) Pierce was the algebraic sparkplug at the time. His knowledge was broad and very deep and he taught a wide variety of courses. His Ph.D. students ranged in topics from lattice theory and Boolean algebra through abelian groups to commutative rings.

Beaumont, Jans and Nunke spent their entire careers at the University of Washington. Pierce served on the faculty there until 1970 when he accepted a position at the University of Hawaii. In 1975 he moved to the University of Arizona where he served until his retirement from teaching in 1991.

I might mention a few of the non-algebraists who were at the University at the time, and who had a great influence on some of us through their professionalism, teaching and in many cases, friendship.

The senior analyst at the time was Edwin Hewitt, whom we all knew, revered and perhaps feared a bit.

There were Victor Klee, Ernie Michael, the probabilist Ron Getoor, Jake Bear and the Chairman Carl B. Allendoerfer, among of course many others.

On a personal note, I remember H.S. Zuckerman particularly as being, in my opinion, a genuine mathematical artist. It is hard for me to classify him by field, but some might have called him an analytic number theorist. We are probably all familiar with the classic text on number theory by Niven and Zuckerman.

3. People (students). Listed later are all the graduate students who received degrees from these algebraists around this time. But first I recall a few of those with whom most of us are familiar, together with a few small details about them.

Bob Wisner, now at New Mexico State University, is the first who comes to mind. He wrote a thesis in 1953 with Beaumont on rings supported by a given abelian group structure. There will be more on this below.

Among those working in abelian groups, I was probably next chronologically, working with Pierce in 1960 on an extension of the quasi-isomorphism invariants for the quotient divisible groups that Beaumont and Pierce had defined in their influential paper on torsion free rings (1961). It was an honor and a pleasure for me to have worked with him.

Among other students of Beaumont in abelian group theory were Jim Armstrong, Frank Cornelius and Bill Wickless, all of whom worked on endomorphism rings or aspects of quasi-endomorphism algebras. In addition, Phill Schultz wrote a thesis under Beaumont, but on a problem suggested by Pierce, involving endomorphisms in an essential way. There is more on this below.

Schultz and Wickless in particular have made many important contributions in subsequent years, some of which I touch on below.

Robert Stringall, Frank Castagna and John Werth, the latter now a well-known computer scientist, wrote theses with Pierce on

p-groups and their endomorphism rings. And there is the distinguished commutative ring theorist Roger Wiegand, also a student of Pierce.

Again in the Beaumont column, we have Eugene Cornelius who proved "quite an astonishing result" on separable groups, in the words of Goebel (Periodica Math. Hung. **32** (1996)); John Koehler who was one of the first to study the type sets of torsion free abelian groups; Charles Murley, of "Murley group" fame; and James Clay who became a leader in the study of near rings.

Jans had many students, 13 in all, but of course most did ring theory and related work. One, however, who joined our community is Charles Vinsonhaler. Chuck joined the faculty of the University of Connecticut (1968), recruited Wickless there and began working on abelian groups. The rest is history, as they say.

Finally, Ron Nunke had just one student according to my informants. He is Ray Mines of New Mexico State University. Ray has become a prominent contributor to our field, as well as the field of constructive mathematics, among others.

In addition to these students in algebra there were strong contingents working under Hewitt and under Klee. Hewitt students W.W. Comfort, K.A. Ross and Karl Stromberg, together with R.R. Phelps (Klee), come to mind. They along with Pierce students Dorothy Christensen and Gloria Hewitt among many others contributed immensely to the mathematical atmosphere in my time.

So these are some of the people who passed through Seattle around mid-century, more or less.

4. The degrees. Here we list the students who received their Ph.D. in Mathematics in algebra from either one of Beaumont, Jans or Pierce. As noted, Nunke's only student was Ray Mines. The students of Jans worked mainly in ring theory, and many of Pierce's students worked on aspects of lattice theory, Boolean algebra or other areas. However, to share the spirit of the time, we list them all.

Beaumont

John Byrne (1953)	James Clay (1966)
Robert Wisner (1953)	William Wickless (1967)
Burnett Toskey (1959)	Phillip Schultz (1968)
Stephen Tellman (1960)	George Williams (1968)
Paul Yearout (1961)	Frank Cornelius (1969)
James Armstrong (1962)	Charles Murley (1970)
John Koehler (1962)	

Jans

Horace Mochizuki (1963)	William Topp (1968)
Ling-Erl Wu (1964)	Charles Vinsonhaler (1968)
Robert Colby (1965)	Jerry Williams (1968)
Stanley Page (1966)	Stuart Seligson (1969)
Denis Floyd (1967)	M. Mohamed-el-Mahdi Abdeljaouad (1970)
D. George McRae (1967)	Lawrence Lindley (1971)
Robert Kurshan (1968)	

Pierce

Dorothy Christensen (1958)	John Werth (1968)
Richard Mayer (1958)	Joel Berman (1970)
James D. Reid (1960)	John Muth (1975) (University of Hawaii)
Gloria Hewitt (1962)	Eliot Jacobson (1983) (University of Arizona)
Robert Stringall (1965)	Ahmed Hadida (1988) (University of Arizona)
Frank Castagna (1967)	
Timothy Cramer (1967)	
Roger Wiegand (1967)	

All degrees were from the University of Washington except where noted.

This strikes me as being a remarkably productive period for these few mathematicians and it serves to illustrate the dynamic atmosphere in place at the time. It seems an interesting coincidence that Beaumont, Pierce and Jans each had exactly 13 students.

5. Problems and results. One of the main themes in Beaumont's early work was the study of the various ring structures that a given additive abelian group supports. (See [**13**] and [**15**] for bibliographic details on the papers mentioned here.) The work with Zuckerman (1951) on additive subgroups of the rationals, with Wisner (1959) on rings with additive group of rank 2, and his own earlier work (1948) on rings whose additive groups are direct sums of cyclic groups reflect this. The paper with Wisner contains seeds of the important later work of Beaumont and Pierce on torsion free groups of rank two (1961). The joint work of Beaumont and Pierce on torsion free rings (1961) and on subrings of algebraic number fields (1961) reflect in varying degrees this interest.

For a discussion on the genesis of the idea of quasi-isomorphism and the role that Beaumont and Pierce played in its development, see [**13**]. There the case is made, we think, that the work of Beaumont and Pierce has had a profound effect on the development of torsion free abelian group theory and a definitive effect on the theory of torsion free rings. The paper on torsion free rings (1961) contains a deep analysis of the interplay between the structure of a ring and that of its additive group resulting, for example, in the discovery of the quotient divisible groups and the introduction of the *q.d.*-invariants which classify these groups up to quasi isomorphism. (Any ring A such that $Q \otimes A$ is a finite dimensional semi-simple Q-algebra has a *q.d.* group as additive group.) Another major result in this paper is the Beaumont-Pierce analog of the Wedderburn Principal theorem. For one application of this, see [**11**]. The paper on subrings of algebraic number fields (1961; see also Pierce's paper on subrings of simple algebras (1960)) introduces, among other things, the concept of the *smallest field of definition* of such a subring, gives an effective characterization of it, and provides a complete classification of the subrings of a given algebraic number field. This smallest field of definition plays a very important role in the structure theory of the underlying additive group.

6. Subsequent results. The people whom I have mentioned have been greatly influenced by the work described, and many of us have continued along similar lines.

The work on algebras of quasi-endomorphisms has been a major influence, and many of the students wrote theses studying various aspects of these algebras. Further work was done, for example, on irreducible groups on the one hand, and on realizing algebras as the algebras of quasi endomorphisms of groups on the other. Which algebras are such? Deep and significant work was done in this direction in a long series of collaborations by Pierce and Vinsonhaler, for example. (See [**15**] for references.) See also [**5**]. This line of investigation has also been enriched by the contributions of many others of course, notably Arnold, both directly and through his influential book [**1**].

As another example, we recall some work that came out of the thesis by Schultz on the notion of "E-ring" (Schultz's terminology [**14**], which has become standard). This is a ring in which the left regular representation is an isomorphism of the ring *onto* the endomorphism ring of its underlying abelian group. These rings later came up in some work that I myself was involved in with a student (Niedzwecki [**12**]) on groups projective over their endomorphism rings. Further work along these lines was done at a seminar at the University of Connecticut resulting in a paper with five authors, no less (Arnold, Pierce, Reid, Vinsonhaler, Wickless [**2**]). Dugas, Mader and Vinsonhaler ([**6**]) established the existence of large E-rings. And Pierce himself wrote a paper on "E-modules", to some extent summarizing the state of the theory at the time, but in typical fashion adding significantly to it both in depth and breadth.

There have been many papers on the general question of which abelian groups support various kinds of ring structures. In addition to the work of Beaumont and coauthors cited previously, see, for example, Wickless ([**16**]) who motivated Beaumont and Lawver ([**3**]), who spurred Reid ([**10**]).

Finally, I should like to mention a little bit some work of Nunke since this report does not yet reflect his influence on many of us and the value of his presence among us at the time. His thesis (cf. [**7**]) was a study of Ext_R for Dedekind R. This contains many innovative ideas. Moreover his work on slender groups ([**8**]) is among the earliest where set theory entered nontrivially. And a small paper of his on endomorphism rings of p-groups (in the proceedings of the Montpelier Conference, 1967 – see [**9**]) is a beautiful application of homological ideas, in my opinion. It generalizes and sets in a wider context the earlier identification of the center of the ring of endomorphisms of a p-group by Charles. We close this account by quoting this elegant result.

The context is this: If A and B are abelian p-groups with endomorphism rings $E(A), E(B)$, respectively, then there are ring homomorphisms $E(A) \to E(\mathrm{Tor}\,(A,B))$ and $E(B) \to E(\mathrm{Tor}\,(A,B))$ given by $\alpha \rightsquigarrow \mathrm{Tor}\,(\alpha,1)$, $\beta \rightsquigarrow \mathrm{Tor}\,(1,\beta)$ for $\alpha \in E(A)$, $\beta \in E(B)$, respectively. Here is the theorem.

Theorem 1 (Nunke). *If A and B are unbounded abelian p-groups, then the ring homomorphisms above are monomorphisms embedding $E(A)$ and $E(B)$ in $E(Tor\,(A,B))$. Each of $E(A)$ and $E(B)$ as so embedded, is the centralizer of the other and their intersection is the center of $E(Tor\,(A,B))$.*

The theorem of Charles is obtained from this marvelously symmetric statement by taking $B = Z(p^\infty)$ and recalling that $\mathrm{Tor}\,(A, Z(p^\infty))$ and A are naturally isomorphic.

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Remark. The principal references here are [**13**] and [**15**] where complete bibliographies for the work of Beaumont and Pierce can be found. The remaining references are to papers cited in the text.

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