



BIOGRAPHY OF WOLFGANG JOSEPH THRON
(1918–2001)

Wolfgang Joseph Thron (or “Wolf” to his friends) was a mathematical scholar, educator and researcher in classical analysis, topology and history of mathematics. For more than half a century he served as an advisor, mentor and role model for students and colleagues and he devoted much of his life promoting cooperation and mathematical collaboration among university faculties and students in the United States, Germany, India, Norway and the Philippines. Wolf died of emphysema at his home in Boulder, Colorado on August 21, 2001. He was eighty-three years old.

Wolf Thron was the younger of two children of Ludwig and Annemarie Joseph Thron, born on August 17, 1918, in Ribnitz, Germany. He grew up in an old “doctors’ house” where his father and grandfather jointly worked in an extended small town medical practice. Receiving a classical education at the Gymnasium in the major port city Rostock, he acquired a life-long interest in history as well as mathematics. His summers in these years were used for international experiences such as hiking in the Austrian Alps, traveling in Italy and sailing a two-mast schooner to England, Sweden and Denmark.

PRINCETON – Due to political events in Germany, Wolf studied at the ETH Zurich in the summer of 1936. The following fall he enrolled at Princeton and graduated with an A.B. degree in 1939. His undergraduate mathematics included courses with S. Bochner (analysis), H.F. Bohnenblust (analysis), A.W. Tucker (algebra), S.S. Wilks (statistics) and H. Weyl (algebraic number theory). On weekends at Princeton he encountered many scientists and mathematicians in the home of his uncle, Hermann Weyl, and aunt, Helene Joseph Weyl.

GRADUATE SCHOOL – A teaching fellowship brought him to Washington State University in Pullman for the year 1939–40. Away from the ivory tower atmosphere of Princeton, Wolf made two important discoveries: he liked America and he liked teaching. In 1940 he enrolled as a graduate student at the Rice Institute where the Mathematics Department was essentially a department of analysis, with complex analysts S. Mandelbrojt (student and son-in-law of Hadamard) and F.E. Ulrich (student of Ahlfors), real analyst H.E. Bray and in differential equations Walter Leighton (student of Marston Morse). He was assigned to work with Leighton who had previously written a master's thesis on continued fractions with H.S. Wall at Northwestern, and Leighton was giving a seminar on that subject in 1940–41. By the end of that year Wolf had obtained enough ideas to develop a thesis, the results of which were published in two papers: one in 1942 (with Leighton) and one in 1943.

HARVARD AND THE ARMY – With a Ph.D. from Rice Wolf began an instructorship at Harvard in 1943, but after one year this was interrupted by a call from Uncle Sam. The years 1944–46 in the U.S. Army included basic training in South Carolina followed by an assignment with military intelligence in Germany. One advantage of being drafted was becoming eligible for U.S. citizenship, which he received shortly after induction.

WASHINGTON UNIVERSITY – At the end of the war he accepted a position at Washington University in St. Louis, which he held for eight years. During that period he pursued interests in point set topology, function theory and continued fractions and published his first book, *The Theory of Functions of a Complex Variable*, a concise

but self-contained treatment of the subject.

UNIVERSITY OF COLORADO – Wolf Thron’s long and productive career at the University of Colorado began in 1954 and, although formal retirement was taken in 1985, he continued lecturing, collaborating and publishing mathematical research until the late 1990’s when health problems made this no longer possible. In the Colorado years he authored two additional books, *Topological Structures* in 1966 and *Continued Fractions: Analytic Theory and Applications* in 1980 with W.B. Jones. He served as Chair of the Mathematics Department from 1972 to 1974 and was the thesis advisor for 21 Ph.D. students (see following list). He organized research seminars in topology and continued fractions (CF), the CF seminar being active during most of the years from 1958 through 1999. In addition to visiting faculty and graduate students, the regular participants in the CF seminar included William B. Jones, Arne Magnus, Burnett Meyer and the periodic visitors from the University of Trondheim, Norway: Lisa Lorentzen, Olav Njåstad and Haakon Waadeland. In 1999 Wolf made a gift to the University of Colorado Foundation which established the W.J. Thron Mathematics Fellowship awarded each year to an outstanding graduate student in the Mathematics Department. With this gift he was enabling and encouraging students of all future generations to study mathematics. He was also expressing his faith in, and commitment to, the University of Colorado - its faculty and its students.

RESEARCH AND SCHOLARSHIP

ANALYSIS – The contributions of W.J. Thron to the analytic theory of continued fractions are of major proportion. Much of that work was devoted to topics in convergence theory such as uniform convergence regions, separate convergence, acceleration of convergence, asymptotic speed of convergence and truncation error analysis. The crown jewel of this work is a convergence region result known as the Thron Parabola Theorem (TPT) involving regions P in the complex plane bounded by parabolas passing through $z = -1/4$ with focus at the origin. The TPT asserts that a continued fraction $K(a(m)/1)$, whose partial numerators $a(m)$ lie in a parabolic region P , is convergent if and only if the sequence $\{|a(m)|\}$ satisfies the Stern/Stolz growth restriction that is a necessary condition for convergence. Thron

gave three independent proofs of the TPT. The first (part of his 1943 doctoral thesis) is based on properties of normal families (the Stieltjes-Vitali Theorem). The second (1958) uses only elementary methods and is completely constructive; it provides uniform convergence and computable truncation error bounds for the approximants of the continued fractions. The third proof (1963) is based on an indirect argument and uses elementary methods. Each of the three methods provided a new approach to convergence theory that subsequently has been exploited by Thron and others.

In 1948 Thron introduced a new family of continued fractions which was later generalized by O. Perron in *Die Lehre von den Kettenbrüchen* (1957) and Perron called them “die Thronschen Kettenbrüche.” During the next four decades these continued fractions (referred to as general T -fractions) have been used to develop a number of areas of classical analysis including the theory of strong moment problems, orthogonal Laurent polynomials, strong Gaussian quadrature, two-point Padé approximants and the representation of special functions. Positive T -fractions (i.e., general T -fractions with positive coefficients) were used by Thron (and collaborators) to solve the strong Stieltjes moment problem in a manner similar to the application of S -fractions by Stieltjes to solve the classical Stieltjes moment problem. Later Wolf (and collaborators) showed that the denominators of positive T -fractions give rise to sequences of orthogonal Laurent polynomials. The strong Hamburger moment problem was solved by Thron and collaborators using orthogonal Laurent polynomials, strong Gaussian quadrature and a subclass of general T -fractions known as APT -fractions.

TOPOLOGY – W.J. Thron wrote a topology text, *Topological Structures*, in the early 1960’s at a time when the main English text available was by J.L. Kelley (*General Topology*, 1955). Thron wrote his text, in part, because he wanted to learn topology, and this was a good way to do it. He also wanted to place topology in a historical context. In *Topological Structures*, when he introduced a new concept, Thron would explain why the concept was of interest, and give several options for definitions. For example, in discussing the concept of compactness he gave four possible definitions, working out the strengths and weaknesses of each. In treating convergence, he used both nets and filters, including a lengthy discussion of the strengths and weaknesses

of each approach. He often preferred nets to filters, even though the use of filters was simpler and cleaner because, for him, the intuitive was as important as the elegant. In order to make the subject more accessible to beginning students, he was very careful to include details normally skipped over. For example, he wanted to see exactly how the subspace topology related to the original space, and so he was painstaking in his treatment of subspaces and of extensions. Finally, he had a deep interest in the layers of structure connected with topological spaces. At the time when he wrote his text there was some confusion about the relationship between proximities and uniformities. He made this relationship beautifully clear, noting that *proximity* was a layer of structure between *uniformity* and *topology*.

TEACHER – For those students who knew him best Wolf Thron was a great teacher and mentor. Over the years he had many graduate students working with him. His peak was reached in the 1960's when he supervised 13 dissertations, often overseeing five students at a time. For many of these students their passion for mathematics began in his first year topology course, a course where mathematics students grew up.

Today in K-12 education the buzz word is “discovery.” We hear about the discovery method of teaching and the discovery method of learning. Wolf Thron used this method 40 years ago in this first year course. While he didn't ask students to create their mathematics in the tradition of R.L. Moore, he instinctively knew the right problems to pose to give students both understanding and ownership of their results. One of his favorite sayings was “it is important to ask the right questions.” And he practiced what he preached. Wolf was an absolute master at setting problems. Most problems were open-ended, some were even stated wrongly so that students were asked to prove something that was, in fact, not true. Students would search for counterexamples as zealously as they searched for proofs. And often the inability to create a counterexample was just the right way of understanding how a proof would proceed. Wolf's contention was that in “real” original work one would not know whether to try for a proof or a counterexample.

Wolf Thron's ability to pose just the right questions was only the tip of the iceberg. He seemingly had an inexhaustible supply of “right”

questions. The students lucky enough to be accepted by him to work for their doctorates would be treated with just the right set of suitable research problems regardless of their mathematical background and degree of sophistication. And as students pursued their dissertation topics he would guide them in matters of judgment. He insisted that his students acquire insight and mathematical “taste.” He wanted them to understand the significance of important theorems and he strongly encouraged them to pick and choose among their results, keeping those that had substance and discarding the ones that were shallow. Students of Thron would leave the university with more than a degree; they would leave with a life-long appreciation of mathematics.

MENTOR – Besides being a superb teacher, Wolf Thron was a mentor to his students during their graduate days and beyond. His advice was always sought, always carefully considered and invariably sound. “Stick to mathematical research. Don’t move into a hybrid field, no matter how trendy, at least not until you have established yourself.” He not only took students under his wing professionally, but he and his wife Ann would literally provide sustenance and care for students who had personal needs.

SERVICE – In 1939 Wolf Thron took part in the first of a series of summer workcamps sponsored by the American Friends Service Committee. These included: 1939, resettling miners in western Pennsylvania; 1941, constructing a dam in New England for a small community; 1948, improving buildings at (African-American) Storer College in Harpers Ferry, West Virginia; 1949, workcamp in Sweden; 1950, an inner city project in Philadelphia; 1951, teaching at Frei Universität in Berlin and building homes for Yugoslavian refugees. Wolf and Ann Lukach met and became friends at the 1949 workcamp orientation session in New York and subsequent workcamps. They were married in 1953 after Ann had completed her M.D. degree at New York University.

INTERNATIONAL CONNECTIONS – A unique activity of Wolf and Ann Thron was spending every fourth year from 1958 through 1991 at a foreign university. They spent 1958–59 at the University of Munich in close proximity with Oskar Perron (with support from AFOSR). In 1962–63 and 1974–75 the Throns’ traveled to

Panjab University in Chandigarh, India, where Wolf was a Fulbright scholar/professor. The Ford Foundation brought them to Mindanao State University in the Philippines in 1966–67 where Wolf participated in a development program and returned to the University of Colorado with a bright doctoral student. Two of his Colorado students accompanied him to the University of Erlangen in 1970–71 during a sabbatical year. The last four foreign tours were held at the University of Trondheim, Norway, (now the Norwegian University of Science and Technology - NTNU) with support from the Norwegian Marshall Fund and the Norwegian Research Council for Science and the Humanities (NAVF) in 1978–79, 1982–83, January–July 1987 and 1990–91. Two additional summers were spent at Institutes for College Teachers in India (Jodhpur in 1967 and Aligarh in 1969) under the auspices of the United States AID program, NSF and the Indian UGC.

HONORS – In 1980 W.J. Thron was elected to The Royal Norwegian Society for Sciences and Letters (Det Kongelige Norske Videnskabers Selskap) for his outstanding creative research in mathematics and for his great inspiration for others to do creative work. At his retirement in 1985 he was awarded the University of Colorado Medal for outstanding contributions to the University and for his distinguished career as a scholar, teacher and research mathematician. In recognition of his leadership role in developing the analytic theory of continued and related topics in classical analysis, an international conference was held at the University of Colorado - Boulder in 1988 to celebrate his 70th birthday and another in 1998 for his 80th. The proceedings of the 1988 conference were published as a special issue of the *Rocky Mountain Journal of Mathematics* **21**, No. 1, Winter 1991 and were dedicated to Professor Emeritus W.J. Thron. In 1989 a group of former students who had known him as their dissertation advisor gathered in Boulder, Colorado, to honor him on his 70th birthday. Some were separated from Boulder by thousands of miles and some by more than 20 years. It was an evening of testimonials to his mathematical genius and generous spirit, a testimonial to their love and respect for the man who shaped their future and who will always live in their minds and hearts.

Ph.D. STUDENTS OF WOLFGANG J. THRON

Leo J. Lange - 1960
Clarence H. Cook - 1962
Charles E. Aull - 1962
J.D. DePree - 1962
Kenneth L. Hillam - 1962
William B. Jones - 1963
David Drake - 1964
Jack L. Hursch, Jr. - 1964
Nicholas Morez - 1965
Nicholas P. Callas - 1966
Fred W. Stevenson - 1966
Terrence E. Dooher - 1966
Ellen E. Reed - 1966
Thomas H. Jefferson, Jr. - 1969
Ronald E. Larson - 1970
Susan J. Zimmermann Andima - 1970
Richard H. Warren - 1971
Richard A. Valent - 1974
R.G. Ori - 1975
S.D. Diesto - 1977
K.C. Chattopadhyay - 1978

BOOKS BY W.J. THRON

Introduction to the Theory of Functions of a Complex Variable, John Wiley & Sons, New York, 1953.

Topological Structures, Holt, Reinhart and Winston, New York, 1966.

Continued Fractions: Analytic Theory and Applications (with W.B. Jones), Encyclopedia of Mathematics and Its Applications, Addison Wesley, Reading, MA, 1980.

BOOKS EDITED BY W.J. THRON

Proceedings of the International Conference on Padé Approximants (with W.B. Jones), Continued Fractions and Related Topics, Rocky Mountain J. Math. **4** (2), Spring 1974.

Analytic Theory of Continued Fractions (with W.B. Jones and H. Waadeland), Lecture Notes in Math., vol. 932, Springer-Verlag, New York, 1982.

Analytic Theory of Continued Fractions II, Lecture Notes in Math., vol. 1199, Springer-Verlag, New York, 1986.

Continued Fractions and Orthogonal Functions (with S.C. Cooper), Lecture Notes in Pure Appl. Math., vol. 154, Marcel Dekker, Inc., New York, 1994.

Authors' Footnote: Fred Stevenson is Professor of Mathematics at the University of Arizona, Ellen Reed teaches mathematics at Trinity School at Greenlawn in South Bend, Indiana and William Jones is Professor Emeritus of Mathematics at the University of Colorado. All three received their Ph.D. degrees under Wolf Thron's supervision, Reed and Stevenson at the University of Colorado and Jones at Vanderbilt University.

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