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TWO SIDED α-DERIVATIONS IN 3-PRIME NEAR-RINGS

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ABSTRACT. The purpose of this paper is to investigate two sided α-derivations satisfying certain differential identities on 3-prime near-rings. Some well-known results characterizing commutativity of 3-prime near-rings by derivations (semi-derivations) have been generalized. Furthermore, examples proving the necessity of the 3-primeness hypothesis are given.

1. Introduction. In this paper, N will denote a zero-symmetric left near-ring. For any $x, y \in N$, the symbol $[x, y]$ will denote the commutator $xy - yx$, while the symbol $x \circ y$ will stand for the anti-commutator $xy + yx$. The symbol $Z(N)$ will represent the multiplicative center of N. Unless otherwise specified, we will use the term *near-ring* to mean zero-symmetric left near-ring. According to **[6]**, a near-ring N is said to be 3-prime if $xNy = \{0\}$ implies $x = 0$ or $y = 0$. N is said to be 2-torsion free if $2x = 0$ implies $x = 0$. An additive mapping $\delta : N \to N$ is said to be a *derivation* if

$$\delta(xy) = x\delta(y) + \delta(x)y \quad \text{for all } x, y \in N,$$

or equivalently, as noted in **[14]**, that

$$\delta(xy) = \delta(x)y + x\delta(y) \quad \text{for all } x, y \in N.$$

An additive mapping $d : N \to N$ is called an (α, β)-*derivation* if there exist functions $\alpha, \beta : N \to N$ such that

$$d(xy) = d(x)\alpha(y) + \beta(x)\, d(y) \quad \text{for all } x, y \in N.$$

Furthermore, an additive mapping $d : N \to N$ is called a two sided α-derivation if d is an $(\alpha, 1)$-derivation as well as a $(1, \alpha)$-derivation.

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Moreover, if d commutes with α, then d is called a *semi-derivation* (see, [**8**]). Clearly, every semi-derivation is a two sided α-derivation, but the converse is not true. Indeed, in Example 2, since $d\alpha \neq \alpha d$, then d is not a semi-derivation; however, d is a two sided α-derivation.

In the case where $\alpha = 1$, a two sided α-derivation is just a derivation, but an example due to [**1**] proves that the converse is not true.

The recent literature contains numerous results on commutativity in prime and semi-prime rings admitting suitably constrained derivations and generalized derivations, and several authors have proved comparable results on near-rings. In fact, the relationship between the commutativity of a 3-prime near-ring N and the behavior of a derivation on N was initiated in 1987 by Bell and Mason [**6**]. In [**13**], Hongan generalizes some of their results by assuming that the commutativity condition is imposed on an ideal rather than on the wall near-ring. More recently, Bell et al. [**5, 11**] generalize several commutativity theorems for the 3-prime near-ring by treating the case of generalized derivations satisfying certain algebraic identities involving semigroup ideals. Some of our results, which deal with conditions on two sided α-derivations, extend earlier commutativity results involving similar conditions on derivations and semi-derivations.

2. Two sided α-derivation associated with a homomorphism.

In the following lemmas, α is a function, not necessarily a homomorphism.

Lemma 2.1. *Let d be a two sided α-derivation. Then, N satisfies the following partial distributive law*:

$$\big(d(x)\alpha(y) + xd(y)\big)\alpha(t) = d(x)\alpha(yt) + xd(y)\alpha(t) \quad \text{for all } t, x, y \in N.$$

Proof. By the definition of d, we have

$$\begin{aligned} d(xyt) &= d(xy)\alpha(t) + xyd(t) \\ &= \big(d(x)\alpha(y) + xd(y)\big)\alpha(t) + xy\,d(t) \quad \text{for all } t, x, y \in N. \end{aligned}$$

On the other hand,

$$\begin{aligned} d(xyt) &= d(x)\alpha(yt) + xd(yt) \\ &= d(x)\alpha(yt) + xd(y)\alpha(t) + xy\,d(t) \quad \text{for all } t, x, y \in N. \end{aligned}$$

From the two expressions of $d(xyt)$, we find that

$$(d(x)\alpha(y)+xd(y))\alpha(t) = d(x)\alpha(yt)+xd(y)\alpha(t) \quad \text{for all } t, x, y \in N. \ \square$$

Lemma 2.2. *Let N be a near-ring. If N admits an additive mapping d, then the following statements are equivalent*:

(i) *d is a $(1, \alpha)$-derivation.*
(ii) *$d(xy) = \alpha(x)\, d(y) + d(x)y$ for all $x, y \in N$.*

Proof.

(i) $\Rightarrow$ (ii). Since d is a $(1, \alpha)$-derivation, then, for all $x, y \in N$, we get

$$\begin{aligned} d(x(y+y)) &= d(x)(y+y) + \alpha(x)\, d(y+y) \\ &= d(x)y + d(x)y + \alpha(x)\, d(y) + \alpha(x)\, d(y) \quad \text{for all } x, y \in N, \end{aligned}$$

and

$$\begin{aligned} d(x(y+y)) &= d(xy) + d(xy) \\ &= d(x)y + \alpha(x)d(y) + d(x)y + \alpha(x)\, d(y) \quad \text{for all } x, y \in N. \end{aligned}$$

Comparing the two expressions of $d(x(y+y))$, we conclude that

$$d(x)y + \alpha(x)\, d(y) = \alpha(x)\, d(y) + d(x)y \quad \text{for all } x, y \in N.$$

Analogously, we can prove the other implication. $\square$

Theorem 2.3. *Let N be a 3-prime near-ring and d a nonzero $(1, \alpha)$-derivation associated with a homomorphism α. Then the following assertions are equivalent*:

(i) *$d(N) \subseteq Z(N)$;*
(ii) *$d([x, y]) = 0$ for all $x, y \in N$;*
(iii) *N is a commutative ring.*

Proof. It is obvious that (iii) implies both (i) and (ii).

(i) $\Rightarrow$ (iii). Assume that $d(x) \in Z(N)$ for all $x \in N$. Then

$$d(xy)\alpha(t) = \alpha(t)\, d(xy) \quad \text{for all } t, x, y \in N,$$

and, using Lemma 2.1, we obtain

$$d(x)\alpha(yt) + xd(y)\alpha(t) = \alpha(t)\, d(x)\alpha(y) + \alpha(t)xd(y) \tag{2.1}$$

for all $t, x, y \in N$. Replacing x by $d(x)$ in (2.1), we get

$$d^2(x)(\alpha(yt) - \alpha(t)\alpha(y)) = 0 \quad \text{for all } t, x, y \in N,$$

so that

$$d^2(x)N(\alpha(yt) - \alpha(t)\alpha(y)) = \{0\} \quad \text{for all } x, y, t \in N. \tag{2.2}$$

Since N is 3-prime, then equation (2.2) implies that

$$d^2(x) = 0 \quad \text{or} \quad \alpha(yt) = \alpha(t)\alpha(y) \quad \text{for all } t, x, y \in N. \tag{2.3}$$

(a) If $d^2(x) = 0$ for all $x \in N$, by definition of d, we have

$$d(x)y + \alpha(x)\, d(y) = d(x)\alpha(y) + xd(y) \quad \text{for all } x, y \in N. \tag{2.4}$$

Replacing x by $d(x)$ in equation (2.4), we get

$$\alpha(d(x))\, d(y) = d(x)\, d(y) \quad \text{for all } x, y \in N,$$

which, because of $d(y) \in Z(N)$, implies that

$$(\alpha(d(x)) - d(x))\, d(y) = 0 \quad \text{for all } x, y \in N. \tag{2.5}$$

Since N is 3-prime and $d \neq 0$, for all $x \in N$, equation (2.5) implies that $\alpha(d(x)) = d(x)$. In this case, substituting xy for x, we have

$$\alpha(d(x)y + \alpha(x)d(y)) = d(x)\alpha(y) + xd(y) \quad \text{for all } x, y \in N,$$

that is,

$$d(x)\alpha(y) + \alpha^2(x)\, d(y) = d(x)\alpha(y) + xd(y) \quad \text{for all } x, y \in N.$$

Therefore,

$$(\alpha^2(x) - x)\, d(y) = 0 \quad \text{for all } x, y \in N, \tag{2.6}$$

in such a way that $\alpha^2 = Id_N$.

Now, replacing t by $\alpha(t)$ in Lemma 2.1, we get

$$d(xy)t = (d(x)\alpha(y) + xd(y))t = d(x)\alpha(y)t + xd(y)t$$

for all $t, x, y \in N$. By virtue of $d(xy)t = td(xy)$, the above expression becomes

$$d(x)\alpha(y)t + xd(y)t = td(x)\alpha(y) + txd(y) \quad \text{for all } t, x, y \in N. \tag{2.7}$$

Substituting x for t and $\alpha(y)$ for y in equation (2.7), we get

$$d(x)\,yx = xd(x)\,y \quad \text{for all } x, y \in N,$$

which implies that

$$d(x)N[y, x] = \{0\} \quad \text{for all } x, y \in N. \tag{2.8}$$

Since N is a 3-prime, then equation (2.8) implies that

$$d(x) = 0 \quad \text{or} \quad x \in Z(N) \text{ for all } x \in N. \tag{2.9}$$

Since $d \neq 0$, we choose $x_0 \in N$ such that $d(x_0) \neq 0$. Then, $x_0 \in Z(N)$. Replacing y by $\alpha(y)$ and x by x_0, respectively, in equation (2.7), we arrive at

$$d(x_0)N[y, t] = \{0\} \quad \text{for all } y, t \in N.$$

By the 3-primeness of N and $d(x_0) \neq 0$, the last expression gives $N \subseteq Z(N)$. By **[6,** Lemma 1.5], we conclude that N is a commutative ring.

(b) Now assume that

$$\alpha(yt) = \alpha(t)\alpha(y) \quad \text{for all } y, t \in N;$$

in this case, $\alpha(yt) = \alpha(t)\alpha(y) = \alpha(y)\alpha(t)$. Letting $x, y, z \in N$, we have

$$\begin{aligned} d(xyz) &= d(xy)z + \alpha(xy)\,d(z) \\ &= (d(x)y + \alpha(x)\,d(y))z + \alpha(x)\alpha(y)\,d(z), \end{aligned}$$

and

$$\begin{aligned} d(xyz) &= d(x)yz + \alpha(x)\,d(yz) \\ &= d(x)yz + \alpha(x)\,d(y)z + \alpha(x)\alpha(y)\,d(z). \end{aligned}$$

Combining the above expressions of $d(xyz)$ we find that

$$(d(x)y + \alpha(x)\,d(y))z = d(x)yz + \alpha(x)\,d(y)z \quad \text{for all } x, y, z \in N. \tag{2.10}$$

Since $d(x) \in Z(N)$ for all $x \in N$, then equation (2.10) becomes

$$d(x)yz + \alpha(x)\,d(y)z = zd(x)y + z\alpha(x)\,d(y) \quad \text{for all } x, y, z \in N, \tag{2.11}$$

which means that

$$d(x)yz = d(x)zy \quad \text{for all } x, y, z \in N,$$

and therefore,

$$d(x)N[y, z] = \{0\} \quad \text{for all } x, y, z \in N.$$

Since $d \neq 0$, the last equation gives $N \subseteq Z(N)$, and thus, N is a commutative ring by [**6,** Lemma 1.5].

(ii) $\Rightarrow$ (iii). We are assuming that

$$d([x, y]) = 0 \quad \text{for all } x, y \in N. \tag{2.12}$$

Substituting xy for y in equation (2.12) and using $[x, xy] = x[x, y]$, we arrive at

$$d(x)xy = d(x)yx \quad \text{for all } x, y \in N. \tag{2.13}$$

Replacing y by yz in equation (2.13), one can easily verify that $d(x)y[x, z] = 0$ for all $x, y, z \in N$, in such a way that

$$d(x)N[x, z] = \{0\} \quad \text{for all } x, z \in N. \tag{2.14}$$

Once again, using the 3-primeness, equation (2.14) shows that

$$[x, z] = 0 \quad \text{or} \quad d(x) = 0 \text{ for all } x, z \in N.$$

It follows that, for each fixed $x \in N$, we have

$$x \in Z(N) \quad \text{or} \quad d(x) = 0. \tag{2.15}$$

Letting $x_0 \in Z(N)$ and using Lemma 2.2, we have

$$\begin{aligned} d(x_0 y) &= d(x_0)\alpha(y) + x_0 d(y) \\ &= d(y x_0) \\ &= \alpha(y)\, d(x_0) + d(y)x_0, \end{aligned}$$

and thus

$$d(x_0)\alpha(y) = \alpha(y)\, d(x_0) \quad \text{for all } y \in N.$$

In the case where $d(x_0) = 0$, the last result is satisfied. Then, we get the following conclusion:

$$d(x)\alpha(y) = \alpha(y)\, d(x) \quad \text{for all } x, y \in N. \tag{2.16}$$

According to equation (2.12), we have $d(xy) = d(yx)$ for all $x, y \in N$, which, because of Lemma 2.2, yields

$$\alpha(x)\,d(y) + d(x)y = d(y)\alpha(x) + yd(x) \quad \text{for all } x, y \in N. \tag{2.17}$$

It now follows from equations (2.16) and (2.17) that

$$d(x)y = yd(x) \quad \text{for all } x, y \in N,$$

which implies that $d(N) \subseteq Z(N)$, and case (i) gives the required result. □

As an application of Theorem 2.3, we obtian the following corollaries.

Corollary 2.4. *Let N be a 2-torsion free 3-prime near-ring and d a nonzero derivation.*

(i) [**6,** Theorem 2]. *If $d(N) \subseteq Z(N)$, then N is a commutative ring.*

(ii) [**2,** Theorem 4.1]. *If $d[x, y] = 0$ for all $x, y \in N$, then N is a commutative ring.*

Corollary 2.5. *Let N be a 2-torsion free 3-prime near-ring and d a nonzero semi-derivation.*

(i) [**9,** Theorem 1]. *If $d(N) \subseteq Z(N)$, then N is a commutative ring.*

(ii) [**9,** Theorem 2]. *If $d([x, y]) = 0$ for all $x, y \in N$, then N is a commutative ring.*

The following example shows the necessity of the 3-primeness in the previous theorems.

Example 2.6. Let S be a 2-torsion free near-ring. Let us define N and $d, \alpha, F : N \to N$ by:

$$N = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \,\middle|\, x, y \in S \right\},$$

$$d\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\alpha\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix}.$$

It is clear that N is not a 3-prime near-ring and d is a nonzero two sided α-derivation satisfying:

(i) $d(N) \subseteq Z(N)$,
(ii) $d([A, B]) = 0$ for all $A, B \in N$,

but, since the addition in N is not commutative, then N cannot be a commutative ring.

3. Two sided α-derivation associated with a function. In this section, we treat the general case where α is a function and not necessarily a homomorphism.

Theorem 3.1. *Let N be a 3-prime near-ring. If N admits a two sided α-derivation d such that $d([x, y]) = [x, y]$ for all $x, y \in N$, then N is a commutative ring or $d = -\alpha + Id$.*

Proof. Suppose that

(3.1) $$d([x, y]) = [x, y] \quad \text{for all } x, y \in N.$$

Substituting xy for y in equation (3.1), one can easily verify that

(3.2) $$d(x)[x, y] + \alpha(x)[x, y] = x[x, y] \quad \text{for all } x, y \in N.$$

Replacing x by $[x, t]$ in equation (3.2), we obtain

(3.3) $$\alpha([x, t])[x, t]y = \alpha([x, t])y[x, t] \quad \text{for all } t, x, y \in N.$$

Substituting yz for y in (3.3), we get

$$\alpha([x, t])yz[x, t] = \alpha([x, t])y[x, t]z \quad \text{for all } t, x, y, z \in N,$$

and therefore, $\alpha([x, t])y[[x, t], z] = 0$, which can be rewritten as

(3.4) $$\alpha([x, t])N\big[[x, t], z\big] = \{0\} \quad \text{for all } t, x, z \in N.$$

In view of the 3-primeness of N, equation (3.4) yields

$$\alpha([x,t]) = 0 \quad \text{or} \quad [x,t] \in Z(N) \quad \text{for all } t, x \in N. \tag{3.5}$$

If $[x,t] \in Z(N)$ for all $x, t \in N$, then replacing t by xt and using the 3-primeness of N, we find that $[x,t] = 0$ for all $x, t \in N$. According to **[6,** Lemma 1.5], we obtain the conclusion that N is a commutative ring.

Assume that there exist $x, t \in N$ such that $[x,t] \notin Z(N)$. In particular, $[x,t] \neq 0$ and $\alpha([x,t]) = 0$, so that

$$d([x,t]zy) = [x,t]zy \quad \text{for all } z, y \in N. \tag{3.6}$$

On the other hand,

$$\begin{aligned} d([x,t]zy) &= d([x,t]z)\alpha(y) + [x,t]zd(y) \\ &= [x,t]z\alpha(y) + [x,t]zd(y) \quad \text{for all } z, y \in N. \end{aligned} \tag{3.7}$$

Now, combining equation (3.6) with equation (3.7), we conclude that

$$[x,t]z\alpha(y) + [x,t]zd(y) = [x,t]zy \quad \text{for all } z, y \in N. \tag{3.8}$$

Substituting $[u,v]$ for y in equation (3.8), we obtain

$$[x,t]z\alpha([u,v]) = 0 \quad \text{for all } u, v, z \in N,$$

so that,

$$[x,t]N\alpha([u,v]) = \{0\} \quad \text{for all } u, v \in N. \tag{3.9}$$

By the 3-primeness of N, equation (3.9) shows that

$$\alpha([u,v]) = 0 \quad \text{for all } u, v \in N.$$

Computing $d([u,v]zy)$ as in equations (3.6) and (3.7), we obtain

$$[u,v]z(\alpha(y) + d(y) - y) = 0 \quad \text{for all } u, v, z, y \in N,$$

which implies that

$$[u,v]N(\alpha(y) + d(y) - y) = \{0\} \quad \text{for all } u, v, y \in N. \tag{3.10}$$

By the 3-primeness of N, equation (3.10) shows that

$$[u,v] = 0 \quad \text{or} \quad \alpha(y) + d(y) - y = 0 \quad \text{for all } u, v, y \in N. \tag{3.11}$$

According to **[6,** Lemma 1.5], equation (3.11) assures that N is a commutative ring or $d = -\alpha + Id$. $\square$

As an application of Theorem 3.1, we obtain the following corollaries.

Corollary 3.2. [**10,** Theorem 1]. *If R is a prime ring admitting a derivation d satisfying $d([x,y]) = [x,y]$ for all $x,y \in R$, then R is commutative.*

Corollary 3.3. [**8,** Theorem 2.2]. *Let N be a 3-prime near-ring. If N admits a nonzero derivation d such that $d([x,y]) = [x,y]$ for all $x,y \in N$, then N is a commutative ring.*

Corollary 3.4. *Let N be a 3-prime near-ring. If N admits a nonzero semi-derivation d such that $d([x,y]) = [x,y]$ for all $x,y \in N$, then N is a commutative ring or $d = -\alpha + Id$.*

Theorem 3.5. *Let N be a 2-torsion free 3-prime near-ring. There is no nonzero two sided α-derivation d such that $d(x \circ y) = 0$ for all $x,y \in N$.*

Proof. Assume that N admits a nonzero two sided α-derivation d, such that

$$d(x \circ y) = 0 \quad \text{for all } x,y \in N. \tag{3.12}$$

Replacing y by xy in equation (3.12), because of $x \circ xy = x(x \circ y)$, we get $d(x)(x \circ y) = 0$, which means that

$$d(x)xy = d(x)y(-x) \quad \text{for all } x,y \in N. \tag{3.13}$$

Substituting yz for y and $-x$ for x in (3.13), we obtain $d(-x)y(xz - zx) = 0$ for all $x,y,z \in N$, and therefore,

$$d(-x)N(xz - zx) = \{0\} \quad \text{for all } x,z \in N. \tag{3.14}$$

In light of the 3-primeness of N, equation (3.14) yields

$$d(x) = 0 \quad \text{or} \quad x \in Z(N) \quad \text{for all } x \in N. \tag{3.15}$$

Let $x \in Z(N)$; since N is 2-torsion free, then equation (3.12) forces $d(xy) = 0$ so that

$$d(x)y + \alpha(x)\, d(y) = 0 \quad \text{for all } y \in N. \tag{3.16}$$

Substituting $y \circ z$ for y in equation (3.16), we get

$$d(x)(y \circ z) = 0 \quad \text{for all } y, z \in N. \tag{3.17}$$

Replacing z by zt and y by $-y$ in equation (3.17), one can easily see that

$$d(x)N[y,t] = \{0\} \quad \text{for all } t, y \in N. \tag{3.18}$$

By the 3-primeness of N, equation (3.18) shows that either $d(x) = 0$ or N is a commutative ring by [**6,** Lemma 1.5]. But, in the latter case, our hypothesis reduces to $d(xy) = 0$ for all $x, y \in N$ and, replacing y by yz in this equation, we get $d(x)Nz = \{0\}$ for all $x, z \in N$, which yields $d = 0$. Hence, in both cases, we conclude that $d = 0$, a contradiction. □

Remark 3.6. Ashraf and Ali [**2,** Corollary 4.1] showed that a 2-torsion free prime near-ring N must be commutative if it admits a derivation d where d satisfies $d(x \circ y) = 0$ for all $x, y \in N$. However, this result is not true. Indeed, the existence of a derivation satisfying the above condition does not assure the commutativity of N. Our aim in the following corollary is to give the corrected result.

Corollary 3.7. *Let N be a 2-torsion free prime near-ring. If there exists a derivation d of N satisfying $d(x \circ y) = 0$ for all $x, y \in N$, then $d = 0$.*

Corollary 3.8. *Let N be a 2-torsion free 3-prime near-ring. Then there exists no nonzero semi-derivation d of N satisfying $d(x \circ y) = 0$ for all $x, y \in N$.*

In [**8,** Theorem 2.4], it is proved that a 3-prime near-ring N must be a commutative ring if it admits a derivation d such that $d(x \circ y) = x \circ y$ for all $x, y \in N$, but this result is less precise. The following result treats the above condition in a more general situation.

Theorem 3.9. *Let N be a 2-torsion free 3-prime near-ring admitting a two sided α-derivation d. If $d(x \circ y) = x \circ y$ for all $x, y \in N$, then $d = -\alpha + Id$.*

Proof. We assume that

$$d(x \circ y) = x \circ y \quad \text{for all } x, y \in N. \tag{3.19}$$

Replacing y by xy in equation (3.19), it is obvious to see that

$$d(x)(x \circ y) + \alpha(x)(x \circ y) = x(x \circ y) \quad \text{for all } x, y \in N. \tag{3.20}$$

Substituting $x \circ t$ for x in equation (3.20), we obtain

$$\alpha(x \circ t)((x \circ t) \circ y) = 0 \quad \text{for all } t, x, y \in N, \tag{3.21}$$

which can be rewritten as $\alpha(x \circ t)(x \circ t)y + \alpha(x \circ t)y(x \circ t) = 0$ for all $t, x, y \in N$. Accordingly,

$$\begin{aligned} \alpha(x \circ t)(x \circ t)y &= -\alpha(x \circ t)y(x \circ t) \\ &= \alpha(x \circ t)y(-(x \circ t)) \quad \text{for all } t, x, y \in N. \end{aligned} \tag{3.22}$$

Putting yz instead of y in equation (3.22), we find that

$$\begin{aligned} \alpha(x \circ t)(x \circ t)yz &= \alpha(x \circ t)y(-(x \circ t))z \\ &= \alpha(x \circ t)yz(-(x \circ t)) \quad \text{for all } t, x, y, z \in N. \end{aligned}$$

Consequently,

$$\alpha(x \circ t)y[-(x \circ t), z] = 0 \quad \text{for all } t, x, y, z \in N,$$

so that

$$\alpha(x \circ t)N[-(x \circ t), z] = \{0\} \quad \text{for all } t, x, z \in N. \tag{3.23}$$

In light of the 3-primeness of N, equation (3.23) shows that

$$\alpha(x \circ t) = 0 \quad \text{or} \quad -(x \circ t) \in Z(N) \tag{3.24}$$

Suppose that there exist $x, t \in N$ such that $-(x \circ t) \in Z(N)$, and set $-(x \circ t) = u$. From equation (3.19), it follows that $d(u \circ k) = u \circ k$ which, because of $d(u) = u$, yields

$$\alpha(u)d(k) + d(u)k + \alpha(u)d(k) = ku \quad \text{for all } k \in N.$$

In view of

$$d(u)k + \alpha(u)d(k) = \alpha(u)d(k) + d(u)k,$$

we then conclude that $\alpha(u)d(k) = 0$ for all $k \in N$, so that

$$\alpha(u)Nd(k) = 0 \quad \text{for all } k \in N. \tag{3.25}$$

If $d = 0$, then our hypothesis reduces to $x \circ y = 0$ for all $x, y \in N$ which leads to $N = \{0\}$, a contradiction. Thus, equation (3.25) forces $\alpha(u) = 0$. Therefore, equation (3.24) reduces to

$$\alpha(x \circ t) = 0 \quad \text{or} \quad \alpha(-(x \circ t)) = 0 \quad \text{for all } x, t \in N. \tag{3.26}$$

If there exist $x, t \in N$ such that $\alpha(-(x \circ t)) = 0$, then once again setting $u = -(x \circ t)$, we get

$$d(ukv) = d(u)kv + \alpha(u)d(kv) = ukv \quad \text{for all } k, v \in N.$$

On the other hand,

$$d(ukv) = d(uk)\alpha(v) + ukd(v) = uk\alpha(v) + ukd(v),$$

and, comparing the last two expressions, we get

$$uk\alpha(v) + ukd(v) = ukv \quad \text{for all } k, v \in N,$$

which implies that

$$uN(\alpha(v) + d(v) - v) = \{0\} \quad \text{for all } v \in N.$$

By the 3-primeness of N, we conclude that

$$x \circ t = 0 \quad \text{or} \quad \alpha(v) + d(v) - v = 0 \quad \text{for all } v \in N.$$

Similarly, if there exist $x, t \in N$ such that $\alpha(x \circ t) = 0$, then using similar techniques as above, we find that

$$x \circ t = 0 \quad \text{or} \quad \alpha(v) + d(v) - v = 0 \quad \text{for all } v \in N.$$

Now, if we assume that $x \circ t = 0$ for all $x, t \in N$, then $t^2 = 0$ for all $t \in N$, and hence,

$$0 = (x \circ t) = txt \quad \text{for all } x, t \in N,$$

that is, $tNt = \{0\}$ for all $t \in N$, which forces $N = \{0\}$, a contradiction. Consequently, equation (3.26) shows that $d = -\alpha + Id$. □

Using Theorem 3.9, the corrected version of [**8,** Theorem 2.4] should be as follows.

Corollary 3.10. *Let N be a 2-torsion free 3-prime near-ring. There is no derivation d of N such that $d(x \circ y) = x \circ y$ for all $x, y \in N$.*

Corollary 3.11. *Let N be a 2-torsion free 3-prime near-ring admitting a semi-derivation d. If $d(x\circ y) = x\circ y$ for all $x, y \in N$, then $d = -\alpha+Id$.*

The following example shows the necessity of the 3-primeness in the previous theorems.

Example 3.12. Let S be a 2-torsion free near-ring. Let us define N, d and $\alpha : N \to N$ by:

$$N = \left\{ \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \;\middle|\; x, y \in S \right\}$$

$$d \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\alpha \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & y \\ 0 & 0 & 0 \end{pmatrix}.$$

It is clear that N is a non 3-prime near-ring and d is a nonzero two sided α-derivation such that:

(i) $d([A, B]) = [A, B]$;
(ii) $d(A \circ B) = 0$;
(iii) $d(A \circ B) = A \circ B$;

for all $A, B \in N$, but neither $d = -\alpha + Id$ nor N is a commutative ring because the addition is not commutative.

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