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A CONTRACTION OF THE PRINCIPAL SERIES BY BEREZIN-WEYL QUANTIZATION

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ABSTRACT. We study a contraction of the principal series representations of a noncompact semisimple Lie group to the unitary irreducible representations of its Cartan motion group by means of the Berezin-Weyl quantization on the coadjoint orbits associated with these representations.

1. Introduction. In the pioneering paper [**19**], Inönü and Wigner introduced the notion of contraction of Lie groups and of Lie group representations on physical grounds: If two physical theories are related by a limiting process, then the associated invariance groups and their representations should also be related by a limiting process called *contraction*. For example, the Galilei group is a contraction, that is, a limiting case, of the Poincaré group and the unitary irreducible representations of the Galilei group are limits of unitary irreducible representations of the Poincaré group [**19**].

The systematic study of the contractions of Lie group representations began with the work of Mickelsson and Niederle [**24**]. In [**24**], a proper definition of the contraction of unitary representations of Lie groups was given for the first time. The non-zero mass representations of the Euclidean group $\mathbf{R}^{n+1} \rtimes SO(n+1)$ and the positive mass-squared representations of the Poincaré group $\mathbf{R}^{n+1} \rtimes SO_0(n,1)$ were obtained by contraction (i.e., as limits in the sense defined in [**24**]) of the principal series representations of $SO_0(n+1,1)$. These results were partially generalized by Dooley and Rice in [**15, 16**] by following an idea of Mackey [**23**]. In [**16**], a contraction of the principal series of a noncompact semisimple Lie group to the unitary irreducible representations of its Cartan motion group was established.

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In fact, a contraction of Lie group representations provides a link between the Harmonic Analysis on two different Lie groups. In particular, contractions allow recovery of some classical formulas of the theory of special functions [**15, 26**]. Contractions also permit transference of results on L^p-multipliers from unitary groups to Heisenberg groups [**14, 27**].

In [**13**], Dooley suggested interpreting contractions of representations in the context of the Kirillov-Kostant method of orbits [**20**] and, in [**12**], Cotton and Dooley showed how to describe contractions of representations by means of adapted Weyl correspondences. The notion of adapted Weyl correspondence was introduced in [**2, 3**]. Given a Lie group G and a unitary irreducible representation π of G on a Hilbert space $\mathcal{H}$, an adapted Weyl correspondence on a coadjoint orbit $\mathcal{O}$ associated with π by the Kirillov-Kostant method of orbits is a linear isomorphism W from a class of functions on $\mathcal{O}$ (called symbols) onto a class of operators on $\mathcal{H}$, which is adapted to π in the following sense: for each element X of the Lie algebra of G, the function $\widetilde{X}$ defined on $\mathcal{O}$ by $\widetilde{X}(\xi) = \langle \xi, X\rangle$ is a symbol and the equality $W(i\widetilde{X}) = d\pi(X)$ holds on a dense subspace of $\mathcal{H}$. A precise definition of the notion of adapted Weyl correspondence can be found in [**6**]. Adapted Weyl correspondences have been constructed in various situations, see the introduction of [**6**].

The approach of [**12**] is particularly efficient in the case when the coadjoint orbits associated with the representations have Kählerian structures. In that case, the representation spaces are reproducing kernel Hilbert spaces, and the Berezin calculus generally provides an adapted Weyl correspondence on the corresponding coadjoint orbits [**11**]. For example, in [**5, 8, 9**], we used Berezin quantization in order to establish contractions of the unitary irreducible representations of a compact semisimple Lie group and of the discrete series of a noncompact semisimple Lie group to the unitary irreducible representations of a Heisenberg group.

In [**12**], the case of the contraction of the principal series of $SL(2,\mathbf{R})$ to the unitary representations of $\mathbf{R}^2 \rtimes SO(2)$ was treated by using Weyl calculus. In [**4**], the more complicated example of the contraction of the principal series of $SO_0(n+1,1)$ to some unitary irreducible representations of $\mathbf{R}^{n+1} \rtimes SO_0(n,1)$ was studied similarly.

More generally, in the present paper, we apply the ideas of [**12**] to the study of the contraction of the principal series representations of a noncompact semisimple Lie group G to the unitary irreducible representations of its Cartan motion group $V \rtimes K$. We obtain very simple parametrizations of the corresponding coadjoint orbits of G and of $V \rtimes K$ by using the method of [**6**] which is based upon the dequantization of the representations by means of the Berezin-Weyl calculus introduced in [**2**]. This allows us to construct adapted Weyl correspondences on these coadjoint orbits. Then we show how the parametrizations of the orbits as well as the adapted Weyl correspondences are related by the contraction process. In particular, we get an infinitesimal version of the results of [**16**] on the contraction of the principal series.

This paper is organized as follows. In Sections 2 and 3, we realize the representations of the principal series of G and the unitary irreducible representations of $V \rtimes K$ in compatible ways, and we compute the corresponding derived representations. In Section 4, we introduce the Berezin-Weyl calculus. In Sections 5 and 6, we dequantize the representations and then we obtain the parametrizations of the associated orbits and the adapted Weyl correspondences. In Section 7, we recover a contraction result of [**16**] in the 'noncompact picture' (in the terminology of [**21**, Chapter 7]). Finally, in Section 8, we show that the adapted Weyl correspondences on the coadjoint orbits of G and of $V \rtimes K$ associated with the representations are related by the contraction process, and we give a contraction result for the derived representations.

2. Principal series representations. In this section, we first introduce some notation. Our main references are [**21**, Chapter 7] and [**31**, Chapter 8]. We obtain a realization of the principal series representations which is convenient for the study of contractions by slightly modifying the standard 'noncompact' realization [**21**, page 169], [**31**, subsection 8.4.8] and we compute the corresponding derived representations.

Let G be a connected noncompact semisimple real Lie group with finite center. Let $\mathfrak{g}$ be the Lie algebra of G. We identify G-equivariantly $\mathfrak{g}$ to its dual space $\mathfrak{g}^*$ by using the Killing form β of $\mathfrak{g}$ defined by $\beta(X,Y) = \mathrm{Tr}\,(\mathrm{ad}\, X \mathrm{ad}\, Y)$ for X and Y in $\mathfrak{g}$. Let θ be a Cartan involution of $\mathfrak{g}$, and let $\mathfrak{g} = \mathfrak{k} \oplus V$ be the corresponding Cartan decomposition of $\mathfrak{g}$. Let K be the connected compact (maximal) subgroup of G with Lie algebra $\mathfrak{k}$. Let $\mathfrak{a}$ be a maximal abelian subalgebra of V, and let M be the centralizer of $\mathfrak{a}$ in K. Let $\mathfrak{m}$ denote the Lie algebra of M. We can decompose $\mathfrak{g}$ under the adjoint action of $\mathfrak{a}$:

$$\mathfrak{g} = \mathfrak{a} \oplus \mathfrak{m} \oplus \sum_{\lambda \in \Delta} \mathfrak{g}_\lambda$$

where Δ is the set of restricted roots. We fix a Weyl chamber in $\mathfrak{a}$ and we denote by Δ^+ the corresponding set of positive roots. We set $\mathfrak{n} = \sum_{\lambda \in \Delta^+} \mathfrak{g}_\lambda$ and $\overline{\mathfrak{n}} = \sum_{\lambda \in \Delta^+} \mathfrak{g}_{-\lambda}$. Then $\overline{\mathfrak{n}} = \theta(\mathfrak{n})$. Let A, N and $\overline{N}$ denote the analytic subgroups of G with algebras $\mathfrak{a}$, $\mathfrak{n}$ and $\overline{\mathfrak{n}}$, respectively. We fix a regular element ξ_1 in $\mathfrak{a}$, that is, $\lambda(\xi_1) \neq 0$ for each $\lambda \in \Delta$ and an element ξ_2 in $\mathfrak{m}$. Let $\xi_0 = \xi_1 + \xi_2$. Denote by $O(\xi_0)$ the orbit of ξ_0 in $\mathfrak{g}^* \simeq \mathfrak{g}$ under the (co)adjoint action of G and by $o(\xi_2)$ the orbit of ξ_2 in $\mathfrak{m}$ under the adjoint action of M.

Let σ be a unitary irreducible representation of M on a complex (finite-dimensional) vector space E. Henceforth, we assume that σ is associated with the orbit $o(\xi_2)$ in the following sense, see [**32**, Section 4]. For a maximal torus T of M with Lie algebra $\mathfrak{t}$, $i\beta(\xi_2, \cdot) \in i\mathfrak{t}^*$ is a highest weight for σ.

Now we consider the unitarily induced representation

$$\widehat{\pi} = \mathrm{Ind}_{\mathrm{MAN}}^{G}(\sigma \otimes \exp(i\nu) \otimes 1_N)$$

where $\nu = \beta(\xi_1, \cdot) \in \mathfrak{a}^*$. The representation $\widehat{\pi}$ lies in the unitary principal series of G and is usually realized on the space $L^2(\overline{N}, E)$ which is the Hilbert space completion of the space $C_0(\overline{N}, E)$ of compactly supported smooth functions $\phi : \overline{N} \to E$ with respect to the norm defined by

$$\|\phi\|^2 = \int_{\overline{N}} \langle \phi(y), \phi(y) \rangle_E \, dy$$

where dy is the Haar measure on $\overline{N}$ normalized as follows. Let $(E_i)_{1 \leq i \leq n}$ be an orthonormal basis for $\overline{\mathfrak{n}}$ with respect to the scalar product defined by $(Y, Z) := -\beta(Y, \theta(Z))$. Denote by $(Y_1, Y_2, \ldots, Y_n)$ the coordinates of $Y \in \overline{\mathfrak{n}}$ in this basis, and let $dY = dY_1 dY_2 \ldots dY_n$ be the Euclidean measure on $\overline{\mathfrak{n}}$. The exponential map exp is a diffeomorphism from $\overline{\mathfrak{n}}$ onto $\overline{N}$ and we set $dy = \log^*(dY)$ where $\log = \exp^{-1}$.

Recall that $\overline{N}MAN$ is an open dense subset of G. We denote by $g = \overline{n}(g)m(g)a(g)n(g)$ the decomposition of $g \in \overline{N}MAN$. For $g \in G$

the action of the operator $\widetilde{\pi}(g)$ is given by

$$(2.1) \qquad (\widetilde{\pi}(g)\phi)(y) = e^{-(\rho+i\nu)\log a(g^{-1}y)}\sigma(m(g^{-1}y))^{-1}\,\phi(\overline{n}(g^{-1}y))$$

where $\rho(H) := 1/2\mathrm{Tr}_{\overline{\mathfrak{n}}}(\mathrm{ad}\,H) = 1/2\sum_{\lambda\in\Delta^+}\lambda$.

Recall that we have the Iwasawa decomposition $G = KAN$. We denote by $g = \widetilde{k}(g)\widetilde{a}(g)\widetilde{n}(g)$ the decomposition of $g \in G$.

In order to simplify the study of the contraction, we slightly modify the preceding realization of $\widehat{\pi}$ as follows. Let I be the unitary isomorphism of $L^2(\overline{N}, E)$ defined by

$$(I\phi)(y) = e^{-i\nu(\log\widetilde{a}(y))}\phi(y).$$

Then we introduce the realization π of $\widehat{\pi}$ defined by $\pi(g) := I^{-1}\widetilde{\pi}(g)I$ for each $g \in G$. We immediately obtain

(2.2)
$$(\pi(g)\phi)(y) = e^{i\nu(\log\widetilde{a}(y)-\log\widetilde{a}(\overline{n}(g^{-1}y)))}e^{-(\rho+i\nu)\log a(g^{-1}y)}\sigma(m(g^{-1}y))^{-1}\,\phi(\overline{n}(g^{-1}y)).$$

Formula (2.2) can be simplified as follows. For $g \in G$ and $y \in \overline{n}$, we can write

$$\begin{aligned} g^{-1}y &= \overline{n}(g^{-1}y)m(g^{-1}y)a(g^{-1}y)n(g^{-1}y) \\ &= \widetilde{k}(\overline{n}(g^{-1}y))\widetilde{a}(\overline{n}(g^{-1}y))\widetilde{n}(\overline{n}(g^{-1}y))m(g^{-1}y)a(g^{-1}y)n(g^{-1}y). \end{aligned}$$

Then we have

$$\widetilde{a}(g^{-1}y) = \widetilde{a}(\overline{n}(g^{-1}y))a(g^{-1}y).$$

Hence, we obtain

$$(2.3) \quad (\pi(g)\phi)(y) = e^{i\nu(\log\widetilde{a}(y)-\log\widetilde{a}(g^{-1}y))}e^{-\rho(\log a(g^{-1}y))}\sigma(m(g^{-1}y))^{-1}\,\phi(\overline{n}(g^{-1}y)).$$

Now we give an explicit formula for the differential $d\pi$ of π. Let us introduce some additional notation. If H is a Lie group and X is an element of the Lie algebra of H, then we denote by X^+ the right-invariant vector field generated by X, that is, $X^+(h) = d/dt(\exp(tX))h|_{t=0}$ for

$h \in H$. We denote by $p_{\mathfrak{a}}$, $p_{\mathfrak{m}}$ and $p_{\overline{\mathfrak{n}}}$ the projection operators of $\mathfrak{g}$ on $\mathfrak{a}$, $\mathfrak{m}$ and $\overline{\mathfrak{n}}$ associated with the decomposition $\mathfrak{g} = \overline{\mathfrak{n}} \oplus \mathfrak{m} \oplus \mathfrak{a} \oplus \mathfrak{n}$. Moreover, we also denote by $\widetilde{p}_{\mathfrak{a}}$ the projection operator of $\mathfrak{g}$ on $\mathfrak{a}$ associated with the decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{a} \oplus \mathfrak{n}$. We need the following lemma.

Lemma 2.1. 1) *For each $X \in \mathfrak{g}$ and each $y \in \overline{N}$, we have*

$$\begin{aligned} \frac{d}{dt}\, a(\exp(tX)y)\Big|_{t=0} &= p_{\mathfrak{a}}(Ad\,(y^{-1})X) \\ \frac{d}{dt}\, m(\exp(tX)y)\Big|_{t=0} &= p_{\mathfrak{m}}(Ad\,(y^{-1})X) \\ \frac{d}{dt}\, \overline{n}(\exp(tX)y)\Big|_{t=0} &= (Ad\,(y)\, p_{\overline{\mathfrak{n}}}(Ad\,(y^{-1})\, X))^{+}(y). \end{aligned}$$

2) *For each $X \in \mathfrak{g}$ and each $g \in G$, we have*

$$\frac{d}{dt}\, \widetilde{a}(\exp(tX)g)\Big|_{t=0} = (\widetilde{p}_{\mathfrak{a}}(Ad\,(\widetilde{k}(g)^{-1})X))^{+}(\widetilde{a}(g)).$$

Proof. To prove 1), we consider the diffeomorphism $\mu : \overline{N} \times M \times A \times N \to \overline{N}MAN$ defined by $\mu(y,m,a,n) = yman$. For $y \in \overline{N}$, $Y \in \overline{\mathfrak{n}}$, $U \in \mathfrak{m}$, $H \in \mathfrak{a}$ and $Z \in \mathfrak{n}$, we have

$$\begin{aligned} (2.4) \quad d\mu(y,e,e,e)&(Y^{+}(y), U, H, Z) \\ &= \frac{d}{dt}\, \exp(tY)y\exp(tU)\exp(tH)\exp(tZ)\Big|_{t=0} \\ &= (Y + \mathrm{Ad}\,(y)(U + H + Z))^{+}(y). \end{aligned}$$

Now, let $X \in \mathfrak{g}$. We can write $\mathrm{Ad}\,(y^{-1})X = Y_0 + U + H + Z$ where $Y_0 \in \overline{\mathfrak{n}}$, $U \in \mathfrak{m}$, $H \in \mathfrak{a}$ and $Z \in \mathfrak{n}$. Then equality (2.4) implies that $d\overline{n}(y)(X^{+}(y)) = (\mathrm{Ad}\,(y)\, Y_0)^{+}(y)$. This proves the last equality of 1). The other equalities are proved similarly. Finally, we prove 2) analogously. □

From this lemma, we immediately deduce the following proposition.

Proposition 2.2. *For $X \in \mathfrak{g}$, $\phi \in C_0(\overline{N}, E)$ and $y \in \overline{N}$, we have*

$$\begin{aligned}(d\pi(X)\phi)(y) &= i\nu(\widetilde{p}_{\mathfrak{a}}(Ad\,(\widetilde{k}(y)^{-1})X))\phi(y)\\ &+ \rho(p_{\mathfrak{a}}(Ad\,(y^{-1})X))\phi(y)\\ &+ d\sigma(p_{\mathfrak{m}}(Ad\,(y^{-1})X))\phi(y)\\ &- d\phi(y)(Ad\,(y)\,p_{\overline{\mathfrak{n}}}(Ad\,(y^{-1})\,X))^{+}(y).\end{aligned}$$

3. Representations of the Cartan motion group. We retain the notation from Section 2. In particular, we have the Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus V$ where V is the orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$ with respect to the Killing form β. We denote by $p^c_{\mathfrak{k}}$ and p^c_V the projections of $\mathfrak{g}$ on $\mathfrak{k}$ and V associated with the Cartan decomposition.

We form the semidirect product $G_0 := V \rtimes K$. The group law of G_0 is given by

$$(v, k).(v', k') = (v + \mathrm{Ad}\,(k)v', kk')$$

for v, v' in V and k, $k' \in K$. The Lie algebra $\mathfrak{g}_0$ of G_0 is the space $V \times \mathfrak{k}$ endowed with the Lie bracket

$$[(w, U), (w', U')]_0 = ([U, w'] - [U', w], [U, U'])$$

for w, w' in V and U, U' in $\mathfrak{k}$.

Recall that β is positive definite on V and negative definite on $\mathfrak{k}$ [**17**, page 184]. Then, by using β, we can identify V^* to V and $\mathfrak{k}^*$ to $\mathfrak{k}$; hence, $\mathfrak{g}_0^* \simeq V^* \times \mathfrak{k}^*$ to $V \times \mathfrak{k}$. Under this identification, the coadjoint action of G_0 on $\mathfrak{g}_0^* \simeq V \times \mathfrak{k}$ is then given by

$$(v, k) \cdot (w, U) = (\mathrm{Ad}\,(k)w, \mathrm{Ad}\,(k)U + [v, \mathrm{Ad}\,(k)w])$$

for v, w in V, k in K and U in $\mathfrak{k}$. This is a particular case of the general formula for the coadjoint action of a semidirect product, see for instance [**25**].

We need the following lemma.

Lemma 3.1. *For each regular element ξ_1 of $\mathfrak{a}$, the space* $\mathrm{ad}\,\xi_1\,(V)$ *is the orthogonal complement of $\mathfrak{m}$ in $\mathfrak{k}$.*

Proof. For each $\lambda \in \Delta^+$, let $E_\lambda \neq 0$ be in $\mathfrak{g}_\lambda$. Note that the space $p^c_{\mathfrak{k}}(\mathfrak{n}) = p^c_{\mathfrak{k}}(\overline{\mathfrak{n}})$ is generated by the elements $E_\lambda + \theta(E_\lambda)$ and hence orthogonal to $\mathfrak{m}$. Now, by applying $p^c_{\mathfrak{k}}$ to the decomposition $\mathfrak{g} = \mathfrak{m} + \mathfrak{a} + \mathfrak{n} + \overline{\mathfrak{n}}$, we get $\mathfrak{k} = \mathfrak{m} + p^c_{\mathfrak{k}}(\mathfrak{n})$. This shows that $p^c_{\mathfrak{k}}(\mathfrak{n})$ is the orthogonal complement of $\mathfrak{m}$ in $\mathfrak{k}$. On the other hand, by applying p^c_V to the preceding decomposition of $\mathfrak{g}$, we obtain $V = \mathfrak{a} + p^c_V(\mathfrak{n})$. Since $p^c_V(\mathfrak{n})$ is generated by the elements $E_\lambda - \theta(E_\lambda)$, the space $\mathrm{ad}\,\xi_1\,(V)$ is then generated by the elements

$$\mathrm{ad}\,\xi_1\,(E_\lambda - \theta(E_\lambda)) = \lambda(\xi_1)(E_\lambda + \theta(E_\lambda))$$

where $\lambda(\xi_1) \neq 0$ for $\lambda \in \Delta$. Hence $\mathrm{ad}\,\xi_1\,(V) = p^c_{\mathfrak{k}}(\mathfrak{n})$ is the orthogonal complement of $\mathfrak{m}$ in $\mathfrak{k}$. $\square$

The coadjoint orbits of the semidirect product of a Lie group by a vector space were described by Rawnsley in [**25**]. For each $(w, U) \in \mathfrak{g}_0^* \simeq \mathfrak{g}_0$, we denote by $O(w, U)$ the orbit of (w, U) under the coadjoint action of G_0. The following lemma shows that, for almost all (w, U), the orbit $O(w, U)$ is of the form $O(\xi_1, \xi_2)$ with $\xi_1 \in \mathfrak{a}$ and $\xi_2 \in \mathfrak{m}$.

Lemma 3.2. 1) *Let $\mathcal{O}$ be a coadjoint orbit for the coadjoint action of G_0 on $\mathfrak{g}_0^* \simeq \mathfrak{g}_0$. Then there exists an element of $\mathcal{O}$ of the form (ξ_1, U) with $\xi_1 \in \mathfrak{a}$. Moreover, if ξ_1 is regular, then there exists a $\xi_2 \in \mathfrak{m}$ such that $(\xi_1, \xi_2) \in \mathcal{O}$.*

2) *Let ξ_1 be a regular element of $\mathfrak{a}$. Then M is the stabilizer of ξ_1 in K.*

Proof. 1) Let $(w, U) \in \mathcal{O}$. For each $k \in K$, we have $(0, k) \cdot (w, U) = (\mathrm{Ad}\,(k)w, \mathrm{Ad}\,(k)U)$. By [**21**, p. 120], we can choose $k \in K$ so that $\mathrm{Ad}\,(k)w \in \mathfrak{a}$. We set $\xi_1 := \mathrm{Ad}\,(k)w$. If we assume that ξ_1 is regular, then by Lemma 3.1 we can write $U = \xi_2 + [\xi_1, v]$ where $\xi_2 \in \mathfrak{m}$ and $v \in V$. Then $(\xi_1, U) = (v, e) \cdot (\xi_1, \xi_2)$. Hence, $\mathcal{O} = O(\xi_1, \xi_2)$.

2) Denote by $K(\xi_1)$ the stabilizer of ξ_1 in K and by $\mathfrak{g}(\xi_1)$ the centralizer of ξ_1 in $\mathfrak{g}$. Since ξ_1 regular, we have $\mathfrak{g}(\xi_1) = \mathfrak{a} \oplus \mathfrak{m}$ [**17**, page 263].

Let $k \in K(\xi_1)$. Then $\mathrm{Ad}\,(k)$ leaves $\mathfrak{g}(\xi_1)$ invariant. Thus, $\mathfrak{g}(\xi_1) \cap V = \mathfrak{a}$ is also invariant under $\mathrm{Ad}\,(k)$. Hence, $k \in M$. This shows that $K(\xi_1) \subset M$. Finally, $K(\xi_1) = M$. $\square$

In the rest of the section, we consider the orbit $O(\xi_1,\xi_2)$ of $(\xi_1,\xi_2)\in\mathfrak{a}\times\mathfrak{m}\subset\mathfrak{g}_0^*\simeq\mathfrak{g}_0$ under the coadjoint action of G_0. As in Section 2, we assume that ξ_1 is a regular element of $\mathfrak{a}$ and that the adjoint orbit $o(\xi_2)$ of ξ_2 in $\mathfrak{m}$ is associated with a unitary irreducible representation σ of M which is realized on a (finite-dimensional) Hilbert space E. Then $O(\xi_1,\xi_2)$ is associated with the unitarily induced representation

$$\widehat{\pi}_0=\mathrm{Ind}_{V\times M}^{G_0}(e^{i\nu}\otimes\sigma)$$

where $\nu=\beta(\xi_1,\cdot)\in\mathfrak{a}^*$ (see [**22, 25**]). By a result of Mackey, $\widehat{\pi}_0$ is irreducible since σ is irreducible [**29**].

Let $O_V(\xi_1)$ be the orbit of ξ_1 in V under the action of K. We denote by μ the K-invariant measure on $O_V(\xi_1)\simeq K/M$. We denote by $\widetilde{\pi}_0$ the usual realization of $\widehat{\pi}_0$ on the space of square-integrable sections of a Hermitian vector bundle over $O_V(\xi_1)$ [**22, 25, 28**]. Let us briefly describe the construction of $\widetilde{\pi}_0$. We introduce the Hilbert G_0-bundle $L:=G_0\times_{e^{i\nu}\otimes\sigma}E$ over $O_V(\xi_1)\simeq K/M$. Recall that an element of L is an equivalence class

$$[g,u]=\{(g.(v,m),e^{-i\nu(v)}\sigma(m)^{-1}u):v\in V,\,m\in M\}$$

where $g\in G_0$, $u\in E$ and that G_0 acts on L by left translations: $g\,[g',u]:=[gg',u]$. The action of G_0 on $O_V(\xi_1)\simeq K/M$ being given by $(v,k)\cdot\xi=\mathrm{Ad}\,(k)\xi$, the projection map $[(v,k),u]\to\mathrm{Ad}\,(k)\xi_1$ is G_0-equivariant. The G_0-invariant Hermitian structure on L is given by

$$\langle[g,u],[g,u']\rangle=\langle u,u'\rangle_E$$

where $g\in G_0$ and $u,\,u'\in E$. Let $\mathcal{H}_0$ be the space of sections s of L which are square-integrable with respect to the measure μ, that is,

$$\|s\|^2_{\mathcal{H}_0}=\int_{O_V(\xi_1)}\langle s(\xi),\,s(\xi)\rangle\,d\mu(\xi)<+\infty.$$

Then $\widetilde{\pi}_0$ is the action of G_0 on $\mathcal{H}_0$ defined by

$$(\widetilde{\pi}_0(g)\,s)(\xi)=g\,s(g^{-1}\cdot\xi).$$

For the study of contractions, it is more convenient to realize $\widehat{\pi}_0$ in the Hilbert space $L^2(\overline{N},E)$ introduced in Section 2. To this aim, we consider the map $\tau : y \to \mathrm{Ad}\,(\widetilde{k}(y))\xi_1$ which is a diffeomorphism from $\overline{n}$ onto a dense open subset of $O_V(\xi_1)$ [**31**, Lemma 7.6.8]. We denote by $k \cdot y$ the action of $k \in K$ on $y \in \overline{N}$ defined by $\tau(k \cdot y) = \mathrm{Ad}\,(k)\tau(y)$ or, equivalently, by $k \cdot y = \overline{n}(ky)$. Then the K-invariant measure on $\overline{N}$ is given by $(\tau^{-1})^*(\mu) = e^{-2\rho(\log \widetilde{a}(y))}dy$ [**31**, Lemma 7.6.8]. We associate with each $s \in \mathcal{H}_0$ the function $\phi_s : \overline{N} \to E$ defined by

$$s(\tau(y)) = [(0, \widetilde{k}(y)),\, e^{\rho(\log \widetilde{a}(y))}\phi_s(y)].$$

For s and s' in $\mathcal{H}_0$, we have

$$\langle s(\tau(y)), s'(\tau(y))\rangle = e^{2\rho(\log \widetilde{a}(y))}\,\langle \phi_s(y),\, \phi_{s'}(y)\rangle_E.$$

This implies that

$$\langle s,\, s'\rangle_{\mathcal{H}_0} = \int_{\overline{N}} \langle \phi_s(y),\, \phi_{s'}(y)\rangle_E\, dy.$$

Moreover, for $s \in \mathcal{H}_0$, $g = (v, k) \in G_0$ and $y \in \overline{N}$, we have

$$\begin{aligned}
(\widetilde{\pi}_0(g)s)(\tau(y)) &= g\, s(g^{-1} \cdot y) = g\, s(\tau(k^{-1} \cdot y)) \\
&= (v, k)\,[(0, \widetilde{k}(k^{-1} \cdot y)),\, e^{\rho(\log \widetilde{a}(k^{-1} \cdot y))}\phi_s(k^{-1} \cdot y)] \\
&= [(v, k\widetilde{k}(k^{-1} \cdot y)),\, e^{\rho(\log \widetilde{a}(k^{-1} \cdot y))}\phi_s(k^{-1} \cdot y)] \\
&= e^{\rho(\log \widetilde{a}(k^{-1} \cdot y))}[(0, \widetilde{k}(y)) \\
&\qquad\qquad \cdot (\mathrm{Ad}\,(\widetilde{k}(y))^{-1}v, m(k, y)),\, \phi_s(k^{-1} \cdot y)] \\
&= e^{\rho(\log \widetilde{a}(k^{-1} \cdot y)) + i\nu(\mathrm{Ad}\,(\widetilde{k}(y))^{-1}v)} \\
&\quad \times [(0, \widetilde{k}(y)),\, \sigma(m(k, y))\phi_s(k^{-1} \cdot y)],
\end{aligned}$$

where we have set $m(k, y) := \widetilde{k}(y)^{-1}k\widetilde{k}(k^{-1} \cdot y) \in M$. Hence, we see that the equality

$$\begin{aligned}
(3.1)\quad &(\pi_0(v, k)\phi)(y) \\
&= e^{i\beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1, v) + \rho(\log \widetilde{a}(k^{-1} \cdot y) - \log \widetilde{a}(y))}\sigma(m(k, y))\phi(k^{-1} \cdot y)
\end{aligned}$$

defines a unitary representation π_0 of G_0 on $L^2(\overline{N}, E)$ which is unitarily equivalent to $\widetilde{\pi}_0$, the intertwining operator between π_0 and $\widetilde{\pi}_0$ being $s \to \phi_s$.

We can simplify formula (3.1) as follows. Let $k \in K$ and $y \in \overline{N}$. Write $k^{-1}y = \overline{n}(k^{-1}y)m(k^{-1}y)a(k^{-1}y)n(k^{-1}y)$. Then $k^{-1}\widetilde{k}(y) = \widetilde{k}(\overline{n}(k^{-1}y))m(k^{-1}y)$. Thus, $m(k,y) = \widetilde{k}(y)^{-1}k\widetilde{k}(k^{-1}\cdot y) = m(k^{-1}y)^{-1}$. We also see that

$$\widetilde{a}(y) = \widetilde{a}(k^{-1}y) = \widetilde{a}(\overline{n}(k^{-1}y))a(k^{-1}y) = \widetilde{a}(k^{-1}\cdot y)a(k^{-1}y).$$

Hence, we obtain

$$\begin{aligned} (3.2)\quad & (\pi_0(v,k)\phi)(y) \\ &= e^{-\rho(\log a(k^{-1}y)+i\beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1,v))}\sigma(m(k^{-1}y))^{-1}\phi(\overline{n}(k^{-1}y)). \end{aligned}$$

The computation of $d\pi_0$ is quite similar to that of $d\pi$ (see Section 2). By using Lemma 2.1, we easily obtain the following result.

Proposition 3.3. *For $(v,U) \in \mathfrak{g}_0$, $\phi \in C_0(\overline{N},E)$ and $y \in \overline{N}$, we have*

$$\begin{aligned} (d\pi_0(v,U)\phi)(y) = {} & i\beta(Ad\,(\widetilde{k}(y))\xi_1,v)\phi(y) \\ & + \rho(p_{\mathfrak{a}}(Ad\,(y^{-1})U))\phi(y) \\ & + d\sigma(p_{\mathfrak{m}}(Ad\,(y^{-1})U))\phi(y) \\ & - d\phi(y)(Ad\,(y)\,p_{\overline{\mathfrak{n}}}(Ad\,(y^{-1})\,U))^+(y). \end{aligned}$$

4. Berezin-Weyl calculus. In this section, we keep the notation of the previous sections. We recall some properties of the Berezin calculus on $o(\xi_2)$ and of the Berezin-Weyl calculus on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ which was introduced in [**2**] as a generalization of the usual Weyl calculus.

The Berezin calculus on $o(\xi_2)$ associates with each operator B on the finite-dimensional complex vector space E a complex-valued function $s(B)$ on the orbit $o(\xi_2)$, which is called the symbol of B (see [**1**]). The following properties of the Berezin calculus can be found in [**2, 10, 11**].

Proposition 4.1. (1) *The map $B \to s(B)$ is injective.*

(2) *For each operator B on E, we have $s(B^*) = \overline{s(B)}$.*

(3) *For each operator B on E, each $m \in M$ and each $\varphi \in o(\xi_2)$, we have*

$$s(B)(\mathrm{Ad}\,(m)\varphi) = s(\sigma(m)^{-1}B\sigma(m))(\varphi).$$

(4) *For* $X \in \mathfrak{m}$ *and* $\varphi \in o(\xi_2)$, *we have* $s(d\sigma(X))(\varphi) = i\beta(\varphi, X)$.

In particular, note that the map s^{-1} is an adapted Weyl transform on $o(\xi_2)$ in the sense of [**6**] (see also Section 5).

Now we introduce the Berezin-Weyl calculus on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ as a slight modification of the usual Weyl calculus for $\operatorname{End}(E)$-valued functions [**18**]. We say that a smooth function $f : (y, Z, \varphi) \to f(y, Z, \varphi)$ is a symbol on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ if, for each $(y, Z) \in \overline{N} \times \overline{\mathfrak{n}}$, the function $\varphi \to f(y, Z, \varphi)$ is the symbol, in the Berezin calculus on $o(\xi_2)$, of an operator on E denoted by $\widehat{f}(y, Z)$. A symbol f on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ is called an S-symbol if the function $\widehat{f}$ belongs to the Schwartz space of rapidly decreasing smooth functions on $\overline{N} \times \overline{\mathfrak{n}}$ with values in $\operatorname{End}(E)$. For each S-symbol f on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$, we define an operator $W(f)$ on $L^2(\overline{N}, E)$ by

(4.1)
$$(W(f)\phi)(y) = (2\pi)^{-n} \int_{\overline{\mathfrak{n}} \times \overline{\mathfrak{n}}} e^{i\langle T, Z\rangle} \widehat{f}(y \exp(T/2), Z)\, \phi(y \exp T)\, dT\, dZ$$

for $\phi \in C_0(\overline{N}, E)$.

As the usual Weyl calculus, the Weyl-Berezin calculus can be extended to much larger classes of symbols. Here we only consider a class of polynomial symbols. For $Z \in \overline{\mathfrak{n}}$, we denote by $(z_1, z_2, \ldots, z_n)$ the coordinates of Z in the basis $(E_i)_{1 \le i \le n}$ of $\overline{\mathfrak{n}}$. We say that a symbol f on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ is a P-symbol if the function $\widehat{f}(y, Z)$ is polynomial in $z_1, z_2, \ldots, z_n$. Let f be the P-symbol defined by $f(y, Z, \varphi) = u(y) z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_n^{\alpha_n}$ where $u \in C^\infty(\overline{N})$. By imitating [**30**, page 105], we get

(4.2)
$$(W(f)\phi)(y) = (i\partial_{z_1})^{\alpha_1} (i\partial_{z_2})^{\alpha_2} \cdots (i\partial_{z_n})^{\alpha_n} (u(y \exp Z/2)\, \phi(y \exp Z))\big|_{Z=0}.$$

In particular, if $f(y, Z, \varphi) = u(y)$ where $u \in C^\infty(\overline{N})$, then

$$(W(f)\phi)(y) = u(y)\, \phi(y) \tag{4.3}$$

and, if $f(y,Z,\varphi)=(v(y),Z)$ where $v\in C^\infty(\overline{N},\overline{\mathfrak{n}})$, then

$$(4.4)\quad (W(f)\phi)(y)=i\Big(\sum_{k=1}^{n}\frac{d}{dt}(E_k,\,v(y\exp(tE_k/2)))\Big|_{t=0}\phi(T)\\ +\frac{d}{dt}\,\phi(y\exp(tv(y)))\Big|_{t=0}\Big).$$

The following lemmas will be needed in Sections 5 and 6.

Lemma 4.2. *Let $X\in\mathfrak{g}$, and let f be the* P*-symbol on $\overline{N}\times\overline{\mathfrak{n}}\times o(\xi_2)$ defined by $f(y,Z,\varphi)=(p_{\overline{\mathfrak{n}}}(\mathrm{Ad}\,(y^{-1})X),Z)$. Then we have*

$$W(f)\phi(y)=-i\rho(p_{\mathfrak{a}}(\mathrm{Ad}\,(y^{-1})X))\,\phi(y)\\ +i(d\phi)(y)(\mathrm{Ad}\,(y)p_{\overline{\mathfrak{n}}}(\mathrm{Ad}\,(y^{-1})X))^{+}(y)$$

for each $\phi\in C_0(\overline{N},E)$.

Proof. We apply formula (4.4) to $f(y,Z,\varphi)=(p_{\overline{\mathfrak{n}}}(\mathrm{Ad}\,(y^{-1})X),Z)$. On the one hand, we have

$$\sum_{k=1}^{n}\frac{d}{dt}(E_k,\,v(y\exp(tE_k/2)))\Big|_{t=0}=-\frac{1}{2}\sum_{k=1}^{n}(E_k,p_{\overline{\mathfrak{n}}}(\mathrm{ad}\,E_k(\mathrm{Ad}\,(y^{-1})X)))\\ =\frac{1}{2}\,\mathrm{Tr}_{\overline{\mathfrak{n}}}(p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,(\mathrm{Ad}\,(y^{-1})X)).$$

But, for each $Y\in\mathfrak{g}$, we have $\mathrm{Tr}_{\overline{\mathfrak{n}}}(p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,(Y))=-2\rho(p_{\mathfrak{a}}(Y))$. This equality can be proved as follows. If $Y\in\overline{\mathfrak{n}}$, then $p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,Y=\mathrm{ad}\,Y$ is a nilpotent endomorphism of $\overline{\mathfrak{n}}$. Thus, $\mathrm{Tr}_{\overline{\mathfrak{n}}}(p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,Y)=0$. If $Y\in\mathfrak{n}$, then, since $[\mathfrak{n},\mathfrak{g}_\lambda]\subset\mathfrak{a}+\sum_{\mu>\lambda}\mathfrak{g}_\mu$ for each $\lambda<0$, we also have that $\mathrm{Tr}_{\overline{\mathfrak{n}}}(p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,Y)=0$. If $X\in\mathfrak{m}$, then $p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,Y=\mathrm{ad}\,Y$ is an endomorphism of $\overline{\mathfrak{n}}$ which is skew-symmetric with respect to $(\cdot,\cdot)$. Thus, $\mathrm{Tr}_{\overline{\mathfrak{n}}}(p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,Y)=0$. Finally, if $Y\in\mathfrak{a}$, then $\mathrm{Tr}_{\overline{\mathfrak{n}}}(p_{\overline{\mathfrak{n}}}\circ\mathrm{ad}\,Y)=\mathrm{Tr}_{\overline{\mathfrak{n}}}(\mathrm{ad}\,Y)=-2\rho(Y)$.

Then we get

$$(4.5)\qquad \sum_{k=1}^{n}\frac{d}{dt}(E_k,\,v(y\exp(tE_k/2)))\Big|_{t=0}=-\rho(p_{\mathfrak{a}}(\mathrm{Ad}\,(y^{-1})X)).$$

On the other hand, we have

$$\frac{d}{dt}\,\phi(y\exp(tv(y)))\Big|_{t=0} = (d\phi)(y)(\mathrm{Ad}\,(y)v(y))^+(y). \tag{4.6}$$

Putting (4.5) and (4.6) together, we get the desired result. □

We can identify the cotangent bundle $T^*\overline{N}$ with $\overline{N}\times\overline{\mathfrak{n}}$ by using the map $j:\overline{N}\times\overline{\mathfrak{n}}\to T^*\overline{N}$ defined by

$$\langle j(y,Z),Y^+(y)\rangle = -\beta(\theta(Z),\mathrm{Ad}\,(y^{-1})Y)$$

for $y\in\overline{N}$ and $Y,\,Z\in\overline{\mathfrak{n}}$. Under this identification, the Liouville 1-form on $T^*\overline{N}$ corresponds to the 1-form α on $\overline{N}\times\overline{\mathfrak{n}}$ given by

$$\alpha_{(y,Z)}(Y^+(y),T) = -\beta(\theta(Z),\mathrm{Ad}\,(y^{-1})Y)$$

for $y\in\overline{N}$ and $Z,\,Y,\,T\in\overline{\mathfrak{n}}$. We denote by $\{\cdot,\cdot\}_1$ the Poisson bracket associated with the symplectic 2-form $d\alpha$ on $\overline{N}\times\overline{\mathfrak{n}}$. We also denote by ω_2 the Kirillov 2-form on $o(\xi_2)$ and by $\{\cdot,\cdot\}_2$ the corresponding Poisson bracket. We form the symplectic product $\overline{N}\times\overline{\mathfrak{n}}\times o(\xi_2)$ and denote by $\{\cdot,\cdot\}_p$ the Poisson bracket associated with the symplectic form $\omega_p := d\alpha\otimes\omega_2$. Let $u,\,v\in C^\infty(\overline{N}\times\overline{\mathfrak{n}})$ and $a,\,b\in C^\infty(o(\xi_2))$. Then, for $f(y,Z,\varphi)=u(y,Z)a(\varphi)$ and $g(y,Z,\varphi)=v(y,Z)b(\varphi)$, we have

$$\{f,\,g\}_p = u(y,Z)v(y,Z)\{a,\,b\}_2 + a(\varphi)b(\varphi)\{u,\,v\}_1.$$

Lemma 4.3. *Let f and g be two* P*-symbols on $\overline{N}\times\overline{\mathfrak{n}}\times o(\xi_2)$ of the form*

$$u(y)+\beta(v(y),\varphi)+\sum_{k=1}^{n} w_k(y)z_k$$

where $u\in C^\infty(\overline{N})$, $v\in C^\infty(\overline{N},\overline{\mathfrak{n}})$ and $w_k\in C^\infty(\overline{N})$ for $k=1,2,\ldots,n$. Then we have

$$[W(f),\,W(g)] = -i\,W(\{f,\,g\}_p).$$

Proof. We can prove this lemma by a case-by-case verification. The computations are easy but tedious. For instance, take $f(y,Z,\varphi)=w(y)z_k$ and $g(y,Z,\varphi)=w'(y)z_l$. For $Y\in\overline{\mathfrak{n}}$ and $u\in C^\infty(\overline{N})$, we set

$Y(u)(y) := (d/dt)u(y\exp(tY))|_{t=0}$ for each $y \in \overline{N}$. We can easily verify that

$$\{f, g\}_p = -E_k(w')(y)w(y)z_l + E_l(w)(y)w'(y)z_k + w(y)w'(y)\{z_k, z_l\}$$

where $\{z_k, z_l\} = \beta(\theta(Z), [E_k, E_l])$. This implies that

$$\begin{aligned} W(-i\{f,\, g\})\phi = \frac{1}{2}E_l E_k(w)w'\phi - \frac{1}{2}wE_kE_l(w')\phi \\ - E_k(w')wE_l(\phi) + E_l(w)w'E_k(\phi) - ww'[E_k, E_l](\phi), \end{aligned}$$

which is precisely $[W(f), W(g)]$. The calculations in the other cases are similar. □

5. Adapted Weyl correspondence for π. In this section, we first compute the Berezin-Weyl symbol of the operator $-id\pi(X)$ for $X \in \mathfrak{g}$. This dequantization process allows us to obtain an explicit symplectomorphism from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ onto a dense open subset of the orbit $O(\xi_0)$ and then to construct an adapted Weyl correspondence on $O(\xi_0)$.

Proposition 5.1. *Let Ψ be the map from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ to $\mathfrak{g}$ defined by*

$$\Psi(y, Z, \varphi) = \mathrm{Ad}\,(\widetilde{k}(y))\xi_1 + \mathrm{Ad}\,(y)(\varphi - \theta(Z)).$$

Then, for each $X \in \mathfrak{g}$, the Berezin-Weyl symbol of the operator $-id\pi(X)$ is the P*-symbol f_X defined by*

$$f_X(y, Z, \varphi) = \beta(\Psi(y, Z, \varphi),\, X).$$

Proof. Let $X \in \mathfrak{g}$. Recall that an explicit expression for $-id\pi(X)$ was given in Proposition 2.2. Then, by Proposition 4.1 and Lemma 4.2, we immediately see that the Berezin-Weyl symbol of the operator $-id\pi(X)$ is the function f_X defined by

$$\begin{aligned} f_X(y, Z, \varphi) = (p_{\overline{\mathfrak{n}}}(\mathrm{Ad}\,(y^{-1})X), Z) \\ + \beta(\varphi, p_{\mathfrak{m}}(\mathrm{Ad}\,(y^{-1})X)) \\ + \nu(\widetilde{p}_{\mathfrak{a}}(\mathrm{Ad}\,(\widetilde{k}(y)^{-1})X)). \end{aligned} \tag{5.1}$$

Now, let $(y, Z, \varphi) \in \overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$. Since the map $X \to f_X(y, Z, \varphi)$ is linear, there exists an element $\Psi(y, Z, \varphi)$ in $\mathfrak{g}$ such that $f_X(y, Z, \varphi) = \beta(\Psi(y, Z, \varphi), X)$ for each $X \in \mathfrak{g}$. More precisely, by using equality (5.1) we get

$$\begin{aligned} f_X(y, Z, \varphi) &= -\beta(\mathrm{Ad}\,(y^{-1})X, \theta(Z)) + \beta(\mathrm{Ad}\,(y^{-1})X, \varphi) \\ &\quad + \beta(\xi_1, \mathrm{Ad}\,(\widetilde{k}(y)^{-1})X) \\ &= \beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1 + \mathrm{Ad}\,(y)(\varphi - \theta(Z)), X). \end{aligned}$$

This gives the desired result. $\square$

We denote by ω the Kirillov 2-form on $O(\xi_0)$ and by $\{\cdot, \cdot\}$ the corresponding Poisson bracket. Let $\widetilde{O}(\xi_0)$ denote the dense open subset $\mathrm{Ad}\,(\overline{N}MAN)\xi_0$ of $\mathfrak{g}$.

Proposition 5.2. *The map Ψ is a symplectomorphism from $(\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2), \omega_p = d\alpha \otimes \omega_2)$ onto $(\widetilde{O}(\xi_0), \omega)$.*

Proof. By [**2**, Proposition 1] and [**7**, Proposition 4.3], the map Ψ_1 from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ onto $\widetilde{O}(\xi_0)$ defined by

$$\Psi_1(y, Z, \varphi) = \mathrm{Ad}\,(y)(\xi_1 + \varphi - \theta(Z))$$

is a diffeomorphism. Note that, if

$$Z' = Z + \theta(\mathrm{Ad}\,(\widetilde{n}(y)^{-1})\xi_1 - \xi_1),$$

then

$$\begin{aligned} \Psi(y, Z, \varphi) &= \mathrm{Ad}\,(\widetilde{k}(y))\xi_1 \\ &\quad + \mathrm{Ad}\,(y)(\varphi - \theta(Z') - \mathrm{Ad}\,(\widetilde{n}(y)^{-1})\xi_1 + \xi_1) \\ &= \mathrm{Ad}\,(\widetilde{k}(y))\xi_1 - \mathrm{Ad}\,(y\widetilde{n}(y)^{-1})\xi_1 \\ &\quad + \Psi_1(y, Z, \varphi) \end{aligned}$$

Thus, since $y\widetilde{n}(y)^{-1} = \widetilde{k}(y)\widetilde{a}(y)$, we obtain $\Psi(y, Z, \varphi) = \Psi_1(y, Z', \varphi)$. Hence, Ψ is a diffeomorphism from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ onto $\widetilde{O}(\xi_0)$.

Now, we show that Ψ is also a symplectomorphism by following the method of [**6**, Theorem 6.3]. Recall that, for $X \in \mathfrak{g}$, $\widetilde{X}$ denotes the function on $O(\xi_0)$ defined by $\widetilde{X}(\xi) = \beta(\xi, X)$. Observe that $f_X \circ \Psi = \widetilde{X}$. Let X and Y in $\mathfrak{g}$. On the one hand, by Proposition 5.1 and Lemma 4.3, we have

$$[W(f_X),\, W(f_Y)] = -iW(\{f_X,\, f_Y\}_p).$$

On the other hand, we have

$$[W(f_X),\, W(f_Y)] = [-id\pi(X), -id\pi(Y)] = -d\pi([X,\, Y]) = -iW(f_{[X,Y]}).$$

Then we get $f_{[X,Y]} = \{f_X,\, f_Y\}_p$. Since $\widetilde{[X, Y]} = \{\widetilde{X}, \widetilde{Y}\}$, we obtain

$$\{\widetilde{X}, \widetilde{Y}\} \circ \Psi = \{\widetilde{X} \circ \Psi,\, \widetilde{Y} \circ \Psi\}_p.$$

Hence, Ψ is a symplectomorphism. $\square$

Now, we obtain an adapted Weyl correspondence on $O(\xi_0)$ by transferring to $O(\xi_0)$ the Berezin-Weyl calculus on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$. We say that a smooth function f on $O(\xi_0)$ is a symbol (respectively, a P-symbol, an S-symbol) on $O(\xi_0)$ if $f \circ \Psi$ is a symbol (respectively, a P-symbol, an S-symbol) for the Berezin-Weyl calculus on $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$.

Proposition 5.3. *Let $\mathcal{A}$ be the space of all* P*-symbols on $O(\xi_0)$, and let $\mathcal{B}$ be the space of differential operators on $C^\infty(\overline{N}, E)$. Then the map $\mathcal{W} : \mathcal{A} \to \mathcal{B}$ defined by the $\mathcal{W}(f) = W(f \circ \Psi)$ is an adapted Weyl correspondence in the sense of [*__6__*, subsection 6.1], that is, the map $\mathcal{W}$ satisfies the following properties*:

(1) *The map $\mathcal{W}$ is a linear isomorphism from $\mathcal{A}$ onto $\mathcal{B}$;*

(2) *the elements of $\mathcal{B}$ preserve a fixed dense domain D of $L^2(\overline{N}, E)$;*

(3) *the constant function* 1 *belongs to $\mathcal{A}$, the identity operator I belongs to $\mathcal{B}$ and $\mathcal{W}(1) = I$;*

(4) *$A \in \mathcal{B}$ and $B \in \mathcal{B}$ implies $AB \in \mathcal{B}$;*

(5) *for each f in $\mathcal{A}$ the complex conjugate $\overline{f}$ of f belongs to $\mathcal{A}$ and the adjoint operator $\mathcal{W}(f)^*$ is an extension of $\mathcal{W}(\overline{f})$;*

(6) *the elements of D are C^∞-vectors for the representation π, the functions $\widetilde{X}$ ($X \in \mathfrak{g}$) are in $\mathcal{A}$ and $\mathcal{W}(i\widetilde{X})\phi = d\pi(X)\phi$ for each $X \in \mathfrak{g}$ and each $\phi \in D$.*

Proof. Properties (1)–(4) are satisfied with $D = C_0(\overline{N}, E)$. Property (5) is a consequence of (2) of Proposition 4.1. Property (6) follows from Proposition 5.1. ◻

6. Adapted Weyl correspondence for π_0. In this section, we use the same method as in Section 5 to get a symplectomorphism from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ onto a dense open subset of the orbit $O(\xi_1, \xi_2) \subset \mathfrak{g}_0$ and then to construct an adapted Weyl correspondence on $O(\xi_1, \xi_2)$.

Proposition 6.1. *Let Ψ_0 be the map from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ to $\mathfrak{g}_0$ defined by*

$$\Psi_0(y, Z, \varphi) = (\mathrm{Ad}\,(\widetilde{k}(y))\xi_1,\, p^c_{\mathfrak{k}}(\mathrm{Ad}\,(y)(\varphi - \theta(Z)))).$$

Then, for each $(v, U) \in \mathfrak{g}_0$, the Berezin-Weyl symbol of the operator $-id\pi(X)$ is the P-symbol $f_{(v,U)}$ defined by

$$f_{(v,U)}(y, Z, \varphi) = \langle \Psi_0(y, Z, \varphi),\, (v, U)\rangle.$$

Proof. The proof is quite similar to that of Proposition 5.1. Let (v, U) be an element of $\mathfrak{g}_0$. By using the explicit expression for $-id\pi_0(X)(v, U)$ given in Proposition 3.3 and Lemma 4.2, we obtain

$$\begin{aligned} f_{(v,U)}(y, Z, \varphi) &= (p_{\overline{\mathfrak{n}}}(\mathrm{Ad}\,(y^{-1})U), Z) + \beta(\mathrm{Ad}\,(\widetilde{k}(y))\,\xi_1, v) \\ &\quad + \beta(\varphi, p_{\mathfrak{m}}(\mathrm{Ad}\,(y^{-1})U)) \\ &= \beta(\mathrm{Ad}\,(y)(\varphi - \theta(Z)), U) + \beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1, v). \end{aligned}$$

This gives the result. ◻

We denote by ω_0 the Kirillov 2-form on $O(\xi_1, \xi_2)$ and by $\{\cdot, \cdot\}_0$ the corresponding Poisson bracket. Let $\widetilde{O}(\xi_1, \xi_2)$ denote the dense open subset of $O(\xi_1, \xi_2)$ defined by

$$\widetilde{O}(\xi_1, \xi_2) = \{(v, k) \cdot (\xi_1, \xi_2) : v \in V,\, k \in K \cap \overline{N}MAN\}.$$

Proposition 6.2. *The map Ψ_0 is a symplectomorphism from $(\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2), \omega_p = d\alpha \otimes \omega_2)$ onto $(\widetilde{O}(\xi_1, \xi_2), \omega_0)$.*

Proof. First we show that, for each $\xi \in \widetilde{O}(\xi_1, \xi_2)$ there exists a unique $(y, Z, \varphi) \in \overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ such that $\Psi_0(y, Z, \varphi) = \xi$. Let $\xi \in \widetilde{O}(\xi_1, \xi_2)$. Then we can write $\xi = (v, k) \cdot (\xi_1, \xi_2)$ with $v \in V$ and $k \in K \cap \overline{N}MAN$. Clearly, the equation $\Psi_0(y, Z, \varphi) = \xi$ is equivalent to

$$\begin{cases} (a) & \mathrm{Ad}\,(\widetilde{k}(y))\xi_1 = \mathrm{Ad}\,k)\xi_1 \\ (b) & p_{\mathfrak{k}}^c(\mathrm{Ad}\,(k^{-1}y)(\varphi - \theta(Z))) = \xi_2 + [\mathrm{Ad}\,(k^{-1})v, \xi_1]. \end{cases}$$

Equation (a) determines y uniquely. Moreover, $m := k^{-1}\widetilde{k}(y)$ is an element of M. We set $n'(y) = m\widetilde{a}(y)\widetilde{n}(y)\widetilde{a}(y)^{-1}m^{-1}$. Then $n'(y) \in N$, and we have

$$k^{-1}y = k^{-1}\widetilde{k}(y)\widetilde{a}(y)\widetilde{n}(y) = m\widetilde{a}(y)\widetilde{n}(y) = n'(y)m\widetilde{a}(y).$$

Thus, setting $Y := \mathrm{Ad}\,(n'(y)m)\varphi - \mathrm{Ad}\,(m)\varphi \in \mathfrak{n}$, we can write

$$\begin{aligned} &p_{\mathfrak{k}}^c(\mathrm{Ad}\,(k^{-1}y)(\varphi - \theta(Z))) \\ &\qquad\qquad = \mathrm{Ad}\,(m)\varphi + p_{\mathfrak{k}}^c(Y - \mathrm{Ad}\,(n'(y)\widetilde{a}(y)m)\theta(Z)). \end{aligned}$$

Hence, using Lemma 3.1, we see that equation (b) is equivalent to

$$\begin{cases} (c) & \mathrm{Ad}\,(m)\varphi = \xi_2 \\ (d) & p_{\mathfrak{k}}^c(Y - \mathrm{Ad}\,(n'(y)\widetilde{a}(y)m)\theta(Z)) = [\mathrm{Ad}\,(k^{-1})v, \xi_1]. \end{cases}$$

Finally, we get $\varphi = \mathrm{Ad}\,(m^{-1})\xi_1$ and, by using Lemma 3.1 again, we see that there exists a unique element Z of $\overline{\mathfrak{n}}$ satisfying equation (d). This proves the existence of a unique element (y, Z, φ) satisfying $\Psi(y, Z, \varphi) = \xi$.

In the same way, we show that Ψ_0 takes values in $\widetilde{O}(\xi_1, \xi_2)$, and we can conclude that Ψ_0 is a bijection from $\overline{N} \times \overline{\mathfrak{n}} \times o(\xi_2)$ onto $\widetilde{O}(\xi_1, \xi_2)$.

By following the same method as in the proof of Proposition 5.2, we show that $\Psi_0^*(\omega_0) = \omega_p$. Since the 2-form ω_p is non-degenerate, this also shows that Ψ_0 is regular. Finally, Ψ_0 is a symplectomorphism. ◻

We can define the notion of symbols (P-symbols, S-symbols) on $O(\xi_1, \xi_2)$ as in Section 5. Then we obtain the following proposition which is analogous to Proposition 5.3.

Proposition 6.3. *Let $\mathcal{A}_0$ be the space of all* P*-symbols on $O(\xi_1, \xi_2)$, and let B be the space of differential operators on $C^\infty(\overline{N}, E)$. Then the map $\mathcal{W}_0 : \mathcal{A}_0 \to \mathcal{B}$ defined by the $\mathcal{W}_0(f) = W(f \circ \Psi_0)$ is an adapted Weyl correspondence in the sense of* [**6**, Section 6.1].

7. The Dooley-Rice contraction revisited. In this section, we introduce the Dooley-Rice contraction maps from G_0 to G, and we show how to use the results of the previous sections in order to get a new version of Theorem 1 of [**16**] for the 'noncompact' realizations of the representations.

We consider the family of maps $c_r : G_0 \to G$ defined by

$$c_r(v,k) = \exp_G(rv)k$$

for $v \in V$, $k \in K$ and indexed by $r \in]0,1]$. One can easily show that

$$\lim_{r \to 0} c_r^{-1}(c_r(g)\, c_r(g')) = g\, g'$$

for each g, $g' \in G_0$. Then the family (c_r) is a group contraction of G to G_0 in the sense of [**24**] (see also [**5**]).

Let $(\xi_1, \xi_2) \in \mathfrak{g}_0$ as in Section 3. Recall that π_0 is a unitary irreducible representation of G_0 associated with (ξ_1, ξ_2). For each $r \in]0,1]$, we set $\xi_r := (1/r)\xi_1 + \xi_2$, and we denote by π_r the principal series representation of G corresponding to ξ_r. Then we have the following contraction result which is analogous to [**16**, Theorem 1].

Proposition 7.1. *For each $(v,k) \in G_0$, $\phi \in C_0(\overline{N}, E)$ and $y \in \overline{N}$, we have*

$$\lim_{r \to 0} \pi_r(c_r(v,k))\phi\,(y) = \pi_0(v,k)\,\phi(y).$$

Proof. By taking into account the explicit expressions for π_r and π_0 given in Sections 2 and 3 (formulas (2.3) and (3.2)), we have just to verify that

$$\lim_{r \to 0} \frac{1}{r} \beta(\xi_1,\, \log \widetilde{a}(y) - \log \widetilde{a}(k^{-1} \exp(-rv)y)) = \beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1, v).$$

But, applying Lemma 2.1, we have

$$\begin{aligned}\frac{d}{dt}\widetilde{a}(y)^{-1}\widetilde{a}(k^{-1}\exp(-tv)y)\Big|_{t=0}&\\ &= \frac{d}{dt}\,\widetilde{a}^{-1}(k^{-1}y)\widetilde{a}(\exp(-t\mathrm{Ad}\,(k^{-1})v)k^{-1}y)\Big|_{t=0}\\ &= -\widetilde{p}_{\mathfrak{a}}(\mathrm{Ad}\,(\widetilde{k}(k^{-1}y)^{-1})\mathrm{Ad}\,(k^{-1})v)\\ &= -\widetilde{p}_{\mathfrak{a}}(\mathrm{Ad}\,(\widetilde{k}(y)^{-1})\,v).\end{aligned}$$

Then we obtain

$$\begin{aligned}\lim_{r\to 0}\frac{1}{r}\beta(\xi_1,\,\log\widetilde{a}(y)-\log\widetilde{a}(k^{-1}\exp(-rv)y)) &= \beta(\xi_1,\,\widetilde{p}_{\mathfrak{a}}(\mathrm{Ad}\,(\widetilde{k}(y)^{-1})v))\\ &= \beta(\xi_1,\,\mathrm{Ad}\,(\widetilde{k}(y)^{-1})v)\\ &= \beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1,\,v).\end{aligned}$$

The result follows. □

8. Contraction of adapted Weyl correspondences. For each $r\in\,]0,1]$, we denote by Ψ_r the symplectomorphism from $\overline{N}\times\overline{\mathfrak{n}}\times o(\xi_2)$ onto $\widetilde{O}(\xi_r)$ introduced in Section 5 and by $\mathcal{W}_r$ the adapted Weyl correspondence on $O(\xi_r)$. In this section, we show how the symplectomorphisms Ψ_r contract to the symplectomorphism $\Psi_0:\overline{N}\times\overline{\mathfrak{n}}\times o(\xi_2)\to\widetilde{O}(\xi_1,\xi_2)$ and how the correspondences $\mathcal{W}_r$ contract to $\mathcal{W}_0$.

For each $r\in\,]0,1]$, we denote by C_r the differential of c_r. Then the family (C_r) is a contraction of Lie algebras from $\mathfrak{g}$ onto $\mathfrak{g}_0$, that is,

$$\lim_{r\to 0} C_r^{-1}([C_r(X),\,C_r(Y)])=[X,\,Y]_0$$

for each $X,\,Y\in\mathfrak{g}_0$. We also denote by $C_r^*:\mathfrak{g}^*\simeq\mathfrak{g}\to\mathfrak{g}_0^*\simeq\mathfrak{g}_0$ the dual map of C_r.

Proposition 8.1. *For each $(y,Z,\varphi)\in\overline{N}\times\overline{\mathfrak{n}}\times o(\xi_2)$, we have*

$$\lim_{r\to 0} C_r^*(\Psi_r(y,Z,\varphi))=\Psi_0(y,Z,\varphi).$$

Proof. Let $(v,U)\in\mathfrak{g}_0$. Since $\mathfrak{k}$ and V are orthogonal with respect to β, we have

$$\begin{aligned}\langle C_r^*(\Psi_r(y,Z,\varphi)),\,(v,U)\rangle\\ &= \langle \Psi_r(y,Z,\varphi),\,C_r(v,U)\rangle\\ &= \langle (1/r)\mathrm{Ad}\,(\widetilde{k}(y))\xi_1+\mathrm{Ad}\,(y)(\varphi-\theta(Z)),\,rv+U\rangle\\ &= \beta(\mathrm{Ad}\,(\widetilde{k}(y))\xi_1,v)+r\beta(p_V^c(\mathrm{Ad}\,(y)(\varphi-\theta(Z)),v))\\ &\quad+\beta(p_{\mathfrak{k}}^c(\mathrm{Ad}\,(y)(\varphi-\theta(Z)),U)).\end{aligned}$$

Then

$$\begin{aligned}\lim_{r\to 0}\langle C_r^*(\Psi_r(y,Z,\varphi)),\,(v,U)\rangle\\ &= \langle \mathrm{Ad}\,(\widetilde{k}(y))\xi_1+p_{\mathfrak{k}}^c(\mathrm{Ad}\,(y)(\varphi-\theta(Z))),\,(v,U)\rangle.\end{aligned}$$

Hence, the result follows. □

Now, let $f:O(\xi_1,\xi_2)\to\mathbf{C}$ be a P-symbol of degree d, that is,

$$\widehat{f\circ\Psi_0}(y,Z)=\sum_{|\alpha|\le d}u_\alpha(y)Z^\alpha$$

where each u_α is in $C^\infty(\overline{N})$. Following [**12**], we say that a family $f_r:O(\xi_r)\to\mathbf{C}$ of symbols approximates f if each f_r is a P-symbol of degree less than or equal to d, that is,

$$\widehat{f_r\circ\Psi_r}(y,Z)=\sum_{|\alpha|\le d}u_\alpha^r(y)Z^\alpha$$

and if, for each α, $u_\alpha^r-u_\alpha$ and all its derivatives $\partial_\gamma(u_\alpha^r-u_\alpha)$ converge uniformly on compacts to zero, as $r\to 0$. Here, for each $v\in C^\infty(\overline{N})$ and each $\gamma=(\gamma_1,\gamma_2,\dots,\gamma_n)$, the derivative $\partial_\gamma v$ is defined by

$$\begin{aligned}\partial_\gamma v(y)=\left(\frac{d}{dt_1}\right)^{\gamma_1}\left(\frac{d}{dt_2}\right)^{\gamma_2}\cdots\left(\frac{d}{dt_n}\right)^{\gamma_n}&(v(y\exp(t_1E_1)\exp(t_2E_2)\\ &\dots\exp(t_nE_n)))\Big|_{t_1=t_2=\dots=t_n=0}.\end{aligned}$$

By using the properties of the Berezin-Weyl calculus, we immediately obtain the following proposition.

Proposition 8.2. *Let f be a* P*-symbol on $O(\xi_1, \xi_2)$. Let (f_r) be a family of* P*-symbols which approximates f. Then, for each $\phi \in C_0(\overline{N}, E)$ and each $y \in \overline{N}$, we have*

$$\lim_{r \to 0} (\mathcal{W}_r(f_r)\phi)(y) = (\mathcal{W}_0(f)\phi)(y).$$

Then we can deduce a contraction result for the derived representations from the contraction of the symplectomorphisms Ψ_r to Ψ_0.

Proposition 8.3. 1) *Let $(v, U) \in \mathfrak{g}_0$. Then the family $(\widetilde{C_r(v, U)})_{r \in]0,1]}$ approximates $\widetilde{(v, U)}$.*

2) *For each $(v, U) \in \mathfrak{g}_0$, $\phi \in C_0(\overline{N}, E)$ and $y \in \overline{N}$, we have*

$$\lim_{r \to 0} (d\pi_r(C_r(v, U))\phi)(y) = (d\pi_0(v, U)\phi)(y).$$

Proof. 1) This follows from Proposition 8.1.

2) Taking Proposition 5.3 and Proposition 6.3 into account, the result is an immediate consequence of Proposition 8.2. □

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