

**REAL HYPERSURFACES IN COMPLEX
PROJECTIVE SPACE WHOSE STRUCTURE
JACOBI OPERATOR SATISFIES $\mathcal{L}_\xi R_\xi = \nabla_\xi R_\xi$**

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ABSTRACT. We classify real hypersurfaces in complex projective space whose structure Jacobi operator satisfies that its Lie derivative in the direction of the structure vector field coincides with its covariant derivative in the same direction.

1. Introduction. Let $\mathbf{C}P^m$, $m \geq 2$, be a complex projective space endowed with the metric g of constant holomorphic sectional curvature 4. Let M be a connected real hypersurface of $\mathbf{C}P^m$ without boundary. Let J denote the complex structure of $\mathbf{C}P^m$ and N a locally defined unit normal vector field on M . Then $-JN = \xi$ is a tangent vector field to M called the structure vector field on M . We also call \mathbf{D} the maximal holomorphic distribution on M , that is, the distribution on M given by all vectors orthogonal to ξ at any point of M .

Jacobi fields along geodesics of a given Riemannian manifold $(\widetilde{M}, \widetilde{g})$ satisfy a very well-known differential equation. This classical differential equation naturally inspires the so-called Jacobi operator. That is, if \widetilde{R} is the curvature operator of \widetilde{M} , and X is any tangent vector field to \widetilde{M} , the Jacobi operator (with respect to X) at $p \in M$, $\widetilde{R}_X \in \text{End}(T_p\widetilde{M})$, is defined as $(\widetilde{R}_X Y)(p) = (\widetilde{R}(Y, X)X)(p)$ for all $Y \in T_p\widetilde{M}$, being a self-adjoint endomorphism of the tangent bundle $T\widetilde{M}$ of \widetilde{M} . Clearly, each tangent vector field X to \widetilde{M} provides a Jacobi operator with respect to X .

Let M now be a real hypersurface in $\mathbf{C}P^m$, R its curvature operator, and let ξ be the structure vector field on M . We will call the Jacobi operator on M with respect to ξ the structure Jacobi operator on M , R_ξ . Then the structure Jacobi operator $R_\xi \in \text{End}(T_pM)$ is given by $(R_\xi(Y))(p) = (R(Y, \xi)\xi)(p)$ for any $Y \in T_pM$, $p \in M$.

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Recently we have studied parallelness and Lie parallelness of the structure Jacobi operator, [3, 4, 5]. Further results about the covariant derivative of the structure Jacobi operator can be seen in [6, 7].

In this paper we will prove the nonexistence of several families of real hypersurfaces in $\mathbf{C}P^m$ by some propositions that allow us to prove the following

Theorem. *Let M be a real hypersurface in $\mathbf{C}P^m$, $m \geq 3$, whose structure Jacobi operator satisfies $\mathcal{L}_\xi R_\xi = \nabla_\xi R_\xi$. Then M is locally congruent either to a tube of radius $\pi/4$ over a complex submanifold of $\mathbf{C}P^m$ or to a tube with radius $r \neq \pi/4$ over $\mathbf{C}P^k$, $0 \leq k \leq m - 1$.*

2. Preliminaries. Throughout this paper, all manifolds, vector fields, etc., will be considered of class C^∞ unless otherwise stated. Let M be a connected real hypersurface in $\mathbf{C}P^m$, $m \geq 2$, without boundary. Let N be a locally defined unit normal vector field on M . Let ∇ be the Levi-Civita connection on M and (J, g) the Kaehlerian structure of $\mathbf{C}P^m$.

For any vector field X tangent to M , we write $JX = \phi X + \eta(X)N$ and $-JN = \xi$. Then (ϕ, ξ, η, g) is an almost contact metric structure on M . That is, we have

$$(2.1) \quad \phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for any tangent vectors X, Y to M . From (2.1) we obtain

$$(2.2) \quad \phi\xi = 0, \quad \eta(X) = g(X, \xi).$$

From the parallelism of J , we get

$$(2.3) \quad (\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi$$

and

$$(2.4) \quad \nabla_X \xi = \phi AX$$

for any X, Y tangent to M , where A denotes the shape operator of the immersion. As the ambient space has constant holomorphic sectional

curvature 4, the equations of Gauss and Codazzi are given, respectively, by

$$(2.5) \quad R(X, Y)Z = g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z + g(AY, Z)AX - g(AX, Z)AY,$$

and

$$(2.6) \quad (\nabla_X A)Y - (\nabla_Y A)X = \eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi$$

for any tangent vectors X, Y, Z to M .

3. Some propositions.

Proposition 3.1. *There exist no real hypersurfaces M in $\mathbf{C}P^m$, $m \geq 3$, whose Weingarten endomorphism satisfies $A\xi = \alpha\xi + U$, $AU = \xi$, $A\phi U = -(1/\alpha)\phi U$, where U is a unit vector field in \mathbf{D} and α is a nonnull function defined on M .*

Proof. The Codazzi equation yields $(\nabla_U A)\phi U - (\nabla_{\phi U} A)U = -2\xi$. If we compute its scalar product with ξ we get

$$(3.1) \quad g(\nabla_U U, \phi U) = -1,$$

and its scalar product with U implies

$$(3.2) \quad g(\nabla_U U, \phi U) = 2.$$

From (3.1) and (3.2) we have a contradiction, and the proof is finished. \square

Proposition 3.2. *There exist no real hypersurfaces M in $\mathbf{C}P^m$, $m \geq 3$, whose Weingarten endomorphism is given by $A\xi = \alpha\xi + \beta U$, $AU = \beta\xi + ((\beta^2 - 1)/\alpha)U$, $A\phi U = -(1/\alpha)\phi U$, the eigenvalues of A in $\mathbf{D}_U = \text{Span}\{\xi, U, \phi U\}^\perp$ are different from $0, -1/\alpha$ and $(\beta^2 - 1)/\alpha$, and if $Z \in \mathbf{D}_U$ satisfies $AZ = \lambda Z$, then $A\phi Z = \lambda\phi Z$, where U , and α are as in Proposition 3.1 and β is a nonnull function defined on M .*

Proof. We get $(\nabla_Z A)\phi Z - (\nabla_{\phi Z} A)Z = -2\xi$. Its scalar product with ϕZ yields

$$(3.3) \quad Z(\lambda) = 0.$$

Scalar product with Z gives

$$(3.4) \quad (\phi Z)(\lambda) = 0.$$

As $g((\nabla_{\phi U} A)Z - (\nabla_Z A)\phi U, \phi Z) = 0$, we obtain $(\lambda + (1/\alpha))g(\nabla_Z \phi U, \phi Z) = 0$. As $\lambda \neq -(1/\alpha)$, from (2.3) we have

$$(3.5) \quad g(\nabla_Z U, Z) = 0.$$

From the Codazzi equation, $g((\nabla_\xi A)Z - (\nabla_Z A)\xi, Z) = 0$. This yields $\xi(\lambda) = \beta g(\nabla_Z U, Z)$. From (3.5), we obtain

$$(3.6) \quad \xi(\lambda) = 0.$$

The Codazzi equation yields $g((\nabla_U A)\phi U - (\nabla_{\phi U} A)U, \phi U) = 0$. Thus,

$$(3.7) \quad \alpha U\left(\frac{1}{\alpha}\right) = \beta^2 g(\nabla_{\phi U} \phi U, U).$$

Similarly, $g((\nabla_\xi A)\phi U - (\nabla_{\phi U} A)\xi, \phi U) = 0$. Therefore,

$$(3.8) \quad \xi\left(\frac{1}{\alpha}\right) = \beta g(\nabla_{\phi U} \phi U, U).$$

From (3.7) and (3.8), we get

$$(3.9) \quad \beta \xi(\alpha) = \alpha U(\alpha).$$

As from the Codazzi equation $g((\nabla_\xi A)U - (\nabla_U A)\xi, \xi) = 0$, $\xi(\beta) = U(\alpha)$ and from (3.9), we have

$$(3.10) \quad \beta \xi(\alpha) = \alpha \xi(\beta).$$

Now $(\nabla_Z A)\phi Z - (\nabla_{\phi Z} A)Z = -2\xi$. The scalar product with ξ gives

$$(3.11) \quad \beta g([\phi Z, Z], U) = 2\lambda^2 - 2\alpha\lambda - 2,$$

and from the scalar product with U we get

$$(3.12) \quad \left(\lambda - \frac{\beta^2 - 1}{\alpha} \right) g([\phi Z, Z], U) = 2\beta\lambda.$$

From (3.11) and (3.12), as $\beta \neq 0$, we have $(\lambda + (1 - \beta^2/\alpha))(\lambda^2 - \alpha\lambda - 1) = \beta^2\lambda$, and with the hypothesis of Proposition 3.2, this can be written as

$$(3.13) \quad (\alpha\lambda + 1)(\lambda^2 - \alpha\lambda - 1) = \beta^2(\lambda^2 - 1).$$

From (3.13), bearing in mind (3.6), we get $\lambda\xi(\alpha)(\lambda^2 - \alpha\lambda - 1) + (\alpha\lambda + 1)(-\lambda\xi(\alpha)) = (\lambda^2 - 1)2\beta\xi(\beta)$. That is, $(\lambda(\lambda^2 - \alpha\lambda - 1) - \lambda(\alpha\lambda + 1))\xi(\alpha) = 2\beta(\lambda^2 - 1)\xi(\beta)$. From (3.10), we obtain $(\lambda(\lambda^2 - \alpha\lambda - 1) - \lambda(\alpha\lambda + 1))\xi(\alpha) = 2(\beta^2/\alpha)(\lambda^2 - 1)\xi(\alpha)$. If we suppose $\xi(\alpha) \neq 0$, $\lambda(\lambda^2 - \alpha\lambda - 1) - \lambda(\alpha\lambda + 1) = 2(\beta^2/\alpha)(\lambda^2 - 1) = (2/\alpha)(\alpha\lambda + 1)(\lambda^2 - \alpha\lambda - 1)$. Therefore, $\alpha\lambda(\lambda^2 - 2\alpha\lambda - 2) = 2(\alpha\lambda + 1)(\lambda^2 - \alpha\lambda - 1)$. This yields $\alpha\lambda^3 + 2\lambda^2 - 2\alpha\lambda - 2 = 0$. Thus, $\lambda^3\xi(\alpha) - 2\lambda\xi(\alpha) = 0$ and, as we suppose $\xi(\alpha) \neq 0$, this gives $\lambda(\lambda^2 - 2) = 0$. As $\lambda \neq 0$, $\lambda^2 = 2$, and by (3.13) $(\alpha\lambda + 1)(1 - \alpha\lambda) = \beta^2$. Thus, $1 - 2\alpha^2 = \beta^2$. Now $-4\alpha\xi(\alpha) = 2\beta\xi(\beta)$ and from (3.10), we get

$$(3.14) \quad \beta^2 + 2\alpha^2 = 0,$$

giving a contradiction. Thus,

$$(3.15) \quad \xi(\alpha) = \xi(\beta) = U(\alpha) = 0.$$

The Codazzi equation yields $g((\nabla_\xi A)Z - (\nabla_Z A)\xi, \xi) = 0$. Thus,

$$(3.16) \quad Z(\alpha) = -\beta g(\nabla_\xi Z, U).$$

As $g((\nabla_\xi A)Z - (\nabla_Z A)\xi, U) = 0$, we get

$$(3.17) \quad Z(\beta) = \left(\lambda - \frac{\beta^2 - 1}{\alpha} \right) g(\nabla_\xi Z, U).$$

From (3.16) and (3.17), we obtain

$$(3.18) \quad \beta Z(\beta) = \left(\frac{\beta^2 - 1}{\alpha} - \lambda \right) Z(\alpha).$$

Bearing in mind (3.13), from (3.3) we get $\lambda Z(\alpha)(\lambda^2 - \alpha\lambda - 1) + (\alpha\lambda + 1)(-\lambda Z(\alpha)) = 2\beta(\lambda^2 - 1)Z(\beta)$. Therefore, by (3.18), $Z(\alpha)(\lambda(\lambda^2 - \alpha\lambda - 1) - \lambda(\alpha\lambda + 1)) = 2(\lambda^2 - 1)((\beta^2 - 1/\alpha) - \lambda)Z(\alpha)$. If we suppose $Z(\alpha) \neq 0$, we get $3\lambda^3 - 2\alpha\lambda^2 - 4\lambda = (2\beta^2 - 2/\alpha)(\lambda^2 - 1) = (2/\alpha)(\alpha\lambda + 1)(\lambda^2 - \alpha\lambda - 1) - (2/\alpha)(\lambda^2 - 1)$. That is, $\alpha(3\lambda^3 - 2\alpha\lambda^2 - 4\lambda) = 2(\alpha\lambda + 1)(\lambda^2 - \alpha\lambda - 1) - 2(\lambda^2 - 1)$. From this we have $Z(\alpha)(3\lambda^3 - 2\alpha\lambda^2 - 4\lambda) + \alpha(-2\lambda^2 Z(\alpha)) = 2\lambda(\lambda^2 - \alpha\lambda - 1)Z(\alpha) + 2(\alpha\lambda + 1)(-\lambda Z(\alpha))$. As we suppose $Z(\alpha) \neq 0$, this yields $\lambda = 0$, so we have a contradiction. This proves

$$(3.19) \quad Z(\beta) = Z(\alpha) = 0.$$

By a linearity argument we also have

$$(3.20) \quad X(\alpha) = X(\beta) = 0$$

for any $X \in \mathbf{D}_U$.

The Codazzi equation yields $g((\nabla_\xi A)\phi U - (\nabla_{\phi U} A)\xi, \xi) = 0$. Thus,

$$(3.21) \quad (\phi U)(\alpha) = \frac{3\beta}{\alpha} + \alpha\beta + \beta g(\nabla_\xi U, \phi U).$$

As $g((\nabla_\xi A)\phi U - (\nabla_{\phi U} A)\xi, U) = -1$, we also get

$$(3.22) \quad (\phi U)(\beta) = \frac{\beta^2 - 1}{\alpha^2} + \beta^2 + \frac{\beta^2}{\alpha} g(\nabla_\xi U, \phi U).$$

As $g((\nabla_\xi A)U - (\nabla_U A)\xi, \phi U) = 1$, we obtain

$$(3.23) \quad \frac{\beta^2}{\alpha} g(\nabla_\xi U, \phi U) - \beta g(\nabla_U U, \phi U) = \frac{\beta^2 - 1}{\alpha^2}.$$

And, as $g((\nabla_U A)\phi U - (\nabla_{\phi U} A)U, U) = 0$, we have

$$(3.24) \quad \beta g(\nabla_U U, \phi U) + \beta^2 - 3 - 2(\phi U)(\beta) + \frac{\beta^2 - 1}{\alpha\beta} (\phi U)(\alpha) = 0.$$

From (3.21), (3.22) and (3.24), it follows that

$$(3.25) \quad \beta g(\nabla_U U, \phi U) - \frac{\beta^2 + 1}{\alpha} g(\nabla_\xi U, \phi U) + \frac{\beta^2 - 1}{\alpha^2} - 4 = 0.$$

From (3.23) and (3.25), we have $g(\nabla_\xi U, \phi U) = -4\alpha$ and $g(\nabla_U U, \phi U) = (1 - \beta^2)/(\alpha^2\beta) - 4\beta$. From (3.21) and (3.22), we conclude

$$(3.26) \quad (\phi U)(\alpha) = 3\beta\left(\frac{1 - \alpha^2}{\alpha}\right)$$

and

$$(3.27) \quad (\phi U)(\beta) = -3\beta^2 + \frac{\beta^2 - 1}{\alpha^2}.$$

Thus, $\text{grad}(\alpha) = 3\beta((1 - \alpha^2)/\alpha)\phi U = k\phi U$. As $g(\nabla_X \text{grad}(\alpha), Y) = g(\nabla_Y \text{grad}(\alpha), X)$ for any $X, Y \in TM$, we have $X(k)g(\phi U, Y) - Y(k)g(\phi U, X) + k(g(\nabla_X \phi U, Y) - g(\nabla_Y \phi U, X)) = 0$. Taking $Y = \xi$, it follows $k(g(\nabla_X \phi U, \xi) - g(\nabla_\xi \phi U, X)) = 0$, for any $X \in TM$. Thus, either $k = 0$ or $g(\nabla_X \phi U, \xi) = g(\nabla_\xi \phi U, X)$ for any $X \in TM$. Suppose $k \neq 0$. If we take $X = U$ we get $-g(U, AU) = g(\nabla_\xi \phi U, U)$. Thus, $4\alpha^2 + \beta^2 = 1$. Then $4\alpha(\phi U)(\alpha) + \beta(\phi U)(\beta) = 0$. From (3.26) and (3.27) we get $9\alpha^2 + \beta^2 = 1$. Both results yield $\alpha = 0$, which is impossible. Thus, $k = 0$. Equivalently $\alpha^2 = 1$. Now (3.26) becomes $(\phi U)(\alpha) = 0$ and (3.27) yields $(\phi U)(\beta) = -(2\beta^2 + 1)$. From (3.15) and the fact that $g((\nabla_\xi A)U - (\nabla_U A)\xi, U) = 0$ we get $U(\beta) = 0$. Thus,

$$(3.28) \quad \text{grad}(\beta) = -(2\beta^2 + 1)\phi U.$$

By the same argument as applied above to $\text{grad}(\beta)$ we get $-(1 + 2\beta^2)(g(\nabla_X \phi U, \xi) - g(\nabla_\xi \phi U, X)) = 0$, for any $X \in TM$. This yields $g(\nabla_X \phi U, \xi) = g(\nabla_\xi \phi U, X)$, for any $X \in TM$. Taking $X = U$, we obtain $4\alpha^2 + \beta^2 = 1$. As $\alpha^2 = 1$, this yields $\beta^2 = -3$, which is impossible and the proof is finished. \square

Proposition 3.3. *There exist no real hypersurfaces M in CP^m , $m \geq 3$, whose Weingarten endomorphism is given by $A\xi = \xi + \beta U$, $AU = \beta\xi + (\beta^2 - 1)U$, $A\phi U = -\phi U$ and there exists $Z \in \mathbf{D}_U$ such that $AZ = -Z$, $A\phi Z = -\phi Z$, where U , \mathbf{D}_U and β are as in Proposition 3.2.*

The proof is similar to the proof of Proposition 3.2.

4. Proof of the Theorem. The condition $\mathcal{L}_\xi R_\xi = \nabla_\xi R_\xi$ is equivalent to the condition $\phi AR_\xi = R_\xi \phi A$.

First, we consider that M is Hopf, that is, $A\xi = \alpha\xi$ for a certain function α on M . Take $X \in \mathbf{D}$ such that $AX = \lambda X$. Then $\phi AR_\xi(X) = (1 + \alpha\lambda)\lambda\phi X = R_\xi(\phi AX) = \lambda(1 + \alpha\mu)\phi X$, where $\mu = (\alpha\lambda + 2/2\lambda - \alpha)$ is the eigenvalue of ϕX , see [2]. From this we get $\alpha\lambda(\lambda - \mu) = 0$. The same reasoning applied to ϕX yields $\alpha\mu(\lambda - \mu) = 0$. From both equations we have $\alpha(\lambda - \mu)^2 = 0$. Thus either $\alpha = 0$ or $\lambda = \mu$. Therefore, if $\alpha = 0$, M is locally congruent to a tube of radius $\pi/4$ over a complex submanifold of $\mathbf{C}P^m$, [1]. If $\alpha \neq 0$, as $\lambda = \mu$, M is locally congruent to a tube of radius $0 < r < \pi/2$, $r \neq \pi/4$ either over a point (this is a geodesic hypersphere) or over a $\mathbf{C}P^k$, $0 < k < m - 1$.

So we now consider that M is not Hopf. Thus, locally, there exists a unit $U \in \mathbf{D}$ and a nonnull function β defined on M such that $A\xi = \alpha\xi + \beta U$. As $\phi AR_\xi(\xi) = 0$, we get $R_\xi(\phi A\xi) = 0 = \beta R_\xi(\phi U)$. Thus, $R_\xi(\phi U) = \phi U + \alpha A\phi U = 0$, and this yields $\alpha \neq 0$ and $A\phi U = -\frac{1}{\alpha}\phi U$.

Analogously, $R_\xi(\phi A\phi U) = \phi AR_\xi(\phi U) = 0$. Thus, $0 = -(1/\alpha)R_\xi(\phi\phi U) = (1/\alpha)R_\xi(U)$. Then, $R_\xi(U) = U + \alpha AU - \beta A\xi - \beta^2 U = 0$. Therefore, $AU = \beta\xi + (\beta^2 - 1/\alpha)U$. This yields \mathbf{D}_U is A -invariant and ϕ -invariant. If $X \in \mathbf{D}_U$ satisfies $AX = \lambda X$, then $\phi AR_\xi(X) = \lambda(1 + \alpha\lambda)\phi X = R_\xi(\phi AX) = \lambda(\phi X + \alpha A\phi X)$. Thus $\lambda^2\phi X = \lambda A\phi X$. If $\lambda \neq 0$, $A\phi X = \lambda\phi X$. If $\lambda = 0$, $AX = 0$ and $\phi AR_\xi(\phi X) = R_\xi(\phi A\phi X)$ yields $\phi A^2\phi X = A\phi A\phi X$. Therefore, $g(\phi A^2\phi X, X) = g(\phi A\phi X, AX) = 0$. So we have $g(A\phi X, A\phi X) = 0$, which gives $A\phi X = 0$. From this, we can assure that the eigenspaces of A in \mathbf{D}_U are ϕ -invariant.

Taking $Z \in \mathbf{D}_U$ such that $AZ = \lambda Z$, as in the proof of Proposition 3.2 we obtain $(\lambda + (1 - \beta^2/\alpha))(\lambda^2 - \alpha\lambda - 1) = \beta^2\lambda$. Now we have the following possibilities:

1. If there exists $Z \in \mathbf{D}_U$ such that $AZ = 0$, M is as in Proposition 3.1 and this kind of real hypersurfaces does not exist.
2. Suppose there exists $Z \in \mathbf{D}_U$ such that $AZ = \lambda Z$, $\lambda \neq 0$. If $\lambda = (\beta^2 - 1/\alpha)$, from the formula above, $\lambda = 0$, giving a contradiction, thus also $\lambda \neq (\beta^2 - 1/\alpha)$. That is, (3.13) is true. From (3.13), $\lambda = -1/\alpha$ if and only if $\lambda^2 = 1$ and $\alpha^2 = 1$. If this is the case, changing, if necessary, ξ by $-\xi$, we suppose $\alpha = 1$. So M is, locally, a real hypersurface either as in Proposition 3.2 or as in Proposition 3.3. As these types of real hypersurfaces do not exist, the proof concludes. \square

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