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PERMANENCE FOR NONAUTONOMOUS N-SPECIES LOTKA-VOLTERRA COMPETITIVE SYSTEMS WITH FEEDBACK CONTROLS

XIAOMEI FENG, ZHIDONG TENG AND LONG ZHANG

ABSTRACT. In this paper, the permanence of nonautonomous n-species Lotka-Volterra competitive systems with feedback controls is studied. Some new criteria on the permanence for all positive solutions are established. The corresponding results given by Chen in [**3**] are improved.

1. Introduction. As we know, ecosystems in the real world are continuously disturbed by unpredictable forces which can result in changes in biological parameters such as survival rates. Of practical interest in the ecosystem is the question of whether or not an ecosystem can withstand those unpredictable forces which persist for a finite period of time. In the language of control variables, we call the disturbance functions *control variables*.

In recent years, population dynamic systems with feedback controls have been studied in many articles, for example, see [**2**–**6, 9**–**12**] and references cited therein. Some important subjects such as persistence, permanence, global asymptotic stability and the existence of positive periodic solutions and positive almost periodic solutions, etc., are extensively investigated.

In [**3**], the author proposed the following n-species nonautonomous Lotka-Volterra competition system with feedback controls

$$
\begin{aligned}
x_i'(t) &= x_i(t)\Big(b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - d_i(t)u_i(t)\Big),\\
u_i'(t) &= r_i(t) - e_i(t)u_i(t) + f_i(t)x_i(t), \quad i = 1, 2, \dots, n.
\end{aligned}
\tag{1}
$$

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By introducing the upper and lower averages of a function due to Ahmad and Lazer [**1**], applying the differential inequality principle and developing a suitable Lyapunov function, the author obtained sufficient conditions which guarantee the permanence and global attractivity of all positive solutions for system (1).

On the other hand, in [**7**], the author introduced a new research method to discuss the permanence and global asymptotic stability of all positive solutions for the n-species nonautonomous Lotka-Volterra competitive systems without feedback controls.

Motivated by the above works [**2**–**7, 9**–**12**], in this paper we continue to discuss the n-species nonautonomous Lotka-Volterra competition system (1) with feedback controls. We will introduce a new research method which is obtained by further developing the analysis technique given by Teng in [**7**]. This method will be completely different from the method which was given by Chen in [**3**]. We will obtain some new sufficient conditions about the permanence of all positive solutions of system (1). We will see that these sufficient conditions improve the corresponding results which are obtained by Chen in [**3**]. We also will see that in some special cases the feedback controls will not influence the permanence of all positive solutions of system (1).

The paper is organized as follows. In the next section, we will give some assumptions, definitions and useful lemmas. In Section 3, some new sufficient conditions which guarantee the permanence of all positive solutions for system (1) are obtained. In Section 4, a suitable example is given to illustrated that our main results are applicable.

2. Preliminaries. In this paper, for system (1) we denote that $x_i(t)$, $1 \leq i \leq n$, is the density of the ith species at time t, $u_i(t)$, $1 \leq i \leq n$, is the control variable and $(x(t), u(t)) = (x_1(t), x_2(t), \dots, x_n(t), u_1(t), u_2(t), \dots, u_n(t))$. We always assume that functions $b_i(t)$, $a_{ij}(t)$, $d_i(t)$, $r_i(t)$, $e_i(t)$ and $f_i(t)$, $i, j = 1, 2, \dots, n$, are defined on $R_+ = [0, \infty)$ and are bounded and continuous, and $a_{ij}(t) \geq 0$, $d_i(t) \geq 0$, $r_i(t) \geq 0$, $e_i(t) \geq 0$ and $f_i(t) \geq 0$ for $t \in R_+$.

System (1) is said to be ω-periodic, if all coefficients $b_i(t)$, $a_{ij}(t)$, $d_i(t)$, $r_i(t)$, $e_i(t)$ and $f_i(t)$, $i, j = 1, 2, \dots, n$, are ω-periodic continuous functions.

For any function $g(t)$ defined on R_+, we denote $g^l = \liminf_{t\to\infty} g(t)$ and $g^u = \limsup_{t\to\infty} g(t)$. Particularly, when $g(t)$ is ω-periodic continuous function, we have $g^l = \min_{t\in R} g(t)$ and $g^u = \max_{t\in R} g(t)$. In addition, we denote $[g] = \omega^{-1}\int_0^\omega g(t)\,dt$.

Throughout this paper, we will introduce the following assumptions.

(H_1) There exist positive constants w_i such that

$$\liminf_{t\to\infty}\int_t^{t+w_i} b_i(s)\,ds > 0, \quad i = 1, 2, \dots, n.$$

(H_2) There exist positive constants λ_i such that

$$\liminf_{t\to\infty}\int_t^{t+\lambda_i} a_{ii}(s)\,ds > 0, \quad i = 1, 2, \dots, n.$$

(H_3) There exist positive constants γ_i such that

$$\liminf_{t\to\infty}\int_t^{t+\gamma_i} e_i(s)\,ds > 0, \quad i = 1, 2, \dots, n.$$

Definition 1. System (1) is said to be permanent if positive constants m_0 and M_0 exist such that

$$m_0 \le \liminf_{t\to\infty} x_i(t) \le \limsup_{t\to\infty} x_i(t) \le M_0, \quad i = 1, 2, \dots, n,$$

for any positive solution $(x(t), u(t)) = (x_1(t), x_2(t), \dots, x_n(t), u_1(t), u_2(t), \dots, u_n(t))$ of system (1).

Lemma 1. *If initial values $x_i(t_0) > 0$ and $u_i(t_0) > 0$, $i = 1, 2, \dots, n$, then solution $(x(t), u(t)) = (x_1(t), x_2(t), \dots, x_n(t), u_1(t), u_2(t), \dots, u_n(t))$ is positive, that is, $x_i(t) > 0$ and $u_i(t) > 0$ on the maximal existence interval.*

Proof. By integrating the first equation of system (1) from t_0 to t, we obtain

$$x_i(t) = x_i(t_0) \exp \int_{t_0}^{t} (b_i(s) - \sum_{j=1}^{n} a_{ij}(s) x_j(s) - d_i(s) u_i(s)) \, \mathrm{d}s.$$

Therefore, if $x_i(t_0) > 0$, then $x_i(t) > 0$ for all $t \geq t_0$ and $i = 1, 2, \ldots, n$.

From the second equation of system (1) and the positivity of $x_i(t)$, we have

$$u_i'(t) \geq -e_i(t) u_i(t) \quad \text{for all } t \geq t_0.$$

Hence,

$$u_i(t) \geq u_i(t_0) \exp(-\int_{t_o}^{t} e_i(s) \mathrm{d}s) \quad \text{for all } t \geq t_0.$$

From this, if $u_i(t_0) > 0$, then we can see $u_i(t) > 0$ for all $t \geq t_0$ and $i = 1, 2, \ldots, n$. This completes the proof of Lemma 1. □

Considering the following nonautonomous logistic equation

$$x'(t) = x(t)(b(t) - a(t)x(t)), \tag{2}$$

where functions $a(t)$ and $b(t)$ are defined on R_+ and are bounded and continuous, and $a(t) \geq 0$ for all $t \geq 0$. We have the following result.

Lemma 2. *Suppose that the following assumptions hold*:

(A_1) *there exists a positive constant w such that*

$$\liminf_{t \to \infty} \int_t^{t+w} b(s) \, ds > 0.$$

(A_2) *There exists a positive constant λ such that*

$$\liminf_{t \to \infty} \int_t^{t+\lambda} a(s) \, ds > 0.$$

Then,

(a) *there exist positive constants m and M such that*

$$m \leq \liminf_{t \to \infty} x(t) \leq \limsup_{t \to \infty} x(t) \leq M$$

for any positive solution $x(t)$ of equation (2).

(b) $\lim_{t\to\infty}(x^{(1)}(t)-x^{(2)}(t))=0$ *for any two positive solutions* $x^{(1)}(t)$ *and* $x^{(2)}(t)$ *of equation* (2).

(c) *If, further,* $a^l>0$, *then* $\limsup_{t\to\infty} x(t)\le (b/a)^u$ *for any positive solution* $x(t)$ *of equation* (2).

(d) *If, further, equation* (2) *is* ω*-periodic, that is,* $a(t)$ *and* $b(t)$ *are* ω*-periodic continuous functions, then equation* (2) *has a unique positive* ω*-periodic solution* $x_0(t)$ *such that* $[b]=[ax_0]$ *and*

$$\lim_{t\to\infty}(x(t)-x_0(t))=0$$

for any positive solution $x(t)$ *of equation* (2).

Further, we consider the following nonautonomous linear equation

$$u'(t)=r(t)-e(t)u(t), \tag{3}$$

where functions $r(t)$ and $e(t)$ are defined on R_+ and are bounded and continuous, and $r(t)\ge 0$ for all $t\ge 0$. We have the following result.

Lemma 3. *Suppose that the following assumption holds*:

(A_3) *there exist a positive constant* γ *such that*

$$\liminf_{t\to\infty}\int_t^{t+\gamma} e(s)\,ds>0.$$

Then,

(a) *There exists a positive constant* U *such that* $\limsup_{t\to\infty} u(t)\le U$ *for any positive solution* $u(t)$ *of equation* (3).

(b) $\lim_{t\to\infty}(u^{(1)}(t)-u^{(2)}(t))=0$ *for any two positive solutions* $u^{(1)}(t)$ *and* $u^{(2)}(t)$ *of equation* (3).

(c) *If, further,* $e^l>0$, *then* $\limsup_{t\to\infty} u(t)\le (r/e)^u$ *for any positive solution* $u(t)$ *of equation*(2).

(d) *If, further, equation* (3) *is* ω*-periodic, that is,* $r(t)$ *and* $e(t)$ *are* ω*-periodic continuous functions, then equation* (2) *has a unique positive* ω*-periodic solution* $u_0(t)$ *such that* $[r]=[eu_0]$ *and*

$$\lim_{t\to\infty} (u(t) - u_0(t)) = 0$$

for any positive solution $u(t)$ *of equation* (3).

Lemmas 2 and 3 can be found in many articles, for example, [**7, 8**].

We further consider the following nonautonomous linear equation

$$u'(t) = r(t) - e(t)u(t) + a(t), \tag{4}$$

where functions $r(t)$, $e(t)$ and $a(t)$ are defined on R_+ and are bounded and continuous, and $r(t)$ and $e(t)$ are nonnegative for all $t \geq 0$.

Let $u(t, t_0, u_0)$ be the solution of equation (4) satisfying initial condition $u(t_0) = u_0$. Further, let $u_0(t)$ be the solution of the following equation

$$u'(t) = r(t) - e(t)u(t) \tag{5}$$

satisfying the initial condition $u(t_0) = 0$. We have the following result.

Lemma 4. *Suppose that assumption* (A_3) *holds. Then for any constants* $\epsilon > 0$ *and* $M > 0$ *there exist constants* $\delta = \delta(\epsilon) > 0$ *and* $T = T(M) > 0$ *such that for any* $t_0 \in R_+$ *and* $u_0 \in R$ *with* $|u_0| \leq M$, *when* $|a(t)| < \delta$ *for all* $t \geq t_0$, *we have*

$$|u(t, t_0, u_0) - u_0(t)| < \epsilon \quad \textit{for all } t \geq t_0 + T.$$

Proof. Firstly, by assumption (A_3), there exist positive constants H and $\Lambda > 0$ such that for any $s \geq \tau \geq 0$ we have

$$\int_\tau^s e(t)\,\mathrm{d}t \geq \Lambda(s - \tau) - H.$$

According to the variation-of-constants formula, we obtain

$$u(t, t_0, u_0) = u_0 \exp\bigg(- \int_{t_0}^t e(s)\,\mathrm{d}s) + \int_{t_0}^t (r(s) + a(s)) \\ \times \exp\bigg(- \int_s^\tau e(\tau)\,\mathrm{d}\tau\bigg)\,\mathrm{d}s$$

and

$$u_0(t) = \int_{t_0}^{t} r(s) \exp\left(- \int_{s}^{\tau} e(\tau) \mathrm{d}\tau \right) \mathrm{d}s.$$

Therefore,

$$\begin{aligned}
|u(t, t_0, u_0) - u_0(t)| &\le u_0 \exp\left(- \int_{t_0}^{t} e(s)\, \mathrm{d}s \right) \\
&\quad + \int_{t_0}^{t} a(s) \exp\left(- \int_{s}^{\tau} e(\tau)\, \mathrm{d}\tau \right) \mathrm{d}s \\
&\le Me^{-\Lambda(t-t_0)+H} + a^M \int_{t_0}^{t} e^{-\Lambda(t-s)+H}\, \mathrm{d}s \\
&\le Me^H e^{-\Lambda(t-t_0)} + a^M e^H \frac{1}{\Lambda} e^{-\Lambda(t-s)}|_{t_0}^{t} \\
&\le Me^H e^{-\Lambda(t-t_0)} + a^M e^H \frac{1}{\Lambda},
\end{aligned}$$

where $a^M = \sup_{t \ge t_0} a(t)$. For any constant $\epsilon > 0$ we choose a constant $\delta = \delta(\epsilon) = \Lambda\epsilon/2e^{\overline{H}}$, and for any constant $M > 0$ we choose a constant $T = T(M) > 0$ such that $Me^H e^{-\Lambda T} < \epsilon/2$. Then when $|a(t)| \le \delta$ for all $t \ge t_0$ we have for all $t \ge t_0 + T$

$$Me^H e^{-\Lambda(t-t_0)} < \frac{\epsilon}{2}, \qquad a^M e^H \frac{1}{\Lambda} < \frac{\epsilon}{2}.$$

Therefore, we finally have

$$|u(t, t_0, u_0) - u_0(t)| < \epsilon \quad \text{for all } t \ge t_0 + T.$$

This completes the proof of Lemma 4. □

If $r(t) \equiv 0$, then equation (4) becomes

$$u'(t) = -e(t)u(t) + a(t). \tag{6}$$

As a consequence of Lemma 4, we have the following corollary.

Corollary 1. *Suppose that assumption* (A_3) *holds. Then for any constants* $\epsilon > 0$ *and* $M > 0$ *there exist constants* $\delta = \delta(\epsilon) > 0$ *and*

$T = T(M) > 0$ such that for any $t_0 \in R_+$ and $u_0 \in R$ with $|u_0| \le M$, when $|a(t)| < \delta$ for all $t \ge t_0$ we have $|u(t, t_0, u_0)| < \epsilon$ for all $t \ge t_0 + T$.

In fact, if $r(t) \equiv 0$, then equation (5) becomes

$$u'(t) = -e(t)u(t). \tag{7}$$

We see that $u_0(t) \equiv 0$ is the solution of equation (7) with initial value $u(t_0) = 0$. Therefore, Corollary 1 is obvious.

3. Main results.

Theorem 1. *Suppose that assumptions* (H_1)–(H_3) *hold. Then there exist constants $M_0 > 0$ and $U_0 > 0$ such that*

$$\limsup_{t\to\infty} x_i(t) < M_0, \qquad \limsup_{t\to\infty} u_i(t) < U_0, \quad i = 1, 2, \dots, n,$$

for any positive solutions $(x(t), u(t)) = (x_1(t), x_2(t), \dots, x_n(t), u_1(t), u_2(t), \dots, u_n(t))$ of system (1).

Proof. From the first equation of system (1) we have

$$\begin{aligned} x_i'(t) &= x_i(t)(b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - d_i(t)u_i(t)) \\ &\le x_i(t)(b_i(t) - a_{ii}(t)x_i(t)) \end{aligned}$$

for all $t \ge 0$ and $i = 1, 2, \dots, n$. Therefore, applying the comparison theorem and conclusion (a) of Lemma 2 we can obtain that there exists some constant $M_0 > 0$ such that for any positive solution $(x(t), u(t))$ of system (1) there is a $T_0 > 0$ such that

$$x_i(t) < M_0 \quad \text{for all } t \ge T_0. \tag{8}$$

From the second equation of system (1) we further have

$$\begin{aligned} u_i'(t) &= r_i(t) - e_i(t)u_i(t) + f_i(t)x_i(t) \\ &\le r_i(t) - e_i(t)u_i(t) + f_i(t)M_0 \end{aligned}$$

for all $t \geq T_0$ and $i = 1, 2, \dots, n$. Similarly, applying the comparison theorem and conclusion (a) of Lemma 3 we can obtain that there exists some constant $U_0 > 0$ such that for any positive solution $(x(t), u(t))$ of system (1) there is a $T_1 \geq T_0$ such that

$$u_i(t) < U_0 \quad \text{for all } t \geq T_1. \tag{9}$$

Finally, from (8) and (9) we obtain

$$\limsup_{t\to\infty} x_i(t) < M_0, \qquad \limsup_{t\to\infty} u_i(t) < U_0, \quad i = 1, 2, \dots, n$$

for any positive solutions $(x(t), u(t)) = (x_1(t), x_2(t), \dots, x_n(t), u_1(t), u_2(t), \dots, u_n(t))$ of system (1). This completes the proof of Theorem 1. □

Let $x_{i0}(t)$ be some fixed positive solution of the following nonautonomous logistic equation

$$x_i'(t) = x_i(t)(b_i(t) - a_{ii}(t)x_i(t)), \quad i = 1, 2, \dots, n.$$

If $a_{ii}^l > 0$, then by conclusion (c) of Lemma 2 we have

$$\limsup_{t\to\infty} x_{i0}(t) \leq \left(\frac{b_i}{a_{ii}}\right)^u. \tag{10}$$

Let $u_{i0}(t)$ be the solution of the following nonautonomous linear equation

$$u_i'(t) = r_i(t) - e_i(t)u_i(t), \quad i = 1, 2, \dots, n, \tag{11}$$

satisfying the initial condition $u_i(0) = 0$. If $e_i^l > 0$, then by conclusion (c) of Lemma 3 we have

$$\limsup_{t\to\infty} u_{i0}(t) \leq \left(\frac{r_i}{e_i}\right)^u. \tag{12}$$

On the permanence of all positive solutions of system (1) we have the following result.

Theorem 2. *Suppose that assumptions* (H_1)–(H_3) *hold and*

$$\liminf_{t\to\infty}\int_t^{t+\omega_i}(b_i(s)-\sum_{j\neq i}^n a_{ij}(s)x_{j0}(s)-d_i(s)u_{i0}(s))\,\mathrm{d}s>0,\quad i=1,2,\dots,n. \tag{13}$$

Then system (1) *is permanent.*

Proof. Firstly, from condition (12) we obtain that there are constants $\epsilon_0>0$, $\eta>0$ and $T_0>0$ such that for all $t\geq T_0$,

$$\int_t^{t+\omega_i}\Big(b_i(s)-a_{ii}(s)\epsilon_0-\sum_{j\neq i}^n a_{ij}(s)(x_{j0}(s)+\epsilon_0) - d_i(s)(u_{i0}(s)+\epsilon_0)\Big)\,\mathrm{d}s>\eta. \tag{14}$$

Since

$$x_i'(t)\leq x_i(t)(b_i(t)-a_{ii}(t)x_i(t)),\quad i=1,2,\dots,n$$

for all $t\geq 0$, by the comparison theorem and conclusion (b) of Lemma 2, we can obtain that there is a $T_1\geq T_0$ such that

$$x_i(t)\leq x_{i0}(t)+\epsilon_0\quad \text{for all } t\geq T_1,\ i=1,2,\dots,n.$$

We consider the equation of $u_i(t)$ in system (1)

$$u_i'(t)=r_i(t)-e_i(t)u_i(t)+f_i(t)x_i(t),\quad i=1,2,\dots,n. \tag{15}$$

By Lemma 4 we have the following conclusion:

For ϵ_0 above and constant $U_0>0$ which is given in Theorem 1 there exist constants $\delta_{i0}=\delta_{i0}(\epsilon_0)>0$ with $\delta_{i0}<\epsilon_0$ and $T_{i0}=T_{i0}(U_0)>0$ such that, for any $t_0\in R_+$ and $u_{i0}\in R$ with $|u_{i0}|\leq U_0$, when $f_i(t)x_i(t)\leq\delta_{i0}$ for all $t\geq t_0$ we have

$$|u_i(t,t_0,u_{i0})-u_{i0}(t)|<\epsilon_0\quad \text{for all } t\geq t_0+T_{i0}, \tag{16}$$

where $u_i(t,t_0,u_{i0})$ is the solution of equation (15) satisfying initial condition $u_i(t_0)=u_{i0}$.

Let $\delta_i = \delta_{i0}/(f_i^M + 1)$, where $f_i^M = \sup_{t\geq 0} f_i(t)$. Then $\delta_i < \epsilon_0$, $i = 1, 2, \dots, n$. For each $i \in \{1, 2, \dots, n\}$, we consider $x_i(t)$. There exist three cases as follows.

Case 1. *There is a constant $T^* \geq T_1$ such that $x_i(t) \leq \delta_i$ for all $t \geq T^*$.*

Case 2. *There is a constant $T^* \geq T_1$ such that $x_i(t) \geq \delta_i$ for all $t \geq T^*$.*

Case 3. *There is an interval sequence $\{[s_k, t_k]\}$ with $T_1 \leq s_1 < t_1 < s_2 < t_2 < \cdots < s_k < t_k < \cdots$ and $\lim_{k\to\infty} s_k = \infty$ such that $x_i(t) \leq \delta_i$ for all $t \in \cup_{k=1}^{\infty}[s_k, t_k]$ and $x_i(t) > \delta_i$ for all $t \neq \cup_{k=1}^{\infty}[s_k, t_k]$ and $t \geq T_1$.*

We consider Case 1. Since $\limsup_{t\to\infty} u_i(t) < U_0$, there exists a $T_2 \geq T^*$ such that

$$x_i(t) < \delta_i \text{ and } u_i(t) < U_0 \quad \text{for all } t \geq T_2.$$

Hence, for any $t \geq T_2$ we have $f_i(t)x_i(t) < \delta_{i0}$ and $u_i(T_2) < U_0$. In (16), we choose $t_0 = T_2$ and $u_{i0} = u_i(T_2)$, since $u_i(t) = u_i(t, T_2, u_i(T_2))$. By (16) we obtain

$$|u_i(t) - u_{i0}(t)| < \epsilon_0 \quad \text{for all } t \geq T_2 + T_{i0}.$$

Therefore, for any $t \geq T_2 + T_{i0}$ we have $x_i(t) < \delta_i < \epsilon_0$ and $u_i(t) < u_{i0}(t) + \epsilon_0$. Since for any $t \geq T_2 + T_{i0}$ we have

$$\begin{aligned} x_i'(t) &= x_i(t)(b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - d_i(t)u_i(t)) \\ &\geq x_i(t)(b_i(t) - a_{ii}(t)\epsilon_0 - \sum_{j\neq i}^{n} a_{ij}(t)(x_{j0}(t) + \epsilon_0) \\ &\quad - d_i(t)(u_{i0}(t) + \epsilon_0)), \end{aligned}$$

integrating this inequality from $T_2 + T_{i0}$ to t, we obtain

$$x_i(t) \geq x_i(T_2 + T_{i0}) \exp \int_{T_2+T_{i0}}^{t} (b_i(s) - a_{ii}(s)\epsilon_0 - \sum_{j\neq i}^{n} a_{ij}(s)(x_{j0}(s) + \epsilon_0) - d_i(s)(u_{i0}(s) + \epsilon_0))\,\mathrm{d}s.$$

Thus, from (14) we directly obtain $x_i(t) \to \infty$ as $t \to \infty$ which leads to a contradiction. Therefore, Case 1 cannot arise.

Now we consider Case 3. For any interval $[s_k,\, t_k]$ we have $x_i(s_k) = x_i(t_k) = \delta_i$ and

$$x_i(t) \leq \delta_i \quad \text{for all } t \in [s_k,\, t_k]. \tag{17}$$

Let $t_k - s_k \leq T_{i0}$. Choose constants

$$h_i = \sup_{t\geq 0} \Big\{ |b_i(t)| + a_{ii}(t)\epsilon_0 + \sum_{j\neq i}^{n} a_{ij}(t)(x_{j0}(t) + \epsilon_0) + d_i(s)U_0 \Big\},$$

and $\sigma_i = \exp[-h_i(T_{i0} + \omega_i)]$. Integrating the first equation of system (1) on interval $[s_k,\, t_k]$ we obtain

$$\begin{aligned} x_i(t) &= x_i(s_k) \exp \int_{s_k}^{t} \Big(b_i(s) - \sum_{j=1}^{n} a_{ij}(s)x_j(s) - d_i(s)u_i(s) \Big)\,\mathrm{d}s \\ &\geq x_i(s_k) \exp \int_{s_k}^{t} \Big(b_i(s) - a_{ii}(s)\epsilon_0 \\ &\qquad - \sum_{j\neq i}^{n} a_{ij}(s)(x_{j0}(s) + \epsilon_0) - d_i(s)U_0 \Big)\,\mathrm{d}s \\ &\geq \delta_i \exp(-h_i T_{i0}) \geq \delta_i \sigma_i. \end{aligned}$$

Let $t_k - s_k > T_{i0}$. From (17) we have

$$f_i(t)x_i(t) < \delta_{i0} \quad \text{for all } t \in [s_k,\, t_k].$$

In (16), we choose $t_0 = s_k$ and $u_{i0} = u_i(s_k)$, since $u_i(s_k) < U_0$ and $u_i(t) = u_i(t, s_k, u(s_k))$ for all $t \geq s_k$. By (16) we obtain

$$|u_i(t) - u_{i0}(t)| < \epsilon_0 \quad \text{for all } t \in [s_k + T_{i0},\, t_k].$$

Therefore, for any $t \in [s_k + T_{i0}, t_k]$ we have $x_i(t) < \delta_i < \epsilon_0$ and $u_i(t) < u_{i0}(t) + \epsilon_0$. For any $t \in [s_k, t_k]$, when $t \le s_k + T_{i0}$, then from the above discussion on the case $t_k - s_k \le T_{i0}$ we have

$$x_i(t) \ge \delta_i \exp(-h_i T_{i0}).$$

In particular, we also have $x_i(s_k + T_{i0}) \ge \delta_i \exp(-h_i T_{i0})$. When $t > s_k + T_{i0}$, we choose an integer $p \ge 0$ such that $t \in (s_k + T_{i0} + p\omega_i, s_k + T_{i0} + (p+1)\omega_i]$. Integrating the first equation of system (1) from $s_k + T_{i0}$ to t we have

$$\begin{aligned}
x_i(t) &= x_i(s_k + T_{i0}) \\
&\quad \times \exp \int_{s_k+T_{i0}}^{t} \Big(b_i(s) - \sum_{j=1}^{n} a_{ij}(s) x_j(s) - d_i(s) u_i(s) \Big) \, \mathrm{d}s \\
&\ge \delta_i \exp(-h_i T_{i0}) \exp \int_{s_k+T_{i0}}^{t} (b_i(s) - a_{ii}(s)\epsilon_0 \\
&\qquad - \sum_{j \ne i}^{n} a_{ij}(s)(x_{j0}(s) + \epsilon_0) - d_i(s)(u_{i0}(s) + \epsilon_0)) \mathrm{d}s, \\
&= \delta_i \exp(-h_i T_{i0}) \exp \Big(\int_{s_k+T_{i0}}^{s_k+T_{i0}+p\omega_i} + \int_{s_k+T_{i0}+p\omega_i}^{t} \Big) \Big[b_i(s) - a_{ii}(s)\epsilon_0 \\
&\qquad - \sum_{j \ne i}^{n} a_{ij}(s)(x_{j0}(s) + \epsilon_0) - d_i(s)(u_{i0}(s) + \epsilon_0) \Big] \, \mathrm{d}s \\
&\ge \delta_i \exp(-h_i T_{i0}) \exp \int_{s_k+T_{i0}+p\omega_i}^{t} [b_i(s) - a_{ii}(s)\epsilon_0 \\
&\qquad - \sum_{j \ne i}^{n} a_{ij}(s)(x_{j0}(s) + \epsilon_0) - d_i(s)(u_{i0}(s) + \epsilon_0)] \, \mathrm{d}s \\
&\ge \delta_i \exp(-h_i T_{i0}) \exp(-h_i \omega_i) \\
&= \delta_i \sigma_i.
\end{aligned}$$

Thus, from the above discussion we obtain

$$x_i(t) \ge \delta_i \sigma_i \quad \text{for all } t \in \cup_{k=1}^{\infty} [s_k, t_k].$$

Further, for any $t \notin \cup_{k=1}^{\infty} [s_k, t_k]$ and $t \ge T_1$, since $x_i(t) > \delta_i$, we directly have $x_i(t) \ge \delta_i \sigma_i$. Therefore, for Case 3 we finally have

$$x_i(t) \ge \delta_i \sigma_i \quad \text{for all } t \ge T_1. \tag{18}$$

Lastly, we consider Case 2. From $x_i(t) \geq \delta_i$ for all $t \geq T^*$, we directly obtain

$$x_i(t) \geq \delta_i \sigma_i \quad \text{for all } t \geq T^*. \tag{19}$$

Choose constant $m_0 = \min_{1 \leq i \leq n}\{\delta_i \sigma_i\}$. Then from (18) and (19) we finally obtain

$$\liminf_{t \to +\infty} x_i(t) \geq m_0, \quad i = 1, 2, \ldots, n$$

for any positive solution $(x(t), u(t)) = (x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_n(t))$ of system (1). This completes the proof of Theorem 2. □

If $r_i(t) \equiv 0$, then system (1) becomes the following system

$$\begin{cases} x_i'(t) = x_i(t)(b_i(t) - \sum_{j=1}^n a_{ij}(t) x_j(t) - d_i(t) u_i(t)), \\ u_i'(t) = -e_i(t) u_i(t) + f_i(t) x_i(t), \quad i = 1, 2, \ldots, n. \end{cases} \tag{20}$$

From Theorem 2, we have the following corollary.

Corollary 2. *Suppose that assumptions* (H_1)–(H_3) *hold and*

$$\liminf_{t \to +\infty} \int_t^{t+\omega_i} (b_i(s) - \sum_{j \neq i}^n a_{ij}(s) x_{j0}(s))\, \mathrm{d}s > 0, \quad i = 1, 2, \ldots, n.$$

Then system (20) *is permanent.*

From (10), (11) and Theorem 2, we have the following corollary.

Corollary 3. *Suppose that assumptions* (H_1)–(H_3) *hold,* $a_{ii}^l > 0$ *and* $e_i^l > 0$, $i = 1, 2, \ldots, n$. *If*

$$\liminf_{t \to \infty} \int_t^{t+\omega_i} \Big(b_i(s) - \sum_{j \neq i}^n a_{ij}(s) \Big(\frac{b_j}{a_{jj}}\Big)^u - d_i(s) \Big(\frac{r_i}{e_i}\Big)^u\Big) \mathrm{d}s > 0,$$
$$i = 1, 2, \ldots, n.$$

Then system (1) *is permanent.*

From (10) and Corollary 2, we have the following corollary.

Corollary 4. *Suppose that assumptions* (H_1)–(H_3) *hold,* $a_{ii}^l > 0$ *and* $e_i^l > 0$, $i = 1, 2, \dots, n$. *If*

$$\liminf_{t\to\infty} \int_t^{t+\omega_i} \bigg(b_i(s) - \sum_{j\neq i}^n a_{ij}(s)\bigg(\frac{b_j}{a_{jj}}\bigg)^u\bigg)\,\mathrm{d}s > 0,$$
$$i = 1, 2, \dots, n,$$

then system (20) *is permanent.*

Suppose that system (1) is ω-periodic. If assumptions (H_1)–(H_3) hold, then from conclusion (d) of Lemmas 2 and 3, we obtain that the following ω-periodic logistic equations

$$x_i'(t) = x_i(t)(b_i(t) - a_{ii}(t)x_i(t))$$

and ω-periodic linear equation

$$u_i'(t) = r_i(t) - e_i(t)u_i(t)$$

for each $i = 1, 2, \dots, n$ have unique ω-periodic solutions $x_{i0}(t)$ and $u_{i0}(t)$, respectively, and $[b_i] = [a_{ii}x_{i0}]$ and $[r_i] = [e_i u_{i0}]$. Therefore, from Theorem 2 and Corollary 2, we have the following corollaries.

Corollary 5. *Suppose that system* (1) *is* ω*-periodic and assumptions* (H_1)–(H_3) *hold. If*

$$\int_0^\omega (b_i(s) - \sum_{j\neq i}^n a_{ij}(s)x_{j0}(s) - d_i(s)u_{i0}(s))\,\mathrm{d}s > 0, \quad i = 1, 2, \dots, n,$$

then system (1) *is permanent.*

Corollary 6. *Suppose that system* (20) *is* ω*-periodic and assumptions* (H_1)–(H_3) *hold. If*

$$\int_0^\omega (b_i(s) - \sum_{j\neq i}^n a_{ij}(s)x_{j0}(s))\,\mathrm{d}s > 0, \quad i = 1, 2, \dots, n,$$

then system (20) *is permanent.*

Corollary 7. *Suppose that system* (1) *is* ω*-periodic, assumptions* (H_1)–(H_3) *hold,* $a_{ii}^l > 0$ *and* $e_i^l > 0$, $i = 1, 2, \dots, n$. *If*

$$[b_i] - \sum_{j\neq i}^{n} \left(\frac{a_{ij}}{a_{jj}}\right)^u [b_j] - \left(\frac{d_i}{e_i}\right)^u [r_i] > 0, \quad i = 1, 2, \dots, n.$$

Then system (1) *is permanent.*

Corollary 8. *Suppose that system* (20) *is* ω*-periodic, assumptions* (H_1)–(H_3) *hold,* $a_{ii}^l > 0$ *and* $e_i^l > 0$, $i = 1, 2, \dots, n$. *If*

$$[b_i] - \sum_{j\neq i}^{n} \left(\frac{a_{ij}}{a_{jj}}\right)^u [b_j] > 0, \quad i = 1, 2, \dots, n.$$

Then system (20) *is permanent.*

Remark 1. From Corollaries 2, 4, 6 and 8, we easily see that for feedback control system (20), the feedback controls do not impact the permanence of all species x_i. This is obviously a very interesting phenomenon.

Remark 2. It is well known that, if a periodic population system with period ω of ordinary differential equations or functional differential equations is permanent, then it must have at least one positive ω-periodic solution. Therefore, when the all conditions of Corollaries 5–8 are satisfied, ω-periodic systems (1) and (20) must have at least one positive ω-periodic solution.

Remark 3. In [**3**], Chen obtained the following result on the permanence for system (1), see [**3**, Theorem 2.1].

Suppose that the following conditions hold

$$M\Big[b_i(t) - \sum_{j\neq i}^{n} a_{ij}(t)x_{j0}(t) - d_i(t)U_{i0}(t)\Big] > 0, \quad i = 1, 2, \dots, n.$$

Then system (1) is permanent, where for any function $g(t)$ defined on R_+,

$$M[g(t)] = \limsup_{s\to\infty} \left\{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} g(u)\, du : t_2 - t_1 \geq s\right\},$$

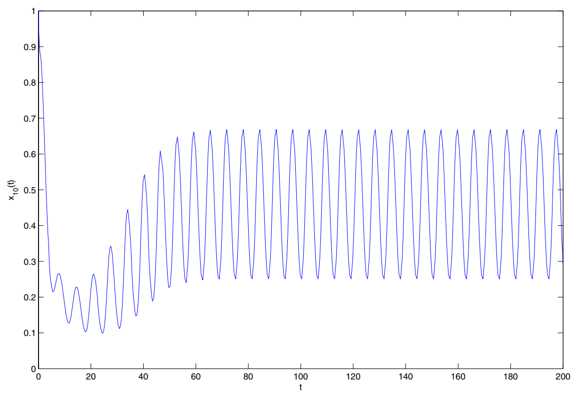

FIGURE 1. Numerical simulation of $x_{10}(t)$.

and $U_{i0}(t)$ is some solution of the following equation

$$u_i'(t) = r_i(t) - e_i(t)u_i(t) + f_i(t)k_i,$$

where constant $k_i = \sup\{x_{i0}(t) + \varepsilon : t \geq t_0\}$ and ε is a small enough positive constant.

However, in condition (13) of Theorem 2, function $u_{i0}(t)$ is some positive solution of equation (11). Obviously, we have that there exists a constant $T > 0$ such that $u_{i0}(t) \leq U_{i0}(t)$ for all $t \geq T$. Therefore, the results given in this paper are an improvement of the corresponding results given in [**3**].

4. An example. In this section we will give an example to illustrate the conclusions obtained in the above sections. We consider the following nonautonomous two-species Lotka-Volterra competitive system with feedback controls.

$$(21)\quad \begin{cases} x_1'(t) = x_1(t)[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - c_1(t)u_1(t)] \\ x_2'(t) = x_2(t)[b_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - c_2(t)u_2(t)] \\ u_1'(t) = f_1(t) - e_1(t)u_1(t) + d_1(t)x_1(t) \\ u_2'(t) = f_2(t) - e_2(t)u_2(t) + d_2(t)x_2(t), \end{cases}$$

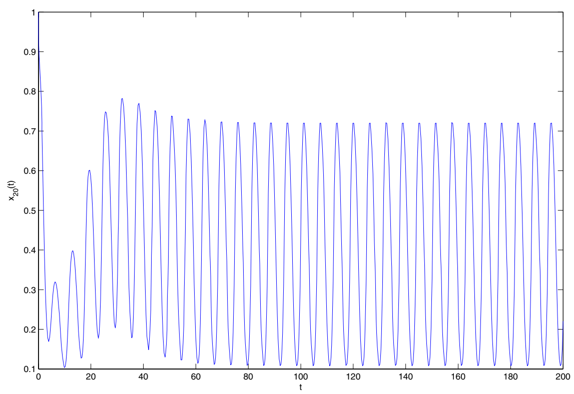

FIGURE 2. Numerical simulation of $x_{20}(t)$.

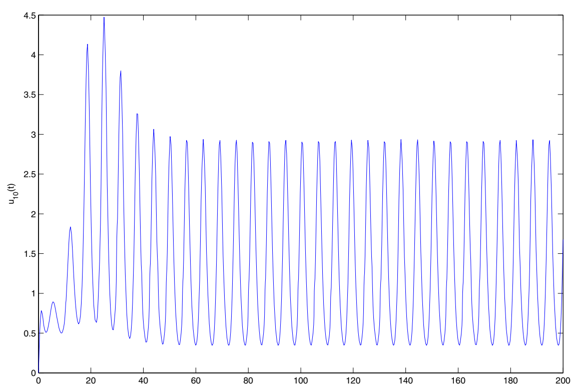

FIGURE 3. Numerical simulation of $u_{10}(t)$.

where

$$
\begin{aligned}
b_1(t) &= \frac{1}{2} + \sin t + te^{-0.15t}, \\
b_2(t) &= 1 + 2\cos t + 2e^{-0.1t}\ln(t+1), \\
a_{11}(t) &= 1 + \sin t + t^2 e^{-0.2t}, \\
a_{22}(t) &= 2(1 + \cos t) + t^2 e^{-0.3t}, \\
a_{12}(t) &= \frac{1}{3}(1 + \sin t) + t^3 e^{-0.3t}, \\
a_{21}(t) &= \frac{1}{4}\cos^2 t + t^2 e^{-0.2t},
\end{aligned}
$$

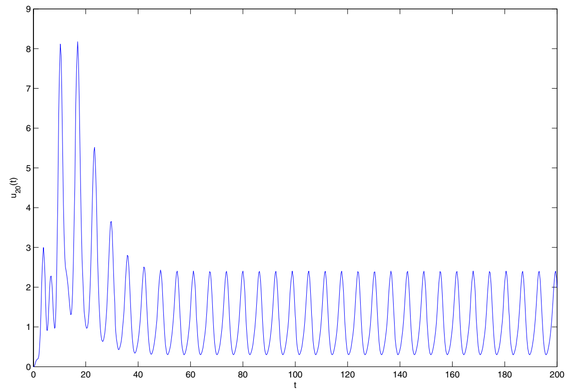

FIGURE 4. Numerical simulation of $u_{20}(t)$.

$$\begin{aligned}
f_1(t) &= \frac{1}{2}(1+\cos t) + te^{-0.2t},\\
f_2(t) &= \frac{1}{3}\sin^2 t + t^2\cos^2 te^{-0.3t},\\
e_1(t) &= \frac{1}{3} + \sin t + t^2 e^{-0.4t},\\
e_2(t) &= \frac{1}{4} + \cos t + 2e^{-0.2t}\ln(t+1),\\
c_1(t) &= \frac{1}{5}(1+\sin t) + t^2 e^{-0.3t},\\
c_2(t) &= \frac{1}{4}(1+\cos t) + t^3 e^{-0.5t}
\end{aligned}$$

and

$$\begin{aligned}
d_1(t) &= 2 + \cos t + \ln(t+1)e^{-0.4t},\\
d_2(t) &= 2 + \sin t + (1+t^2)e^{-0.3t}.
\end{aligned}$$

Obviously, $a_{ii}(t)$, $f_i(t)$, $c_i(t)$ and $d_i(t)$, $i = 1, 2$, are nonnegative for all $t \in R_+$. Choose the constants $\omega_i = \lambda_i = \gamma_i = 2\pi$, $i = 1, 2$. Then we easily obtain

$$\liminf_{t\to\infty} \int_t^{t+2\pi} b_i(s)\,ds > 0, \quad i = 1, 2,$$

$$\liminf_{t\to\infty}\int_t^{t+2\pi} a_{ii}(s)\,ds > 0, \quad i = 1, 2$$

and

$$\liminf_{t\to\infty}\int_t^{t+2\pi} e_i(s)\,ds > 0, \quad i = 1, 2.$$

Therefore, assumptions (H_1)–(H_3) hold.

We consider the following four initial value problems of differential equations, respectively,

$$x_1'(t) = x_1(t)(b_1(t) - a_{11}(t)x_1(t)), \quad x_1(0) = 1, \tag{22}$$
$$x_2'(t) = x_2(t)(b_2(t) - a_{22}(t)x_2(t)), \quad x_2(0) = 1, \tag{23}$$

$$u_1'(t) = f_1(t) - e_1(t)u_1(t), \quad u_1(0) = 0 \tag{24}$$

and

$$u_2'(t) = f_2(t) - e_2(t)u_2(t), \quad u_2(0) = 0. \tag{25}$$

Let $x_{10}(t)$, $x_{20}(t)$, $u_{10}(t)$ and $u_{20}(t)$ be the solutions of problems (22)–(25), respectively, and further let

$$I_1 = \liminf_{t\to\infty}\int_t^{t+2\pi}(b_1(s) - a_{12}(s)x_{20}(s) - c_1(s)u_{10}(s))\,ds$$

and

$$I_2 = \liminf_{t\to\infty}\int_t^{t+2\pi}(b_2(s) - a_{21}(s)x_{10}(s) - c_2(s)u_{20}(s))\,ds.$$

Since the precise expressions of $x_{10}(t)$, $x_{20}(t)$, $u_{10}(t)$ and $u_{20}(t)$ are quite complex, it is hard to determine the values of I_1 and I_2. Therefore, here we will use the method of numerical simulation.

Applying Matlab software, we can obtain the numerical simulations of $x_{10}(t)$, $x_{20}(t)$, $u_{10}(t)$ and $u_{20}(t)$, respectively, see Figures 1–4.

Furthermore, by numerical calculation, for $t \geq 200$ large enough, we can obtain

$$\int_t^{t+2\pi} (b_1(s) - a_{12}(s)x_{20}(s) - c_1(s)u_{10}(s))\, ds \doteq 0.29$$

and

$$\int_t^{t+2\pi} (b_2(s) - a_{21}(s)x_{10}(s) - c_2(s)u_{20}(s))\, ds \doteq 4.34$$

Therefore, we have $I_1 > 0$ and $I_2 > 0$. Thus, by Theorem 2 we finally obtain that species x_1 and x_2 in system (21) are permanent.

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Department of Mathematics, Xinjiang University, Urumqi 830046, P.R. China
Email address: xiaomei_0529@126.com

Department of Mathematics, Xinjiang University, Urumqi 830046, P.R. China
Email address: zhidong@xju.edu.cn

Department of Mathematics, Xinjiang University, Urumqi 830046, P.R. China
Email address: longzhang_xj@sohu.com