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LIFTING MCKAY GRAPHS AND RELATIONS TO PRIME EXTENSIONS

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ABSTRACT. We give an algorithmic procedure for constructing the quivers and homogeneous relations of the Brauer correspondents of blocks of the abelian noncyclic defect group in the almost simple groups in the ATLAS. This is one step in a program to compute the structure of the indecomposable projectives for these blocks. As an illustration, we determine an explicit tilting complex for the nonprincipal 3-block of the central extension of PSL(3,4) by C_2.

0. Introduction. One of the important outstanding problems in the theory of modular group representations is the determination of the structure of the projective indecomposables of blocks B in simple or almost simple groups. Recent work on the Broué conjecture (the conjecture that blocks of abelian defect group are derived equivalent to the Brauer correspondent b, cf. [**3, 12, 18**] leads in many cases to explicit tilting complexes P^*, cf. [**1**]. In these cases, the desired block is Morita equivalent to the endomorphism ring of P^* in the homotopy category of complexes of projective modules over the Brauer correspondent b of the block B. Both the verification that the complex P^* is a tilting complex producing the desired block and the calculation of the endomorphism ring of the complex in the derived category can be done more efficiently if the structure of b is well understood. This can be done in a particularly compact fashion by giving the quiver and relations of the block b.

There is, furthermore, a natural grading on b, compatible with the filtration by the power of the radical in the group algebra of the defect group. It is induced by the image V of a splitting of $\mathrm{Rad}\,(b)/\mathrm{Rad}^2(b)$ into $\mathrm{Rad}\, b$, a homogeneous basis for the subspace of weight d being

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given by monomials of degree d in basis vectors of V. If we could find a homogeneous basis for the endomorphism ring of the tilting complex, then we could try to transfer the grading to the blocks B of the larger group, as was done for the cyclic case in [**24**]. This would also mean that the Donald-Flanigan deformation guaranteed in [**11**] could be made homogeneous, with parameter-ring $K[t]$.

In this paper we concentrate on the noncyclic case where the defect group Q of b is $C_q \times C_q$ or $C_q \times C_q \times C_q$. This covers most of the noncyclic blocks of abelian defect group in the ATLAS groups and their subgroups [**4**]. We first note a general result about the construction of the quiver of b in Section 2 and then give an algorithm for constructing the relations in Section 3, followed by some examples. In Section 4 we show that the differentials on the tilting complex can be chosen to be homogeneous. Finally, in Section 5 we give an explicit tilting complex for the nonprincipal 3-block of the central extension of PSL(3,4) by C_2.

1. Quivers and relations for blocks with normal defect group. Before discussing the case of abelian defect group in particular, we first review what is known in general about the quivers and relations of blocks of normal defect group. By a fundamental result of Külshammer [**8**], all such blocks are isomorphic to $M_n(K^\gamma[Q \rtimes G'])$, where K is a sufficiently large field of characteristic q, Q is the defect group, G' is a q' group acting faithfully on Q and γ is a Hochschild cohomology two-cycle for G' into K^*. The matrix algebra does not affect the quivers and relations, and we incorporate γ into the group theory by passing to a central extension G of G' with kernel N, as in [**14**], where N is cyclic of order equal to the order of γ in $H^2(G', K^*)$. The original block is thus Morita equivalent to one of the blocks of $Q \rtimes G$; these blocks all have the same K-dimension and are in one-to-one correspondence with the irreducible characters of the abelian group N.

We recall the definition of the McKay graph, which we will use to obtain the quiver of the desired block of $Q \rtimes G$.

Definition. Let G be a finite group, let K be a sufficiently large field of characteristic q not dividing $|G|$ and let X be any character of G (not necessarily irreducible). The McKay graph $D(G, X)$ is the directed graph with

(1) Vertices X_i labeled by elements of $\mathrm{Irr}\,(G)$, $i = 1, \dots, r$.

(2) For each pair of vertices X_i, X_j, a number $n_{ij} = (X_i, X \cdot X_j)_G$ arrows from X_j to X_i, where $X \cdot X_j$ is the class function formed by pointwise multiplication, that is, $(X \cdot X_j)(g) = X(g) \cdot X_j(g)$ and n_{ij} is the number of constituents of X_i in $X \cdot X_j$, i.e., $X \cdot X_j = \sum_{i=1}^{r} n_{ij} X_i$.

We recall that the quiver of a block B is a directed graph whose vertices correspond to the isomorphism classes of projective indecomposables $[e_i B]$, where the number of arrows from $[e_j B]$ to $[e_i B]$ is $\dim_k e_i((\mathrm{Rad}\, B)/\mathrm{Rad}\,(B)^2)e_j$. This is sometimes called the Ext-quiver of the block.

The main result of [**20**] states that if χ is the character of the action of G on J/J^2, $J = \mathrm{Rad}\,(KQ)$ and $\mathrm{char}\, K \nmid |G|$, then the quiver of $K[Q \rtimes G]$ is the McKay graph $D(G, \chi)$. The connected components of $D(G, \chi)$ are in one-to-one correspondence with the blocks of $K[Q \rtimes G]$. Let $y_1, \dots, y_s$ be a complete set of primitive idempotents for KG, and let $y_1, \dots, y_m$ be a completion to a basis of KG using matrix units. Then from [**11**] we know that there exists a subvector space $V = \langle x_1, \dots, x_r \rangle$ of $K[Q]$ which is a G-module and is isomorphic as a G-module to J/J^2. If $r_1, \dots, r_t$ is a set of relations for $K[Q]$ as a quotient of the free tensor algebra on V, then a complete set of relations for the algebra $K[Q \rtimes G]$ as a quotient of the path algebra of the quiver is given by the relations $y_i r_j y_k$, where $1 \le i \le s$ and $1 \le k \le m$ for $j = 1, \dots, t$.

In practice, when Q is not abelian, writing the relations of $K[Q]$ in terms of the basis of V is not trivial because the basis elements of V are not simple linear compositions of group elements like $h - e$, but rather are obtained by some Maschke averaging. Thus, it is not easy to convert group relations to algebra relations. However, when Q is abelian, the relations of $K[Q]$ are of the simple form $x_i^{p^n} = 0$ and $x_i x_j - x_j x_i = 0$. The algebra $K[Q]$ has a homogeneous basis, given by the monomials in the generators $x_1, \dots, x_r$. In the sequel, we will abbreviate $K[Q]$ by KQ.

2. Lifting McKay graphs. We consider the case of a McKay graph $D(G, X)$ where X is an irreducible character of G induced from an irreducible character W of a normal subgroup H, where $[G : H] = p$. Letting a be an element of $G - H$, and letting $W_i = W^{a^i}$, $i = 0, \dots, p-1$, we have $X = \sum_{i=0}^{p-1} W_i$. We note that the permutation α of the irreducibles induced by conjugation by a induces graph isomorphism $D(H,W) \overset{\sim}{\to} D(H,W_i)$, $i = 0,\dots,p-1$. The set of functions from the conjugacy classes of H which are $\mathbf{Z}$-linear combinations of irreducible characters will be denoted by $\mathbf{Z}\mathrm{Irr}\,(H)$.

Lemma 2.1. *Let $X \in \mathrm{Irr}\,(G)$ be a character induced from an irreducible character W of H, i.e., $X = W^G$. Then the McKay graph $D(G,X)$ is completely determined by the McKay graph $D(H,W)$ and the mapping of $\mathrm{Irr}\,(G)$ to $\mathbf{Z}\mathrm{Irr}\,(H)$ induced by restriction. If $R', R'' \in \mathrm{Irr}\,(G)$, then the number of arrows from R' to R'' in $D(G,X)$ is the total number of arrows from summands of R'_H to summands of R''_H in $D(H,W)$.*

Proof. By a result of Frobenius $X \cdot R' = (W \cdot R'_H)^G$. We now apply Frobenius reciprocity:

$$(R'', X \cdot R')_G = (R'', (W \cdot R'_H)^G)_G = (R''_H, W \cdot R'_H)_H. \qquad \square$$

In the sequel, in our situation of $H \trianglelefteq G$ and $[G:H] = p$ a prime, we let $\mathrm{Irr}\,(H) = S_H \cup C_H$, where S_H are the characters fixed under conjugation by G. Let $\mathrm{Irr}\,(G) = S_G \cup C_G$ be the corresponding partition into characters which have split and characters which have collapsed.

Example 1. $G = C_5 \rtimes C_8$, $H = C_5 \rtimes C_4$, the generalized quaternion group.

$$G = \left\langle a, b \mid a^8 = b^5 = e, a^{-1}ba = b^2 \right\rangle,$$
$$H = \left\langle c, b \mid c^4 = b^5 = e, c^{-1}bc = b^{-1} \right\rangle,$$
$$X = X_9 \in C_G, \quad W = X_5 \in C_H, \quad \deg X = 4, \quad \deg W = 2.$$

Here $c = a^2$.

See Figure 1 for the diagram of the McKay graphs.

3. Lifting relations. The second author set up a database [**19**], using GAP [**5**], of blocks of noncyclic abelian defect group with defect

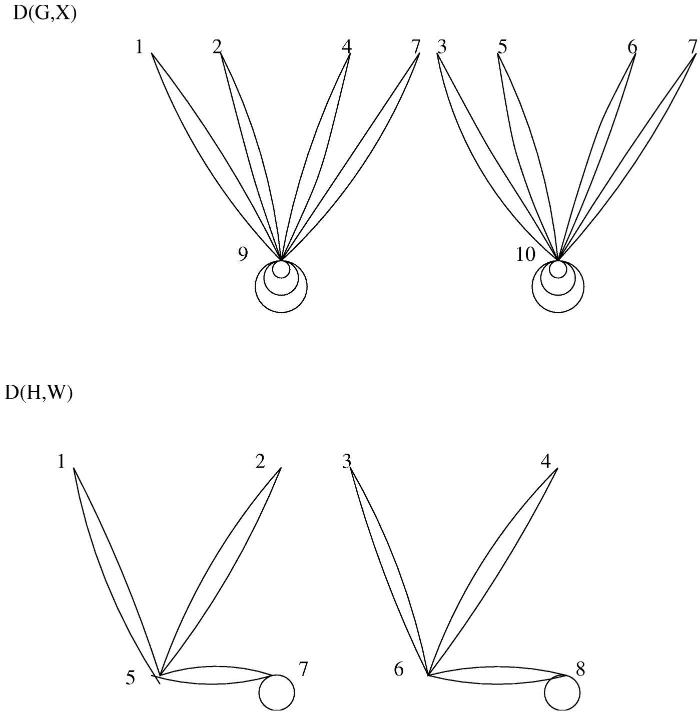

FIGURE 1.

at least 2 in the ATLAS groups and their subgroups. There are none for primes $q > 13$, and only 10 for $q = 11, 13$. For $q = 3, 5$, and 7, the blocks are further subdivided. The main database, SortAt_q, located at [**19**], consists of blocks with elementary abelian defect group. There are 874 blocks in this database for $q = 3$, 360 for $q = 5$ and 81 for $q = 7$. There is a separate list, CyclicSortAt_q, of blocks of abelian defect group containing C_{q^2}, most of which have cyclic defect, with the remainder being products of cyclic blocks.

In the interesting case where the defect group Q is elementary abelian, it usually has prime rank $d(= 2, 3)$, and $d = 2$ is much more common than $d = 3$. Notable exceptions are the O'Nan group, whose principal 3-block is elementary of rank 4, and extensions of $\mathrm{PSL}(2, 81)$. Since $|Q| = q^d$, the integer d is the defect of the block.

TABLE 1. Reynolds groups for 3-blocks in the database with elementary abelian defect group, $k(B) \leq 24$.

d	$k(B)$	$\ell(B)$	# of blocks	Reynolds group
2	6	2	15	$(C_3 \times C_3) : (C_8 : C_2), b_2$
2	6	2	8	$(C4 \times (C_3 \times C_3) : C_2) : C2, b_3$
2	6	2	16	$(C_3 \times C_3) : C_2$
2	6	4	74	$(C_3 \times C_3) : C_4$
2	6	5	74	$(C_3 \times C_3) : Q$
2	9	1	35	$C_3 \times C_3$
2	9	2	48	$C_3 \times S_3$
2	9	4	141	$S_3 \times S_3$
2	9	5	182	$(C_3 \times C_3) : D_4$
2	9	7	89	$(C_3 \times C_3) : (C_8 : C_2)$
2	9	8	4	$(C_3 \times C_3) : C_8$
3	15	2	2	$(C_3)^3 : C_2$
3	15	5	1	$(C_3 \times (C_3 \times C_3) : C_4) : C_2$
3	15	7	1	$(C_3 \times (C_3 \times C_3) : Q) : C_2$
3	18	4	4	$C_3 \times (C_3 \times C_3) : C_4$
3	18	8	4	$S_3 \times (C_3 \times C_3) : C_4$
3	18	10	3	$S_3 \times (C_3 \times C_3) : Q$
4	18	14	2	$(C_3)^4 : (C_2.(C_2)^4) : D_5$
4	24	14	1	$(((C_3 \times C_3) : C_4) \times ((C_3 \times C_3) : C_4)).C_2$
4	24	16	1	$(C_3)^4 : (C_2.((C_2)^4) : D_5).C_2$
4	24	20	1	$(C_3)^4 : (C_{80} : C_2), b_2$
4	24	22	1	$(C_3)^4 : (C_{40} : C_4)$

A study of the Brauer correspondents b of the blocks with elementary abelian undertaken by Leabovich [**9**] and Berrebi [**2**] has determined that in almost all cases either H is itself abelian, or the Brauer correspondent is isomorphic to the principal block of the normalizer, and thus we are reduced to the abelian case, or $b \xrightarrow{\sim} M_t(b')$, where $b' \xrightarrow{\sim} K[Q \rtimes G]$, with $[G : H] = 2$ and H abelian. For example, the normalizer for blocks of the symmetric groups S_n are of the form $(Q \rtimes G) \times S_{n-2p}$. Then $b \xrightarrow{\sim} M_t(b')$, where b' is a block of $K[Q \rtimes G]$ and t is the degree of a block of S_{n-2p} of defect zero. In general, as was shown by Reynolds [**14**], if b is a block isomorphic to $M_t(K^\gamma[Q \rtimes G'])$, we can find a central extension G of G' with cyclic kernel N such that b is Morita equivalent to a block of $Q \rtimes G$. The group $Q \rtimes G$, with N

TABLE 2. Reynolds groups for 5-blocks.

d	$k(B)$	$\ell(B)$	# of blocks	Reynolds group
2	10	6	1	$(C_5 \times C_5) : (C_3 : (C_8 : C_2)), b_2$
2	11	5	5	$(C_5 \times C_5) : C_2.(C_4 \times C_4).C_2, b_2$
2	13	10	8	$(C_5 \times C_5) : (C_8 : C_2)$
2	13	12	1	$(C_5 \times C_5) : (C_3 : C_8)$
2	14	12	6	$(C_5 \times C_5) : C_{12}$
2	16	10	8	$(C_5 \times C_5) : (D_4 Y C_4)$
2	16	12	9	$(C_5 \times C_5) : (S_3 \times C_4)$
2	16	14	1	$(C_5 \times C_5) : (Q Y C_4)$
2	20	14	19	$(C_5 \times C_5) : (C_4 \times C_4) : C_2$
2	26	24	3	$(C_5 \times C_5) : C_{24}$

minimal, will be called the Reynolds group. These are listed in Tables 1, 2 and 3. When $q = 3$ and $d = 3$, the Reynolds group is generally a direct product. For nilpotent blocks, which are Morita equivalent to KQ, we take the Reynolds group to be Q. Those blocks for which the decomposition matrix was not known and no other information was available are not included.

Let the Reynolds group of b be $Q \rtimes G$, where G is a q'-group and the action of G on $Q = C_q \times \cdots \times C_q$ induces an action on $\mathrm{Rad}(KQ)/\mathrm{Rad}^2(KQ)$ with character χ. In Table 1–Table 3, we give $Q \rtimes G$, sorted as in the database by the invariants $k(B)$ and $\ell(B)$ of the block B. If the block which is Morita equivalent to b is not the principal block of $K[Q \rtimes G]$, then we indicate the block by writing b_j. Note that in all cases given here $K[Q \rtimes G]$ has at most three blocks.

In this paper, we treat the common case when one of the following holds:

Case 1. The character χ is a sum of linear characters.

Case 2. The group G has an abelian normal subgroup H of index p such that $\chi_H = \phi_1 + \cdots + \phi_p$ is a sum of linear characters permuted by G and KH has a set of primitive idempotents permuted by a, where $a \in G$ is an element of p power order whose residue in G/H generates G/H. Let $N = \ker(\chi)$.

TABLE 3. Reynolds groups for 7-blocks.

d	$k(B)$	$\ell(B)$	# of blocks	Reynolds group
2	16	12	1	$(C_7 \times C_7) : (C_3 \times D_{16})$
2	22	18	3	$(C_7 \times C_7) : (C_3 \times (C_3 : C_4))$
2	22	21	1	$(C_7 \times C_7) : (C_3 \times Q_8)$
2	25	18	3	$(C_7 \times C_7) : (S_3 \times C_6)$
2	25	21	2	$(C_7 \times C_7) : (C_3 \times D_8)$
2	26	18	1	$(C_7 \times C_7) : (C_3 \times C_6)$
2	26	24	3	$(C_7 \times C_7) : C_{24}$
2	27	21	8	$(C_7 \times C_7) : (C_3 \times SL(2,3))$
2	27	24	1	$(C_7 \times C_7) : (C_3 \times GL(2,3))$
2	35	27	7	$(C_7 \times C_7) : (C_3 \times (C_3 : D_4))$
2	49	48	3	$(C_7 \times C_7) : C_{48}$

In Case 1, the relations are all of the form $e_i x_j^q$ or $e_i(x_j x_k - x_k x_j)$ for $j, k = 1, \ldots, p$, $i = 1, \ldots, [G : N]$, as described in [**11**].

Case 2 is the case treated in detail by this paper. Clearly,

$$N = \bigcap_{i=1}^{n} \ker(\phi_i).$$

(Even if H had not been assumed abelian, since the derived group H' of H is in each $\ker(\phi_i)$, we could have gotten $H' \subset N$, so that H/N would have been abelian.) Since the ϕ_i are conjugate via a, $\phi_1(a^p) = \phi_j(a^p)$ for $j = 2, \ldots, p$.

Let $e_1, \ldots, e_r$ be the block idempotents for H. Since conjugation by a is an automorphism, a permutes these idempotents of H. Any ϕ_j, for $j = 1, \ldots, p$, induces a permutation τ_j on the linear characters θ_i of H by setting $\tau_j(i) = k$ if and only if $\phi_j \cdot \theta_i = \theta_k$. As was proven in [**11**], this precisely corresponds to the condition

$$e_i x_j = x_j e_k.$$

Since τ_j corresponds to multiplication by ϕ_j, the τ_j commute because character multiplication commutes. In particular, in the relation $e_i(x_j x_k - x_k x_j)$, both monomials have the same idempotent e_ℓ acting on the left, where $\ell = \tau_j \circ \tau_k(i) = \tau_k \circ \tau_j(i)$.

The problem of finding quivers and relations for the case of H abelian and $G = H \rtimes C_p$ was first treated by [**13**, Section 2]. We consider the nonsplit case and also construct the relations in a mechanical, algorithmic fashion which has been machine implemented for the case of H abelian and $p = 2$. We recall that C_H is the set of characters of H on which a acts nontrivially and refer to the set $\overline{C}_H$ of the corresponding idempotents as "collapsing idempotents." Similarly, S_H is the set of fixed characters, and the set $\overline{S}_H$ containing the corresponding idempotents will be called "splitting idempotents."

To give the quiver and relations of $K[Q \rtimes G]$, we proceed as follows:

1) Choose one primitive idempotent $e_{\hat{\imath}}$ of each nontrivial conjugacy class $[e_i]$ of idempotents of H under the action of a. For each idempotent e_k in H, choose a natural number $\ell(k)$, $1 - p < \ell(k) \leq p - 1$ such that $e_{\hat{k}} = a^{-\ell(k)} e_k a^{\ell(k)}$, and set $E_{k\hat{k}} = a^{\ell(k)}$, $E_{\hat{k}k} = a^{-\ell(k)}$.

2) Construct the quiver $D(G, \chi)$ from $D(H, \phi_1)$ by collapsing and splitting as in Lemma 2.1, labeling each collapsed vertex by the idempotent $e_{\hat{k}}$ representing its conjugacy class of idempotents.

3) For each splitting idempotent e_i, let $e_{i1}, e_{i2}, \ldots, e_{ip}$ be the idempotents into which it splits. (If $p = 2$, we abbreviate e_{i1} by e_i^+ and e_{i2} by e_i^-.) Let θ be a nontrivial lifting of 1_H. By standard results in Clifford theory, if θ_i' is the character associated to e_{i1}, then the character θ_{ij}' associated to e_{ij} is a multiple of θ_i' by a power of θ, and we may choose the numbering so that $\theta_{ij} = \theta_i' \theta^{j-1}$ is associated to e_{ij}.

Lemma 3.2. *Let Q be an elementary abelian q-group of prime rank p. Let G be a q'-group acting irreducibly on Q, and assume that G is a p-extension of an abelian group H. Let $R = \{r_j\}$ be the set of all relations of KQ, i.e., $x_\ell^q = 0$, $\ell = 1, \ldots, p$, and $x_t x_s - x_s x_t = 0$ for $1 \leq t < s \leq p$. Each relation r_j determines a unique permutation σ_j such that $e_i r_j = r_j e_{\sigma_j(i)}$. Set $k = \sigma_j(i)$. Each relation $e_i r_j e_k$ of $K[Q \rtimes H]$ determines the following relations of the quiver of $K[Q \rtimes G]$:*

(a) *If e_i does not split and is a representative of its conjugacy class and e_k does not split, then we get a relation $e_i r_j E_{k\hat{k}} = e_i r_j a^\ell$, where $\ell = \ell(k)$ is chosen so that $a^{-\ell} e_k a^\ell$ is the conjugacy class representative $e_{\hat{k}}$.*

(b) *If e_i does not split and is a representative of its conjugacy class and e_k does split, we get a relation*

$$e_i r_j e_k,$$

which splits into p relations since e_k is splitting.

(c) *If e_i splits and e_k does not, then we get relations $e_{i\ell} r_j e_k a^\ell$, where $\ell = \ell(k)$ is chosen so that $a^{-\ell} e_k a^\ell$ is the conjugacy class representative $e_{\hat{k}}$.*

(d) *If e_i, e_k are both splitting, we get the p^2 relations $e_{it} r_j e_{kt'}$.*

Proof. Let $\{r_j\}$ be the complete set of relations such that

$$K[Q] \xrightarrow{\sim} K\langle x_1, \ldots, x_p\rangle / \{r_j\},$$

as given in the statement of the lemma. Since the quiver of $K[Q]$ is just p loops, then by the lemma in [**11**, Section 5], substituting 0 for the deformation parameter, we know that the relations of $K[Q \rtimes H]$ and $K[Q \rtimes G]$, respectively, are obtained from $\{r_j\}$ by multiplying on the left by a primitive idempotent corresponding to a vertex and, in the case of $K[Q \rtimes G]$, on the right by a matrix unit.

Fix a relation r_j of $K[Q]$. If it is monomial in $x_1, \ldots, x_p$, then we get σ_j as a composition of τ_i. If it is of the form $x_s x_t - x_t x_s$, then $\sigma_j = \tau_t \tau_s = \tau_s \tau_t$. We previously defined a partition $\mathrm{Irr}\,(H) = S_H \cup C_H$, according to whether the character was fixed under conjugation by G and would thus split, or whether it had nontrivial conjugates, and a similar partition of the idempotents $\{e_1, \ldots, e_r\} = \overline{S}_H \cup \overline{C}_H$. For any primitive idempotent e_i for which $i = \hat{\imath}$, we have the following logical possibilities:

(a) If $e_i, e_k \in \overline{C}_H$, $k = \sigma_j(i)$, then there is a relation $e_i r_j$ of the quiver of $K[Q \rtimes H]$. Let $\ell = \ell(k)$ be the integer such that $e_{\hat{k}} = a^{-\ell} e_k a^\ell$. Then the corresponding relation of $K[Q \rtimes G]$ will be

$$0 = e_{\widehat{i}} r_j E_{k\widehat{k}} = e_{\widehat{i}} r_j a^\ell.$$

(b) If $e_i \in \overline{C}_H$ and $e_k \in \overline{S}_H$, we multiply by the various idempotents which split it. These relations are distinct since they end at different points.

(c) If $e_i \in \overline{S}_H$ and $e_k \in \overline{C}_H$, then we proceed as in (a) to show that the matrix unit given in the lemma of [**11**] is a power of a.

(d) If $e_i, e_k \in \overline{S}_H$, then there are p^2 possible relations.

Corollary 3.3. *If a splitting idempotent $e_s \in \overline{S}_H$ can be inserted in $e_i r_j$, then the a^ℓ at the end in cases* (a) *and* (c) *can be conjugated forward until it reaches e_s when it can be converted into a coefficient. Since H is abelian, case* (d) *occurs only when $p = 2$, and there are only p nonzero relations in* (d).

Proof. Conjugation by a introduces coefficients in two ways. First of all, $ax_pa^{-1} = \phi_1(a^p) \cdot x_1$. Second, when the power of a reaches an element of $\overline{S}_H$, it is converted into a coefficient by the formula $a \cdot e_{st} = e_{st} \cdot a = \theta'_{st}(a)e_{st}$.

We must show that when H is abelian, case (d) occurs only for $p = 2$, and that then there are only p nonzero relations.

If r_j is a relation for which case (d) occurs, and $m(x_1, \ldots, x_p)$ is a monomial occurring in r_j, then we have $\sigma_j(i) = m(\tau_1, \ldots, \tau_p)(i) = k$, which on characters becomes $m(\phi_1, \ldots, \phi_p) \cdot \theta_i = \theta_k$. Since $\theta_i, \theta_k \in S_H$, we get that $m(\phi_1, \ldots, \phi_p) = \theta_i^{-1}\theta_k \in S_H$, showing that $m(\phi_1 \ldots, \phi_p)$ is invariant under the action of G.

This cannot occur for the monomial relation $x_t^q = 0$ because the cycles in the McKay graph $D(H, \phi_1)$ are of length r dividing $|H|$, so $\phi_t^r = \phi_{t'}^r$ for all t, t'. If we had ϕ_t^q invariant under the action of G so that $\phi_t^q = \phi_{t'}^q$, then since $|H|$ is a q' integer, we would have $\phi_t = \phi_{t'}$ for all t', a contradiction since the irreducibility of the action of G on Q implies that the restriction of this action to H is a sum of distinct conjugates.

If r_j is the relation $x_t x_s - x_s x_t$, then $\phi_t\phi_s$ is invariant. Then, acting by a^{s-t}, we get $\phi_t\phi_s = \phi_s\phi_{2s-t}$, so, multiplying by ϕ_s^{-1}, $t \equiv 2s - t \pmod{p}$. Since p is a prime, $t \neq s$, and $1 \leq s, t \leq p$, we have $2t \equiv 2s \pmod{p}$ only when $p = 2$.

The monomial x_1x_2 is an eigenvector for G since x_1 and x_2 are eigenvectors for H, with characters ϕ_1 and ϕ_2, while $a^{-1}x_1x_2a = x_2(\phi_1(a^2)x_1) = \phi_1(a^2)x_1x_2$. Thus, if η is the linear character of G with $\eta \big|_H = \phi_1\phi_2$ and $\eta(a) = \phi_1(a^2)$, then for any idempotent e_{is}

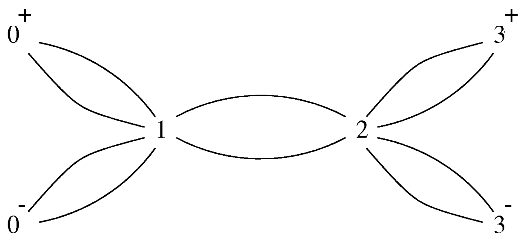

FIGURE 2.

with character ϕ_{is}, we have $e_{is}r_j = r_j e_{ks'}$, where s' is so chosen that $\eta \cdot \theta_{is} = \theta_{ks'}$. Thus, there is only one value s'' of s' for which the relation $e_{is}r_j e_{ks''}$ is nonzero.

Example 2. Consider the Brauer correspondent of the principal block of the Janko group J_2 for $q = 5$. It is a semi-direct product of $C_5 \times C_5$ by D_6, the dihedral group of order 12, and the action of D_6 on J/J^2 is determined by the unique two-dimensional representation without a kernel. The McKay graph is (see Figure 2).

Let $\alpha_{01}^{\pm}$, α_{12}, $\alpha_{23}^{\pm}$ be the arrows with increasing exponent and $\beta_{10}^{\pm}$, β_{21}, $\beta_{32}^{\pm}$ the arrows in the opposite direction. The relations are derived as follows from the following cycles in H, $e_i x_1 = x_1 e_{i+1}$ and $e_i x_2 = x_2 e_{i-1}$.

$$
\begin{array}{ll}
e_1x_1^5, e_1x_2^5 = 0: & \alpha_{01}^{+}\alpha_{12}(a_{23}^{+}\beta_{32}^{+} - \alpha_{23}^{-}\beta_{32}^{-})\beta_{21} = 0 \\
e_2x_1^5, e_2x_2^5 = 0: & \beta_{32}^{\pm}\beta_{21}(\beta_{10}^{+}\alpha_{01}^{+} - \beta_{10}^{-}\alpha_{01}^{-})\alpha_{12} = 0 \\
e_1x_1^5 = 0: & \alpha_{12}(\alpha_{23}^{+}\beta_{32}^{+} - \alpha_{23}^{-}\beta_{32}^{-})\beta_{21}\beta_{10}^{\pm} = 0 \\
e_2x_2^5x_2^5 = 0: & \beta_{21}(\beta_{10}^{+}\alpha_{01}^{+} - \beta_{10}^{-}\alpha_{01}^{-})\alpha_{12}\alpha_{23}^{\pm} = 0 \\
e_2x_1^5 = 0: & (\alpha_{23}^{+}\beta_{32}^{+} - \alpha_{23}^{-}\beta_{32}^{-})\beta_{21}(\beta_{10}^{+}\alpha_{01}^{+} - \beta_{10}^{-}\alpha_{01}^{-}) = 0 \\
e_1x_2^5 = 0: & (\beta_{10}^{+}\alpha_{01}^{+} - \beta_{10}^{-}\alpha_{01}^{-})\alpha_{12}(\alpha_{23}^{+}\beta_{32}^{+} - \alpha_{23}^{-}\beta_{32}^{-}) = 0 \\
e_0^{\pm}(x_1x_2 - x_2x_1) = 0: & \alpha_{01}^{+}\beta_{10}^{-} = 0,\ \alpha_{01}^{-}\beta_{10}^{+} = 0 \\
e_3^{\pm}(x_1x_2 - x_2x_1) = 0: & \beta_{32}^{+}\alpha_{23}^{-} = 0,\ \beta_{32}^{-}\alpha_{23}^{+} = 0 \\
e_1(x_1x_2 - x_2x_1) = 0: & \alpha_{12}\beta_{21} = \beta_{10}^{+}\alpha_{01}^{+} + \beta_{10}^{-}\alpha_{01}^{-} \\
e_2(x_1x_2 - x_2x_1) = 0: & \beta_{21}\alpha_{12} = \alpha_{23}^{+}\beta_{32}^{+} + \alpha_{23}^{-}\beta_{32}^{-}.
\end{array}
$$

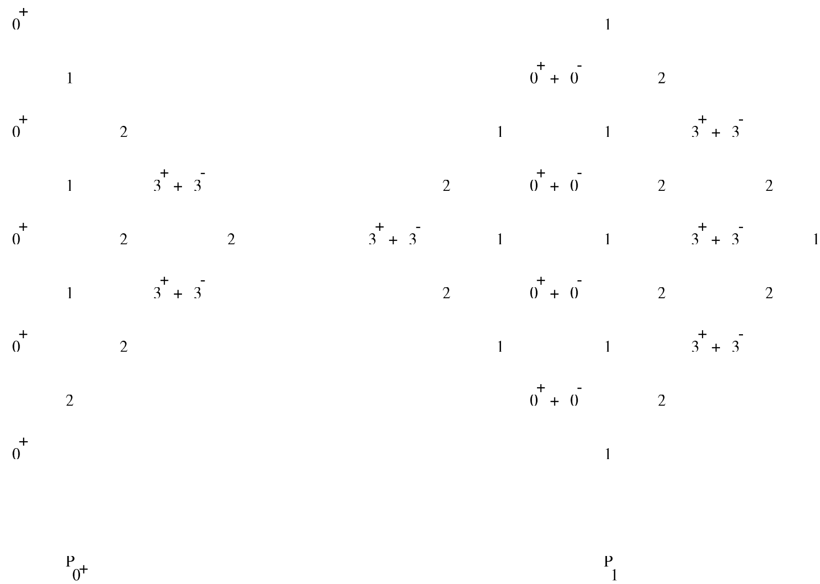

FIGURE 3.

The projective modules needed to build the tilting complexes and the resolution of the Green correspondents in Okuyama's method are then constructed by laying out the x_1^i, x_2^j in a grid. We give only P_{0^+} and P_1, the other projective modules being similar. The Cartan matrix, whose columns give the number of copies of every simple in each projective, is

$$C = \begin{bmatrix} 5 & 0 & 2 & 2 & 4 & 4 \\ 0 & 5 & 2 & 2 & 4 & 4 \\ 2 & 2 & 5 & 0 & 4 & 4 \\ 2 & 2 & 0 & 5 & 4 & 4 \\ 4 & 4 & 4 & 4 & 9 & 8 \\ 4 & 4 & 4 & 4 & 8 & 9 \end{bmatrix}$$

Note that our algorithm makes it quite natural to write the projectives in the rectangular form which was used with such good effect by Holloway [**6**] in his study of $SL(2, q^n)$.

4. Homogeneous maps in tilting complexes. Let us now consider the indecomposable projectives in the case where $p = 2$ and the normalizer of the defect group $Q = C_q \times C_q$ is of the form $Q \rtimes G$, where G has a normal abelian subgroup H of index 2. (A sample case with $p = 3$ was given in [**20**].) An examination of the Reynolds groups for blocks in our database [**19**] given in the tables in Section 3 shows that either G is abelian or this condition is fulfilled with $p = 2$ for all but three blocks of defect 4 with $q = 3$, two blocks with $q = 5$ and two blocks for $q = 7$. If $\operatorname{Rad}(KQ) = J$, the action of G on J/J^2 is given by $\chi = \phi_1 + \phi_2$. We assume that a is an element of order 2^c of G not lying in H, which permutes ϕ_1 and ϕ_2. We let x_1 and x_2 be eigenvectors for the actions of ϕ_1 and ϕ_2 on J/J^2. Set $\zeta = \phi_1(a^2) = \phi_2(a^2)$, where ζ is a primitive $2^{c'}$ root of unity for some integer $0 \le c' \le c$. We may choose the x_i so that $ax_1 = \zeta x_2 a$, $x_1 a = a x_2$.

Whereas in calculating the relations in S3 we found it convenient to conjugate a to a splitting idempotent and convert it into a coefficient, in giving a homogeneous basis of the indecomposable projectives it will be more efficient to conjugate the element a to a collapsing idempotent.

Let $A = K[Q \rtimes H]$, $\widetilde{A} = K[Q \rtimes G]$. If, as before, we let $e_1, \dots, e_r$ be the block idempotents of H, which are primitive because H is abelian, then the indecomposable projective $e_i A$ has a homogeneous basis $\{e_i x_1^j x_2^k\}_{j,k=0}^{p-1}$.

Lemma 4.1. *If $m_{jk} = x_1^j x_2^k$, then $a m_{jk} a^{-1} = \zeta^j m_{kj}$.*

Proof.

$$a x_1^j x_2^k a^{-1} = (a x_1 a^{-1})^j (a x_2 a^{-1})^k = (\zeta x_2)^j x_1^k = \zeta^j x_1^k x_2^j. \qquad \square$$

Definition. Let $\rho_{jk} = \tau_1^j \tau_2^k$, so that $e_i m_{jk} = m_{jk} e_{\rho_{jk}(i)}$.

Lemma 4.2. *Let $e_i^{\pm}$ be an idempotent corresponding to a splitting character θ_i. Then a homogeneous basis of the indecomposable projective $e_i^{\pm}\widetilde{A}$ is a union of the following sets:*

$$B_i^C = \left\{ \frac{1}{2} e_i^{\pm} \left(m_{jk} \pm \zeta^j m_{kj} a \right) \mid e_{\rho_{jk}(i)} \in \overline{C}_H \right\}$$

$$B_i^S = \left\{ e_i^{\pm} m_{jk} e_\ell^{\pm} \mid \rho_{jk}(i) = \ell, e_\ell \in S_H \right\} \cup \{ e_i^{\pm} m_{jj} \}.$$

Each element in these bases lies in a Peirce component of $\widetilde{A}$ with respect to the primitive idempotents.

Proof. For any idempotent e_i which splits into e_i^+ and e_i^-, we have

$$e_i^+ = \frac{1}{2}(e+a)e_i = \frac{1}{2}e_i(e+a)$$

and

$$e_i^- = \frac{1}{2}(e-a)e_i = \frac{1}{2}e_i(e-a).$$

Thus,

$$e_i^{\pm} m_{jk} = \frac{1}{2}e_i(e \pm a)m_{jk} = \frac{1}{2}e_i(m_{jk} \pm am_{jk}) = \frac{1}{2}e_i\left(m_{jk} \pm \zeta^j m_{kj}a\right).$$

Case 1. $\ell = \rho_{jk}(i)$ is the index of a collapsing idempotent. Let ℓ' be the index of the conjugate idempotent so that $e_{\ell'} = ae_\ell a^{-1}$. Then

$$ae_i m_{jk} e_\ell a^{-1} = \zeta^j e_i m_{kj} \cdot (ae_\ell a^{-1}) = \zeta^j e_i m_{kj} e_{\ell'},$$

so we see that $\rho_{kj}(i) = \ell'$. Thus,

$$\frac{1}{2}e_i^{\pm}(m_{jk} \pm \zeta^j m_{kj}a) = \frac{1}{2}(m_{jk} \pm \zeta^j m_{kj}a)e_\ell.$$

Case 2. $\ell = \rho_{jk}(i)$ is the index of a splitting idempotent. If $j = k$, with $\ell = \rho_{jk}(i)$, then

$$0 \neq ae_i m_{jj} e_\ell a^{-1} = \zeta^j e_i m_{jj}(ae_\ell a^{-1}),$$

so we conclude that $e_\ell \in \overline{S}_H$, since otherwise we would have

$$e_i m_{jj} e_\ell (ae_\ell a^{-1}) = 0.$$

Finally, if $j \neq k$ and $\ell = \rho_{jk}(i)$, then

$$e_i^{\pm} m_{jk} e_\ell^{\pm} = \frac{1}{4}e_i^{\pm} m_{jk}(e \pm a)e_\ell = \frac{1}{4}e_i^{\pm}(m_{jk} \pm \zeta^j am_{kj})e_\ell.$$

The two different $\pm$ signs are independent, so these represent four different maps.

Lemma 4.3. *Let e_i be an idempotent corresponding to a collapsing character θ_i with $i = \hat{\imath}$. A homogeneous basis of the projective indecomposable $e_i\widetilde{A}$ is a union of the following sets:*

$$B_i^S = \left\{ \frac{1}{2}(e_i m_{jk} \pm m_{kj} a) e_\ell^\pm \mid \rho_{jk}(i) = \ell,\ e_\ell \in \overline{S}_H \right\}$$

and

$$B_i^C = \left\{ e_i m_{jk} \mid \rho_{jk}(i) = \ell = \hat{\ell}, e_\ell \in C_H \right\} \cup \{ e_i m_{jk} a \mid \rho_{jk}(i) = \ell \neq \hat{\ell},\ e_\ell \in C_H \}.$$

Proof. The proof for B_i^S is similar to the previous lemma. As for B_i^C, if $\rho_{jk}(i) = \ell = \hat{\ell}$, the chosen representative of its conjugacy class, then $e_i m_{jk} = e_{\hat{\imath}} m_{jk}$ ends with the idempotent e_ℓ, whereas if $\rho_{jk}(i) = \ell \neq \hat{\ell}$, then we must multiply by a matrix unit, which is equivalent to a times a constant since $\zeta = \phi_1(a^2)$ is a constant. □

Now suppose that we have a tilting complex in which all the differentials are homogeneous and the irreducible components have length 2, as in the cyclic defect case or the elementary tilting complexes used by Okuyama [**12**]. Then we can choose representatives of the endomorphisms which are homogeneous in each degree. Furthermore, the set of homotopies also has a basis homogeneous in each degree.

5. The Broué conjecture for the faithful 3-block of the covering group of $PSL(3,4)$. We now give, as an application, a verification of the Broué conjecture for a block with defect group $C_3 \times C_3$. To the best of our knowledge, guided by the list of proven cases maintained at Bristol, this particular block has not been treated as of the time of this writing. Let L be the central extension with kernel C_2 of $PSL(3,4)$. Then Q is a 3-Sylow subgroup, isomorphic to $C_3 \times C_3$. The centralizer of Q is $Q \times C_2$. The centralizer has two blocks, each stable under the action of the normalizer. These correspond to the two blocks of the normalizer. In this case, since the defect group is abelian and thus contained in the centralizer, we consider $N_L(Q)/C_L(Q)$, and calculation shows that it is isomorphic to Q_8, the quaternion group of order 8. Both blocks of the normalizer are isomorphic as algebras, and thus both are isomorphic to the group algebra of the unique semi-direct product $(C_3 \times C_3) \rtimes Q_8$ in which the center of Q_8 acts nontrivially.

We are considering the nonprincipal block of the normalizer. This is the Brauer correspondent of the faithful 3-block B of L, i.e., the block consisting of ordinary characters which are nontrivial on the center $Z(L)$ of order 2. A calculation of the decomposition matrix of this block at L shows that it is identical, up to permutation of rows and columns, with the decomposition matrix of the principal block of the Mathieu group M_{22}. The centralizers of the cyclic subgroups of the defect group Q are identical to the centralizer of Q itself, so the restriction map induces a stable equivalence. Calculating the Green correspondent of the simples of the block B with the *C-MeatAxe* developed by Ringe produced modules for the Brauer correspondent b of dimensions 1, 1, 4, 4, 6, with the same structure as the Green correspondents of M_{22}. Thus, the same sequence of two elementary tilting complexes used by Okuyama [**12**] to settle the Broué conjecture for M_{22} will work for our block B.

We want now to give the tilting complex explicitly, as a complex of b modules with homogeneous differentials. In the notation of Section 3, we have $G = C_4 \rtimes C_4 = C_2.Q_8$, $H = C_4 \times C_2$, and the character χ is an irreducible character of G whose restriction to H is the sum of two linear characters ϕ_1 and ϕ_2 with common kernel $N = Z(G) \xrightarrow{\sim} C_2$. The McKay graph $D(H, \phi_1)$ is the disjoint union of two cycles of length four. The element a has order 8, and it acts on each of the two cycles by leaving two antipodal points fixed and exchanging the other two points. The element a^2 is in H, and we have

$$\phi_1(a^2) = \phi_2(a^2) = -1.$$

Suppose 1 and 3 are the indices of the idempotents of the nonprincipal block of H left fixed by a, and let 2 and 4 be the indices of the idempotents interchanged by a. Choose 2 to be the representative of its orbit under the action of a, so that $\hat{2} = \hat{4} = 2$. Then the idempotents of G will be $e_1^{\pm}$, $e_3^{\pm}$ and e_2. These primitive idempotents then determine four indecomposable projective modules of dimension 9, which we will denote by $P_1^{\pm}$ and $P_3^{\pm}$, and a single indecomposable projective module of dimension 18, which we will designate by P_2. The two Green correspondents G_1^+ and G_1^- of dimension 1 are a pair of split simples, which we will identify with $e_1^{\pm}$. The two Green correspondents of dimension 4, G_3^+ and G_3^-, have as tops a matched pair of split simples, which we will identify with $e_3^{\pm}$.

Using the rectangular representation of the projectives as in Section 3, we see that in fact G_3^+, which has composition factors 3^+, 2, 3^-, is the quotient of P_3^+ by a submodule whose projective cover is $P_1^+ \oplus P_1^-$, and similarly G_3^- is a quotient of P_3^-. The Green correspondent G_2, with composition factors 2, 1^+, 1^-, 2, is the quotient of P_2 by a submodule with projective cover $P_3^+ \oplus P_3^-$. We can represent these as

$$G_3^+ = \begin{array}{cc} 3^+ & \\ & 2 \\ 3^- & \end{array} \qquad G_3^- = \begin{array}{cc} 3^- & \\ & 2 \\ 3^+ & \end{array} \qquad G_2 = \begin{array}{ccc} 2 & & \\ & 1^+ \oplus 1^- & \\ & & 2. \end{array}$$

We now apply Okuyama's method of elementary tilting complexes from [**12**]. For those who have difficulty obtaining this extremely important paper, the method is outlined, with an extension, in [**18**]. We choose the index set $I_0 = \{3^+, 3^-\}$. Denote the right regular representation of b by M. Letting g be the endomorphism corresponding to left multiplication by x_1, letting h be the endomorphism of M corresponding to left multiplication by x_2, and letting α be the automorphism of modules given by left multiplication by a, we get a tilting complex $T_1 = (P_1^+)^{(1)} \oplus (P_1^-)^{(1)} \oplus P_2^{(1)} \oplus (P_3^+)^{(1)} \oplus P_3^{(1)}$.

$$\begin{array}{lccc}
(P_1^+)^{(1)}: & P_3^+ \oplus P_3^- & \xrightarrow{\begin{bmatrix} g^2+h^2 \\ g^2-h^2 \end{bmatrix}} & P_1^+ \\
(P_1^-)^{(1)}: & P_3^+ \oplus P_3^- & \xrightarrow{\begin{bmatrix} g^2-h^2 \\ g^2+h^2 \end{bmatrix}} & P_1^- \\
(P_2)^{(1)}: & P_3^+ \oplus P_3^- & \xrightarrow{\begin{bmatrix} h-\alpha g \\ h+\alpha g \end{bmatrix}} & P_2 \\
(P_3^+)^{(1)}: & & P_3^+ & \\
(P_3^-)^{(1)}: & & P_3^- &
\end{array}$$

The indecomposable projectives with indices in I_0 occur in degree (-1) and the indecomposable projectives with indices in $\{1^+, 1^-, 2\}$ occur in degree 0.

Each elementary tilting complex determines a linear combination of columns of the decomposition matrix D_b of b, with those in odd degrees multiplied by (-1). Multiplying negative rows by (-1) then produces the decomposition matrix of $B_1 = \mathrm{End}_{D^b(b)}(T)$, from which the Cartan matrix can be calculated.

$$D_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 & 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = D_{B_1}.$$

By [**12**, Lemma 1.3 (i)], the images $G_1^{+(1)}$ and $G_1^{-(1)}$ of G_1^+ and G_1^- remain simple in B_1. Furthermore, by [**12**, Lemma 1.3 (ii)], $G_3^{+(1)}$ and $G_3^{-(1)}$ are now simple. This leaves only to calculate the Green correspondent $G_2^{(1)}$, using the resolution method in Section 4 of [**15**]. A projective resolution over b of G_2 begins with the map in $P_2^{(1)}$, and thus a projective cover of this resolution in B_1 has $P_2^{(1)}$ as its top. The projective cover of the Heller translate over B_1 has $(P_3^+)^{(1)} \oplus (P_3^-)^{(1)}$ as its top. Calculating the Cartan matrix from D_{B_1} and noting that there are unique maps in each direction between $(P_3^{\pm})^{(1)}$ and P_2, we discover that $G_2^{(1)}$ has, as before, composition factors $2^{(1)}$, $1^{+(1)}$, $1^{-(1)}$, $2^{-(1)}$. The Heller translate of $G_2^{(1)}$ has socle $2^{(1)}$ and all the other simples as composition factors (with multiplicity 1), so it satisfies the conditions of Lemma 1.3 (ii) [**12**]. Therefore, a second elementary tilting complex with index set $I_1 = \{2\}$ will complete the solution. Taking mapping cones, we can represent the new tilting complex as a complex of projectives from b, with homogeneous maps:

$$P_2 \stackrel{h+g\alpha}{\longrightarrow} P_1^+$$
$$P_2 \stackrel{h-g\alpha}{\longrightarrow} P_1^-$$
$$P_3^+ \oplus P_3^- \stackrel{\begin{bmatrix} h-\alpha g \\ h+\alpha g \end{bmatrix}}{\longrightarrow} P_2$$
$$P_3^- \stackrel{h+\alpha g}{\longrightarrow} P_2$$
$$P_3^+ \stackrel{h-\alpha g}{\longrightarrow} P_2.$$

Conclusion. We have concentrated here on an algorithmic construction of the indecomposable projectives of the Brauer correspondent. We hope that by using the decomposition matrix to find combinatorial tilting complexes, we will get a method for obtaining quivers and relations for the group blocks themselves in the abelian defect case. The method we are pursuing is outlined in [**18**]. However, it requires a solid knowledge of the projective modules of the correspondent to which this paper is intended to contribute. It also involves questions of "folding" tilting complexes, as described in [**16, 21, 22, 23, 24**]. We hope that, as in the cyclic case [**17**], the Green correspondent will aid in choosing the proper "folding."

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