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CONNECTEDNESS IN FUZZY TOPOLOGICAL SPACES

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1. **Two independent "good extensions" of connectedness for fuzzy topological spaces.** X will be an arbitrary set and $I = [0, 1]$ the unit interval. By $[X, \delta]$ we denote a fuzzy topological space (fts for short) in the terminology of [**1**], i.e., δ is a family of fuzzy subsets on X with the following properties:

(i) for all α constant, $\alpha \in \delta$;
(ii) if $\mu, \nu \in \delta$, then $\mu \wedge \nu \in \delta$; and
(iii) if $\forall j \in J$, $\mu_j \in \delta$, then $\bigvee_{j \in J} \mu_j \in \delta$.

Several reasons why we have adopted this different notion of fts can be found in [**1**]. Let us however restate the most important one. It is clear that topological spaces provide the most natural framework in which one can define continuity. Now maps which should always be continuous are constant maps. Luckily in topology this is indeed the case. However in fuzzy topology this is the case if and only if one adopts this alternative definition of fts.

Let us now define the extensions of connectedness. If μ is a fuzzy set on X which is everywhere strictly positive, i.e., $\mu \in I^X$ and for all $x \in X$, $\mu(x) > 0$, then we shall write that $\mu \gg 0$ (on X). Suppose now first that $\mu \gg 0$. We shall say that the pair (ξ_1, ξ_2) of open fuzzy sets is a $(c1)$-separation of μ or that they $(c1)$-separate μ if and only if

(i) $\xi_1 \neq \mu, \xi_2 \neq \mu$,
(ii) $\xi_1 \vee \xi_2 = \mu$, and
(iii) $\xi_1 \wedge \xi_2 = 0$.

Second suppose that for some $\varepsilon > 0$ we have $\mu \geqq \varepsilon$. We shall say that the pair (ξ_1, ξ_2) of open fuzzy sets is a $(c2)$-separation of μ or that they $(c2)$-separate μ if and only if there exists some $\varepsilon' \in [0, \varepsilon]$ such that

(i) $\xi_1 \neq \mu, \xi_2 \neq \mu$,
(ii) $\xi_1 \vee \xi_2 = \mu$, and
(iii) $\xi_1 \wedge \xi_2 \leqq \mu - \varepsilon'$.

DEFINITION 1.1. An fts will be called $(c1)$ if and only if no clopen fuzzy set $\mu \gg 0$ can be $(c1)$-separated. An fts will be called $(c2)$ if and only if no clopen fuzzy set $\mu \geqq \varepsilon > 0$ can be $(c2)$-separated.

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Let us now first make precise the meaning of "good extension" (see also [3]). If $(X, \mathscr{T})$ is a topological space, then the family of lower semicontinuous functions from X to the unit inrerval equipped with the usual topology is a fuzzy topology on X which carries exactly the same information as $\mathscr{T}$ itself. We denote this fuzzy topology $\omega(\mathscr{T})$. Actually $(X, \mathscr{T})$ and $(X, \omega(\mathscr{T}))$ are the same structured spaces but in the first the structure is given by the open sets and in the second by the lower semicontinuous functions. The following diagram makes things precise

$$\begin{tikzcd} \mathrm{Top} \arrow[r, "\tilde{\omega}"] \arrow[dr, "\varphi\circ\tilde{\omega}"'] & \tilde{\omega}(\mathrm{Top}) \arrow[d, "\varphi"] \\ & \mathrm{Fuz} \end{tikzcd}$$

where Top and Fuz are respectively the category of topological and of fuzzy topological spaces, φ is the inclusion functor and $\tilde{\omega}$ the isomorphism $\tilde{\omega}(X, \mathscr{T}) = (X, \omega(\mathscr{T}))$ and if f is a morphism $\tilde{\omega}(f) = f$. Thus Top is naturally equivalent with a full subcategory of Fuz. Now we shall say that a property P' for the objects of Fuz is a good extension of a property P for the objects of Top if and only if for all objects X in Top "X has P if and only if $\varphi \circ \tilde{\omega}(X)$ has P'". In our case, very simply, this means that for $(c1)$ and $(c2)$ to be good extensions a topological space $(X, \mathscr{T})$ should be connected if and only if $(X, \omega(\mathscr{T}))$ is $(c1)$ if and only if it is $(c2)$.

Before showing that this is indeed the case, we would like to point out that there exists a natural strengthening of $(c1)$ and $(c2)$ which however turns out not to be a "good extension". Suppose we were to ask that for all μ clopen, $\mu \gg 0$, there exist no $\xi_1, \xi_2 \in \delta$ such that $\xi_1 \neq \mu$, $\xi_2 \neq \mu$, $\xi_1 \vee \xi_2 = \mu$ and $\xi_1 \wedge \xi_2(x) < \mu(x)$ for all $x \in X$. Then it is easily seen that this condition implies both $(c1)$ and $(c2)$ but, as follows from the following counterexample, it is no longer a good extension.

COUNTEREXAMPLE. Let $X = \{(0, y): -1 \leqq y \leqq 1\} \cup \{(x, \sin 1/x): 0 < x \leqq 1\}$ equipped with the usual topology of $\mathbf{R}^2$. Let $\mu = 1$ and define ξ_1 and ξ_2 as

$$\xi_1: X \to I: (x, y) \to 1 - x,$$

$$\xi_2: X \to I: (x, y) \to \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \neq 0. \end{cases}$$

Then, although X is connected, we have that ξ_1 and ξ_2 are lower semicontinuous, $\xi_1 \neq \mu$, $\xi_2 \neq \mu$, $\xi_1 \vee \xi_2 = \mu$ and for all $(x, y) \in X$ $\xi_1 \wedge \xi_2(x, y) < 1$.

PROPOSITION 1.1. *$(c1)$ and $(c2)$ are good extensions of connectedness.*

PROOF. Let $(X, \mathscr{T})$ be a topological space. First we deal with $(c1)$.

Suppose $(X, \mathscr{T})$ is not connected. Then, if (Y_1, Y_2) is an open separation of X, $(1_{Y_1}, 1_{Y_2})$ is a $(c1]$-separation of 1 so that $(X, \omega(\mathscr{T}))$ is not $(c1)$. Conversely, if μ is clopen, $\mu \gg 0$, and (ξ_1, ξ_2) is a $(c1)$-separation of μ, then $Y_1 = \xi_1^{-1}]0, 1]$ and $Y_2 = \xi_2^{-1}]0, 1]$ are an open separation of X.

Now consider $(c2)$. Again if (Y_1, Y_2) is an open separation of X, then $(1_{Y_1}, 1_{Y_2})$ is a $(c2)$-separation of 1. Conversely, suppose μ is clopen, $\mu \geqq \varepsilon > 0$ and let $\xi_1, \xi_2 \in \omega(\mathscr{T})$ be such that $\xi_1 \neq \mu$, $\xi_2 \neq \mu$, $\xi_1 \vee \xi_2 = \mu$ and for some $\varepsilon' \in (0, \varepsilon]$, $\xi_1 \wedge \xi_2 \leqq \mu - \varepsilon'$. Put $\mu' = \varepsilon$, $\xi_1' = \varepsilon \cdot \xi_1/\mu$, and $\xi_2' = \varepsilon \cdot \xi_2/\mu$. Then, since $\mu \geqq \varepsilon$, these are well defined fuzzy sets, and since in the present case open means lower semicontinuous and clopen means continuous, we have that μ' is clopen and $\xi_1', \xi_2' \in \omega(\mathscr{T})$. Further clearly $\xi_1' \neq \mu'$, $\xi_2' \neq \mu'$, $\xi_1' \vee \xi_2' = \mu'$ and $\xi_1' \wedge \xi_2' \leqq \varepsilon(1 - \varepsilon')$, so that if we put $\varepsilon'' = (\varepsilon/2)(2 - \varepsilon')$, we have that $Y_1 = \xi_1'^{-1}]\varepsilon'', 1]$ and $Y_2 = \xi_2'^{-1}]\varepsilon'', 1]$ are an open separation of X.

REMARK. Translated into topological terms $(c2)$, for example, says that if X is connected, no continuous function to the unit interval can be written as the nontrivial supremum of two lower semicontinuous functions whose infimum is uniformly bounded away from that continuous function.

There is no implication between $(c1)$ and $(c2)$ as the following two counterexamples show.

COUNTEREXAMPLES. (A). $(c1) \nRightarrow (c2)$. This is shown putting $X = I$ and $\delta = \{\text{constants}\} \cup \{\mu \in I^X\colon\ \mu \geqq 1/2\}$. Since the only clopen sets are constants, it is easily seen that (X, δ) is $(c1)$. However it is not $(c2)$ since if we let Y_1 and Y_2 be two nonempty complementary sets, then $(1 - (1/2)1_{Y_1}, 1 - (1/2)1_{Y_2})$ is a $(c2)$-separation of 1.

(B). $(c2) \nRightarrow (c1)$. To show this we put $X = I\backslash\{0\}$. Then we define

$$\mu\colon\ X \to I\colon\ x \to x/2$$
$$\mu_{\mathbf{Q}} = \mu \wedge 1_{\mathbf{Q} \cap X}$$
$$\mu_{\mathscr{I}} = \mu \wedge 1_{\mathscr{I} \cap X}$$

where $\mathscr{I} = \mathbf{R}\backslash\mathbf{Q}$, and let δ be the fuzzy topology generated by the subbase

$$\sigma = \{\text{constants}\} \cup \{\mu, \mu^c, \mu_{\mathbf{Q}}, \mu_{\mathscr{I}}\},$$

Then μ is clopen, $\mu \gg 0$, and $(\mu_{\mathbf{Q}}, \mu_{\mathscr{I}})$ are a $(c1)$-separation of μ. Thus (X, δ) is not $(c1)$. However it is $(c2)$. Indeed it is easily seen that ν is clopen if and only if either there exist $\alpha, \beta \in [0, 1/2]$ such that $\nu = (\alpha \vee \mu) \wedge \beta$ or there exist $\alpha, \beta \in [1/2, 1]$ such that $\nu = (\alpha \vee \mu^c) \wedge \beta$.

Clearly no clopen set of the second type can be $(c2)$-separated. Therefore let $\alpha_0, \beta_0 \in [\varepsilon, 1/2]$ where $\varepsilon > 0$, $\alpha_0 < \beta_0$ and let $\nu = (\alpha_0 \vee \mu) \wedge \beta_0$ (if $\alpha_0 \geqq \beta_0$, then ν is a constant which is a trivial case). Then let (ξ_1, ξ_2) be a $(c2)$-separation of ν.

It is tedious but trivial to check that the only open fuzzy sets in δ which are smaller than 1/2 are of the following kind, where α, β and γ denote constant fuzzy sets (also smaller than 1/2 of course), $\alpha \vee (\mu_{\mathbf{Q}} \wedge \beta) \vee (\mu_{\mathscr{I}} \wedge \gamma)$. thus

$$\xi_1 = \alpha_1 \vee (\mu_{\mathbf{Q}} \wedge \beta_1) \vee (\mu_{\mathscr{I}} \wedge \gamma_1)$$
$$\xi_2 = \alpha_2 \vee (\mu_{\mathbf{Q}} \wedge \beta_2) \vee (\mu_{\mathscr{I}} \wedge \gamma_2).$$

Now if $\alpha_0 = 0$, then, even if $\xi_1 \wedge \xi_2 = 0$, there cannot be an $\varepsilon' \in (0, \varepsilon]$ such that $\xi_1 \wedge \xi_2 < \nu - \varepsilon'$ since in this case $\inf_{x \in X} \nu(x) = 0$. On the other hand if $\alpha_0 > 0$, then either $\alpha_2 \leqq \alpha_1 = \alpha_0$ or $\alpha_1 \leqq \alpha_2 = \alpha_0$. Let $\alpha_2 \leqq \alpha_1 = \alpha_0$; then we have

$$\nu/_{[0, 2\alpha_0]} = \xi_1/_{[0, 2\alpha_0]} = \alpha_0$$

so that for some $\varepsilon' \in (0, \varepsilon]$ we must have

$$\xi_2/_{[0, 2\alpha_0]} \leqq \alpha_0 - \varepsilon'.$$

On $[0, 2\alpha_2]$ this can be so only if $\alpha_2 \leqq \alpha_0 - \varepsilon'$, and on $[2\alpha_2, 2\alpha_0]$ this can be so only if both $\beta_2 \leqq \alpha_0 - \varepsilon'$ and $\gamma_2 \leqq \alpha_0 - \varepsilon'$ from which it follows that $\xi_2 \leqq \alpha_0 - \varepsilon'$ and consequently $\xi_1 = \mu$ in contradiction with the fact that ξ_1 and ξ_2 $(c2)$-separate ν. This shows that (X, δ) is indeed $(c2)$.

If (X, δ) is $(c1)$ (resp. $(c2)$) and $\delta' \subset \delta$ is a coarser fuzzy topology, then (X, δ') is also $(c1)$ (resp. $(c2)$). This follows at once from the definition. Combining this with proposition 1 we have that if $(X, \iota(\delta))$ (see **[1]**) is connected, then (X, δ) is both $(c1)$ and $(c2)$. (Let us recall that if (X, δ) is a fuzzy topology, then $\iota(\delta)$ is the smallest topology on X such that $\omega(\iota(\delta))$ is finer than δ. It is also the smallest topology on X making all $\mu \in \delta$ lower semicontinuous. The functor $\bar{\iota}$ from Fuz to Top associated with ι is a left inverse of $\bar{\omega}$). The converse of course is not true. The first counterexample gives a space which is (c1) but, since it is not (c2), for which $(X, \iota(\delta))$ is not connected. Analogously the second conuterexample provides us with a space which is (c2) but for which again $(X, \iota(\delta))$ is not connected.

PROPOSITION 1.2. *If f: $(X, \delta) \to (X', \delta')$ is fuzzy continuous and onto, and if (X, δ) is* (c1) *(resp.* (c2)), *then (X', δ') is also* (c1) *(resp.* (c2)).

PROOF. This is straightforward.

2. **An alternative characterization of $\iota(\delta)$.** The property that if a space has a connected dense subspace, then it is connected itself, is particularly useful in showing that connectedness is productive. Before we can show that (c1) and (c2) are indeed preserved when taking products, we have to define fuzzy denseness.

DEFINITION 2.1. A subset Y of X is fuzzy dense in (X, δ) if and only if for all $\mu \in \delta$ $\sup_{x \in X}\mu(x) = \sup_{y \in Y}\mu(y)$.

Related to this definition is the notion of a closure operator. For notational reasons let us agree to put $\sup_{y \in Y}\mu(y) = u_Y(\mu)$ for all $Y \subset X$ and $\mu \in I^X$. Recall that a subspace of (X, δ) is a subset $Y \subset X$ equipped with the fuzzy topology $\delta_Y = \{\mu|_Y : \mu \in \delta\}$. Now consider the following mapping $^-: 2^X \to 2^X : Y \to \bar{Y}$ where $\bar{Y}$ = largest set Z in X for which for all $\mu \in \delta$, $u_Z(\mu) = u_Y(\mu)$, i.e., $(\bar{Y}, \delta_{\bar{Y}})$ is the largest subspace of (X, δ) in which Y is fuzzy dense. This definition makes sense since, if we put $\mathscr{L}_Y = \{Z \subset X: \forall \mu \in \delta\ u_Z(\mu) = u_Y(\mu)\}$, then first $\mathscr{L}_Y \neq \emptyset$, since $Y \in \mathscr{L}_Y$, and second it is easily seen that any union of subsets in $\mathscr{L}_Y$ again is in $\mathscr{L}_Y$. We can now show the following result.

THEOREM 2.1. *The mapping $Y \to \bar{Y}$ is a closure operator on X and the topology associated with it is exactly $\iota(\delta)$.*

PROOF. (A). $^-$ is a closure operator.

(i) $\bar{\phi} = \phi$ since, if $x \in \bar{\phi}$, then $u_{\bar{\phi}}(1) \geqq 1(x) = 1 > u_\phi(1) = 0$.

(ii) If $Y \subset X$, then $Y \subset \bar{Y}$ by definition.

(iii) $\overline{Y_1 \cup Y_2} = \bar{Y}_1 \cup \bar{Y}_2$. Indeed let $A \subset B \subset X$ and let $x \in \bar{A}$. Then putting $B = A \cup (B\backslash A)$ and $\mu \in \delta$, we have

$$\begin{aligned} u_B(\mu) &= u_A(\mu) \vee u_{B\backslash A}(\mu) \\ &= u_{A \cup \{x\}}(\mu) \vee u_{B\backslash A}(\mu) \\ &= u_{B \cup \{x\}}(\mu), \end{aligned}$$

which shows that $x \in \bar{B}$. Thus $\bar{Y}_1 \cup \bar{Y}_2 \subset \overline{Y_1 \cup Y_2}$. Conversely let $x \in \overline{Y_1 \cup Y_2}$. Then $u_{Y_1}(\mu) \vee u_{Y_2}(\mu) \geqq \mu(x)$ so that, for example, $u_{Y_1}(\mu) \geqq \mu(x)$ and consequently $x \in \bar{Y}_1$.

(iv) $\bar{\bar{Y}} = \bar{Y}$. Let $x \in \bar{\bar{Y}}$. Then for all $\mu \in \delta$ $u_Y(\mu) = u_{\bar{Y}}(\mu) = u_{\bar{Y} \cup \{x\}}(\mu)$, but since $\bar{Y}$ is the largest set having this property, we have $\bar{Y} \cup \{x\} = \bar{Y}$ or $x \in \bar{Y}$.

(B). The topology, $\mathscr{T}^-$, associated with $^-$, is $\iota(\delta)$. As usual let I_r denote the unit interval equipped with the topology $\mathscr{T}_r = \{]\varepsilon, 1]: \varepsilon \in I\} \cup \{I\}$. Then a map to I with the usual topology is lower semicontinuous if and only if to I_r it is continuous. Suppose $\mu \in \delta$; we shall show that $\mu: (X, \mathscr{T}^-) \to I_r$ is continuous thus showing that $\mathscr{T}^- \supset \iota(\delta)$. Let $Y \subset X$. Then it suffices to show that $\mu(\bar{Y}) \subset \overline{\mu(Y)}$. Clearly $\overline{\mu(Y)} = [0, u_Y(\mu)]$. Let $y \in \mu(\bar{Y})$. Then there exists an $x \in \bar{Y}$ such that $y = \mu(x)$, and since then $u_Y(\mu) = u_{Y \cup \{x\}}(\mu)$, we have that $\mu(x) \leqq u_Y(\mu)$ showing that $y \in \overline{\mu(Y)}$. Conversely let $Y \in \mathscr{T}^-$ and $x \in Y$. Then, since $x \notin \overline{Y^c}$, there exists some $\mu \in \delta$ such that $u_{Y^c}(\mu) < \mu(x)$. Choose ε such that $u_{Y^c}(\mu) < \varepsilon < \mu(x)$. Then clearly $x \in \mu^{-1}[\varepsilon, 1] \subset Y$, which shows that $Y \in \iota(\delta)$. This proves the theorem.

3. **The product theorem for** $(c1)$ **and** $(c2)$. If $(X_j, \delta_j)_{j\in J}$ is a family of *fts*, then the product fuzzy topology on $\prod_{j\in J} X_j$ is defined as the coarsest fuzzy topology making all projections fuzzy continuous, ([2], [4]), and is denoted $\prod_{j\in J}\delta_j$.

THEOREM 3.1. *If $(X_j, \delta_j)_{j\in J}$ is a family of fuzzy topological spaces, then $(\prod_{j\in J} X_j, \prod_{j\in J}\delta_j)$ is $(c1)$ (resp. $(c2)$) if and only if for all $j \in J$ (X_j, δ_j) is $(c1)$ (resp. $(c2)$).*

The only if part follows at once from the fact that the projections are fuzzy continuous (see proposition 1.2). To show the if-part we need some lemmas.

LEMMA 3.1. *If (X, δ) is a fts and (Y, δ_Y) is a $(c1)$ (resp. $(c2)$) subspace, then for any Z such that $Y \subset Z \subset \bar{Y}$ we have that (Z, δ_Z) is $(c1)$ (resp. $(c2)$).*

PROOF. We shall only prove this for $(c1)$. Suppose on the contrary that there exists some μ clopen in Z, $\mu \gg 0$ and α, $\beta \in \delta_Z$ a$(c1)$-separation of μ.Then obviously $\mu/_Y$ is clopen in Y, $\alpha/_Y \in \delta_Y$ and $\beta/_Y \in \delta_Y$, $\alpha/_Y \vee \beta/_Y = \mu/_Y$ and $\alpha/_Y \wedge \beta/_Y = 0$. Further it follows from the denseness of Y in Z that both $\alpha/_Y \neq 0$ and $\beta/_Y \neq 0$. This shows that $(\alpha/_Y, \beta/_Y)$ is a $(c1)$-separation of $\mu/_Y$ which is in contradiction with the fact that (Y, δ_Y) is $(c1)$.

LEMMA 3.2. *If $(Z_j, \delta_{Z_j})_{j\in J}$ is a family of $(c1)$ (resp. $(c2)$) subspaces of (X, δ) such that $\bigcap_{j\in J} Z_j \neq \emptyset$, then, if $Z = \bigcup_{j\in J} Z_j$, (Z, δ_Z) is $(c1)$ (resp. $(c2)$).*

PROOF. We shall only show this for $(c1)$. Let $x_0 \in \bigcap_{j\in J} Z_j$. Suppose there exists some μ clopen in (Z, δ_Z), $\mu \gg 0$ (on Z), such that (α, β) is a $(c1)$-separation of μ. Since $\mu(x_0) > 0$, we have, for example, $\beta(x_0) > 0$, and since $\alpha \neq 0$, there exists some $j \in J$ such that $\alpha/_{Z_j} \neq 0$. Since $x_0 \in Z_j$, $\beta/_{Z_j} \neq 0$. Clearly then $(\alpha/_{Z_j}, \beta/_{Z_j})$ is a $(c1)$-separation of $\mu/_{Z_j}$ which contradicts the fact that (Z_j, δ_{Z_j}) is $(c1)$.

PROOF OF THEOREM 3.1. We shall again show the if part only for $(c1)$. Let $x^0 = (x_j^0)_{j\in J}$ be some fixed point in $X = \prod_{j\in J} X_j$ and let $Z = \{x\colon x_i = x_i^0\ \forall i \neq j\}$. Let $\delta = \prod_{j\in J}\delta_j$. It is easily seen that (Z, δ_Z) is fuzzy homeomorphic with (X_j, δ_j) and thus (Z, δ_Z) is $(c1)$. Suppose now that if x^0 and x differ by at most $n-1$ coordinates, they belong to some $(c1)$-subspace of (X, δ). Then suppose x^0 and x differ by n coordinates. Choose y such that x^0 and y differ by $n-1$ coordinates and y and x only by one. Then x^0 and x are in some $(c1)$ subspace (S, δ_s) and y and x as well in (T, δ_T), say. Since $y \in S \cap T$, it follows from lemma 3.2 that $(S \cup T, \delta_{S\cup T})$ is $(c1)$. This shows that if we put $Y =$ union of all $(c1)$ subspaces containing x^0, and $D = \{x\colon x^0$ and x differ by at most finite coordinates$\}$, then $Y \supset D$ and Y is $(c1)$ from lemma 3.2.

Further it is easily seen that D is fuzzy dense in X. Indeed let μ be some base element in $\prod_{j\in J}\delta_j$, i.e.,

$$\mu: \prod_{j\in J} X_j \to I$$
$$(x_j)_j \to \mu_{j_1}(x_{j_1}) \wedge \ldots \wedge \mu_{j_n}(x_{j_n}),$$

where for all $i = 1, \ldots, n$, $\mu_{j_i} \in \delta_{j_i}$. Then for any $x = (x_j)_{j\in J} \in X$ put $\tilde{x} = (\tilde{x}_j)_{j\in J}$ where $\tilde{x}_j = x_j^0$ if $j \notin \{j_1, \ldots, j_n\}$ and $\tilde{x}_{j_i} = x_{j_i}$ if $i = 1, \ldots, n$. Then $\tilde{x} \in D$ and $\mu(x) = \mu(\tilde{x})$ so that $u_D(\mu) = u_X(\mu)$, which proves that D is fuzzy dense. Now since $Y \supset D$, we have also $\bar{Y} = X$ and it follows from lemma 3.1 that (X, δ) is $(c1)$.

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