INTRODUCTION TO THIS ISSUE

This special issue of the Journal is devoted to survey articles based on some of the lecture series given at a Seminar on Nonlinear Eigenvalue Problems held in Santa Fe, New Mexico, during a four-week period in the summer of 1971. This seminar was sponsored by the Rocky Mountain Mathematical Consortium, and was funded by the National Science Foundation.

Nonlinear eigenvalue problems have been studied in the past on different levels of abstraction. On a concrete level, such problems often arise when specific phenomena in classical and modern physics, especially continuum mechanics, are put into the form of integral or differential equations. These problems generally involve a real-valued parameter, and are such that the uniqueness and stability properties of the solutions depend on the value of the parameter. Thus, for example, a layer of viscous fluid heated from below gives rise to a system of partial differential equations for the determination of the steady-state velocity and temperature distributions in the fluid (the Bénard problem). At all values of the Rayleigh number (our parameter in this case), there exists a solution with zero velocity. This is the only solution for small values of the parameter, but others (cellular convections of various types) arise when the parameter is increased. Roughly speaking, the appearance of new solutions when the parameter changes is termed a bifurcation phenomenon. The determination of the structure of the various solution branches, in particular (but not restricted to) their structure near the bifurcation points, is a prime goal of the study of nonlinear eigenvalue problems.

But even without immediate physical motivation or application, nonlinear eigenvalue problems for integral and differential equations have been the object of considerable research. It has been found appropriate in many cases to formulate these problems as equations in Banach spaces and to use functional analytic tools in their treatment. This has given impetus to the construction of successful theories of nonlinear eigenvalue problems in these spaces. In this context, such problems take the form of investigating the behavior of solutions $u(\lambda)$ of $F(u, \lambda) = 0$, where F is a function from $B_1 \times R$ into B_2, the B_i being Banach spaces and R the real line. The papers in this issue are directed primarily, though not exclusively, to the highly important case when F is linear in λ. It has been realized, during the last few decades, that certain topological methods may be applied to the study of problems considered here. An account of these methods, together with their further development, is given in the paper of Paul Rabinowitz, and these methods are also employed in the other papers of the issue. The survey of Morse theory in Hilbert spaces by Erich Rothe is presented with an eye to its potential applicability.

There is a constant interplay among the research activities on the different levels of abstraction mentioned above. In fact, concrete problems constantly provide incentive for the development of analogous abstract theories, and a well-recognized criterion of success for an abstract theory is its ability to shed new light on concrete problems. A major objective of the seminar was to further this interplay and to narrow the dangerously wide gap between researchers earnestly concerned with problems from physics, and those versed in abstract tools. The interchange is seen in the papers in this issue. For example, the applied problems discussed by Klaus Kirchgässner and Hansjörg Kielhöfer, and by Melvin Berger are handled, at least in large part, by methods originally developed for more abstract situations. And Duane Sather's survey of results about branching phenomena in Hilbert space concerns a theory whose origins include Lyapunov's investigations into the equilibrium states of a rotating mass.

It is hoped that these papers will provide an account of the present state of affairs in some of the important branches of this rapidly developing subject.

PAUL C. FIFE
University of Arizona