

Regular version of holomorphic Wiener function

By

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1. Introduction

In the previous paper [8], we suggested a definition of the skeletons of holomorphic Wiener functions as follows: Let (B, H, μ, J) be an almost complex abstract Wiener space. For an L^p -holomorphic Wiener function F , we defined the skeleton of F by

$$F(h) := \int_B F(z+h) \mu(dz), \quad h \in H.$$

Also we suggested a definition of the contraction operation,

$$F(\sqrt{t} z) := T_{-\log t} F(z), \quad 0 < t \leq 1,$$

Where $\{T_t\}_{t \geq 0}$ is the Ornstein-Uhlenbeck semigroup. Then we gave several reasons why these notions should be defined as above.

However we have to say that our reasoning was somewhat weak, because we did not describe exactly for what elements of B , holomorphic Wiener functions are well-defined. For example, in the theory of Dirichlet spaces, each function of finite energy has a nice version, so-called the quasi-continuous version, which is uniquely defined up to the sets of capacity 0. By this version, we could establish a calculus beyond “almost everywhere”. In fact, to study the skeletons and the contraction operation, we need a calculus beyond almost everywhere.

“Without capacity, can we carry out a calculus beyond almost everywhere?” This is a question that was raised by K. Itô, when he tried to reconstruct the Malliavin calculus without topology ([5]). Note that without topology, we cannot define a capacity. Eventually, he could solve the question, establishing a new class of exceptional sets (, which he called *strictly null sets*), and accordingly, nice versions of Malliavin’s smooth functions (, which he called *regular versions*).

In this paper, we exactly develop K. Itô’s idea in our context. Namely, we define a class of exceptional sets, which we shall call *holomorphically exceptional sets* and show that each holomorphic Wiener function has a nice version

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which is unique up to the holomorphically exceptional sets. Holomorphically exceptional sets will turn out to be not only of μ -measure 0 but also of μ_t -measure 0 for $0 \leq t \leq 1$. Here μ_t denotes the induced measure of μ by the mapping $z \mapsto \sqrt{t}z$. Moreover, we can show that each one point set $\{h\}$ for $h \in H$ is not a holomorphically exceptional set. In other words, for the regular version \tilde{F} of each holomorphic Wiener function F , we can directly evaluate the skeleton $\tilde{F}(h)$ as well as the contraction $\tilde{F}(\sqrt{t}z)$. In addition, it holds that

$$\tilde{F}(h) = \int_B F(z+h) \mu(dz), \quad h \in H,$$

$$\tilde{F}(\sqrt{t}z) = T_{-\log t} F(z), \quad \mu\text{-a.e. } z.$$

Finally, we show that the B -valued Brownian motion does not hit any holomorphically exceptional set with probability 1. As a result, we see that the stochastic processes obtained by substituting a B -valued Brownian motion into the regular versions are always continuous conformal martingales. This fact may be interpreted as the “*fine continuity*” of the regular versions with respect to the B -valued Brownian motion.

All these results need essentially no topology of the space B , because we adopted K. Itô's approach.

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2. Holomorphic polynomial

In this section, we introduce our framework, almost complex abstract Wiener space, and basic properties of holomorphic polynomials. See [7] [8] for details.

Let (B, H, μ, J) be an almost complex abstract Wiener space, i.e., B is a real separable Banach space (whose dimension is infinite), H is a real separable Hilbert space continuously and densely imbedded in B , μ is a Gaussian measure satisfying

$$\int_B \exp(\sqrt{-1}\langle l, z \rangle) \mu(dz) = \exp\left(-\frac{1}{4}\|l\|_{H^*}^2\right), \quad l \in B^* \subset H^*.$$

and $J: B \rightarrow B$ is an isometry such that $J^2 = -\text{id}$ and that $J|_H: H \rightarrow H$ is also an isometry (see [7]). Let B^{*c} be the complexification of the dual space B^* . Then defining

$$B^{*(1,0)} := \{\varphi \in B^{*c} \mid J^* \varphi = \sqrt{-1} \varphi\},$$

$$B^{*(0,1)} := \{\varphi \in B^{*c} \mid J^* \varphi = -\sqrt{-1} \varphi\},$$

we see that $B^{*c} = B^{*(1,0)} \oplus B^{*(0,1)}$. The Hilbert spaces H^{*c} , $H^{*(1,0)}$ and $H^{*(0,1)}$ are similarly defined.

A function $G : B \rightarrow C$ is called a *holomorphic polynomial*, if it is expressed in the form

$$G(z) = g(\langle \varphi_1, z \rangle, \dots, \langle \varphi_n, z \rangle), \quad z \in B, \tag{1}$$

where $n \in \mathbb{N}$, $g : C^n \rightarrow C$ is a polynomial with complex coefficients and $\varphi_1, \dots, \varphi_n \in B^{*(1,0)}$. The class of all holomorphic polynomials is denoted by \mathcal{P}_h . The requirement $\varphi_i \in B^{*(1,0)}$ in (1) is not essential. Indeed, for any $\varphi_i \in H^{*(1,0)}$, $i = 1, \dots, n$, we may regard them as elements of $B^{*(1,0)}$ by replacing the Banach space B .

Since each $G \in \mathcal{P}_h$ is everywhere defined and is essentially a holomorphic function on a finite dimensional complex space, the following relations are easy to check ([8]).

$$G(z') = \int_B G(z + z') \mu(dz), \quad \forall z' \in B$$

$$G(\sqrt{t} z) = T_{-\log t} G(z), \quad \forall z \in B$$

In this paper, we consider more general contraction operations than those treated in [8]. We define a class of linear operators on H by

$$\mathcal{L} := \{A : H \rightarrow H \mid \|A\| \leq 1 \text{ and } A^*A \text{ commutes with } J\}.$$

It is known that any bounded linear operator on H can be uniquely extended to a measurable linear operator on B ([2]). Hence if $G \in \mathcal{P}_h$ has an expression (1), then the Wiener function $G(Az)$ for $A \in \mathcal{L}$ has the expression

$$G(Az) = g(\langle A^* \varphi_1, z \rangle, \dots, \langle A^* \varphi_n, z \rangle),$$

which may be no longer a holomorphic polynomial. Note that if A^* commutes with J^* (or equivalently, A commutes with J), we have $A^* \varphi_i \in H^{*(1,0)}$, $i = 1, \dots, n$. Therefore $G(Az)$ is, in this case, essentially a holomorphic polynomial.

Following [2], we define a linear operator $\Gamma(A^*)$ on $L^1(\mu)$ for each $A \in \mathcal{L}$ by¹

$$\Gamma(A^*)G(z) := \int_B G(Az + \sqrt{I_H - A^*A} z') \mu(dz').$$

Since $\sqrt{I_H - A^*A}$ commutes with J , it is easy to see that the integrand of this expression is holomorphic in z' , if $G \in \mathcal{P}_h$. Hence we have

¹In [2], they define $\Gamma(A)$ instead of $\Gamma(A^*)$.

$$G(Az) = \Gamma(A^*)G(z), \quad \mu\text{-a.e. } z,$$

for each $G \in \mathcal{P}_h$.

3. Holomorphically exceptional set and regular version of holomorphic Wiener function

An L^p -holomorphic Wiener function is defined as an $L^p(B \rightarrow C, \mu)$ -limit of a certain sequence of holomorphic polynomials. The class of all L^p -holomorphic Wiener functions is denoted by \mathcal{H}^p . Namely, \mathcal{H}^p is the L^p -closure of \mathcal{P}_h . Throughout the paper, we always assume $1 < p < \infty$.

Our purpose is to establish a way to get good versions of L^p -holomorphic Wiener functions so that we may directly evaluate their skeletons and the contraction. Although being named as holomorphic Wiener functions, they are, in general, neither continuous nor differentiable even in Malliavin's sense. And hence it seems impossible to get their good versions by, for example, Dirichlet space method, that is, capacity.

Now we introduce a class of exceptional sets, which we developed K. Itô's idea ([5]) in our context. Its outward appearance is very similar to Itô's definition, but the complex structure inside it will bring us another world.

Definition 1. For a sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\sum_n \|G_n\|_{L^p(\mu)} < \infty$, we define a subset $N^p(\{G_n\})$ of B by

$$N^p(\{G_n\}) := \{z \in B \mid \sum_n |G_n(z)| = \infty\}.$$

A set $A \subset B$ is called an L^p -holomorphically exceptional sets, if it is a subset of a set of the type $N^p(\{G_n\})$. We denote the class of all L^p -holomorphically exceptional sets by \mathcal{N}_h^p . If an assertion holds outside of an L^p -holomorphically exceptional set, we say that it holds "*a.e.*(\mathcal{N}_h^p)".

Remark 1. Any countable union of L^p -holomorphically exceptional sets is again an L^p -holomorphically exceptional set. Indeed, let $N_k = N^p(\{G_{n,k}\})$, $k \in \mathbb{N}$, with $\sum_n \|G_{n,k}\|_{L^p(\mu)} < \infty$. We may assume $\sum_n \|G_{n,k}\|_{L^p(\mu)} = 2^{-k}$. Let $\{G_n\}$ be a renumbered sequence of $\{G_{n,k}\}$. Then we see $\sum_n \|G_n\|_{L^p(\mu)} < \infty$ and $\bigcup_k N_k \subset N^p(\{G_n\})$.

Theorem 1. (i) For any $h \in H$, the one point set $\{h\}$ is not an L^p -holomorphically exceptional set.
(ii) For any $z \in B \setminus H$, the one point set $\{z\}$ is an L^p -holomorphically exceptional set.

Proof. (i) Take any sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\sum_n \|G_n\|_{L^p(\mu)} < \infty$. What we should prove is $\sum_n |G_n(h)| < \infty$ for each $h \in H$. Since $G_n \in \mathcal{P}_h$, we can write

$$G_n(h) = \int_B G_n(z+h)\mu(dz) = \int_B G_n(z)M(h, z)\mu(dz),$$

where $M(h, z) = \exp(2\langle h, z \rangle - |h|_H^2)$ is the Cameron-Martin density, which has every moment. Consequently, taking $q > 1$ so that $1/p + 1/q = 1$, we have

$$\sum_n |G_n(h)| \leq \sum_n \int_B |G_n(z)M(h, z)|\mu(dz) \leq \sum_n \|G_n\|_{L^p(\mu)} \|M(h, \cdot)\|_{L^q(\mu)} < \infty.$$

(ii) Since J satisfies $J^2 = -\text{id}$ and it is isometric, it is easy to see $\langle h, Jh \rangle_H = 0$ and hence $\|e^{J\theta}h\|_H = \|h\|_H$, for any $h \in H$, where $e^{J\theta}h = (\cos \theta)h + (\sin \theta)Jh$. We therefore have

$$\|h\|_H = \sup\{|\langle h, \varphi \rangle_{H^*c}|; \varphi \in H^{*(1,0)}, \|\varphi\|_{H^*c} = 1\}, \quad h \in H.$$

Nothing that B^* is dense in H^* , for any $z \in B \setminus H$,

$$\sup\{|\langle \varphi, z \rangle_B|; \varphi \in B^{*(1,0)}, \|\varphi\|_{H^*c} = 1\} = \infty.$$

Take a sequence $\{\varphi_n\} \subset B^{*(0,1)}$ so that

$$|\langle \varphi_n, z \rangle_B| \geq n \text{ and } \|\varphi_n\|_{H^*c} = 1,$$

and put

$$G_n(z') = \frac{1}{n^2} \langle \varphi_n, z' \rangle_B, \quad z' \in B.$$

Then we have $\sum_n \|G_n\|_{L^p(\mu)} < \infty$ and $\{z\} \subset N^p(\{G_n\})$.

For each $A \in \mathcal{L}$, we denote by μ_A the induced measure of μ by the mapping $A: B \rightarrow B$. In particular, for $t \in [0, 1]$, we denote the Gaussian measure $\mu_{\sqrt{t}I_H}$ simply by μ_t . Namely, μ_t is the induced measure of μ by the mapping $B \ni z \mapsto \sqrt{t}z \in B$. Note that μ_t and μ are mutually singular, if $t < 1$.

Theorem 2. *Let $A \in \mathcal{L}$. Then any L^p -holomorphically exceptional set is of μ_A -measure 0, in particular, it is of μ_t -measure 0 for $0 \leq t \leq 1$.*

Proof. Take any sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\sum_n \|G_n\|_{L^p(\mu)} < \infty$. What we should prove is $\mu_A(N^p(\{G_n\})) = 0$. We estimate the following integration.

$$\int_B \sum_n |G_n(z)|\mu_A(dz) = \sum_n \int_B |G_n(Az)|\mu(dz)$$

Since $\Gamma(A^*)$ is a contraction on $L^p(\mu)$ ([2]), we have

$$\text{R.H.S.} = \sum_n \|\Gamma(A^*)G_n\|_{L^1(\mu)} \leq \sum_n \|\Gamma(A^*)G_n\|_{L^p(\mu)} \leq \sum_n \|G_n\|_{L^p(\mu)} < \infty.$$

Thus we see $\mu_A(N^p(\{G_n\})) = 0$.

Definition 2. $F \in \mathcal{H}^p$ is called p -regular, if there exists a sequence $\{G_n\} \subset \mathcal{P}_h$ such that

$$\|G_n - F\|_{L^p(\mu)} \rightarrow 0 \text{ and } G_n \rightarrow F \text{ a.e.}(\mathcal{N}_h^p), \quad \text{as } n \rightarrow \infty.$$

Theorem 3. (i) If $\{G_n\} \subset \mathcal{P}_h$ satisfies $\sum_n \|G_{n+1} - G_n\|_{L^p(\mu)} < \infty$, we put

$$F(z) := \text{Lim}_n G_n(z) := \begin{cases} \lim_n G_n(z), & z \in N^p(\{G_{n+1} - G_n\}) \\ 0, & z \in N^p(\{G_{n+1} - G_n\}) \end{cases}$$

Then $F \in \mathcal{H}^p$ and it is p -regular.

(ii) If $F_1, F_2 \in \mathcal{H}^p$ are p -regular and $F_1 = F_2$, μ -a.e., then $F_1 = F_2$, a.e. (\mathcal{N}_h^p) .

Proof. (i) Since $G_n \rightarrow F$, a.e. (\mathcal{N}_h^p) , what we should prove is $\|G_n - F\|_{L^p(\mu)} \rightarrow 0$. On the other hand, since $\{G_n\}$ is a Cauchy sequence in $L^p(\mu)$, there exists its limit $F' \in \mathcal{H}^p$. Now, $\|G_n - F'\|_{L^p(\mu)} \rightarrow 0$ implies that some subsequence $\{G'_n\} \subset \{G_n\}$ converges to F' , μ -a.e. Of course $G'_n \rightarrow F'$, a.e. (\mathcal{N}_h^p) , and hence, $G'_n \rightarrow F$, μ -a.e. Then we have $F = F'$, μ -a.e., and consequently, $\|G_n - F\|_{L^p(\mu)} = \|G_n - F'\|_{L^p(\mu)}$ converges to 0.

(ii) We have only to show that if $F = 0$, μ -a.e. and it is p -regular, then $F = 0$, a.e. (\mathcal{N}_h^p) . Since F is p -regular, there exists a sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\|G_n - F\|_{L^p(\mu)} = \|G_n\|_{L^p(\mu)} \rightarrow 0$ and that $G_n \rightarrow F$, a.e. (\mathcal{N}_h^p) . By taking a subsequence if necessary, we may assume $\sum_n \|G_n\|_{L^p(\mu)} < \infty$. Then we see $G_n \rightarrow 0$ outside of $N^p(\{G_n\})$. On the other hand, since $G_n \rightarrow F$, a.e. (\mathcal{N}_h^p) , we have $F = 0$, a.e. (\mathcal{N}_h^p) .

Theorem 4. For any $F \in \mathcal{H}^p$, there exists a p -regular version \tilde{F} of F , that is, $\tilde{F} = F$, μ -a.e. and \tilde{F} is p -regular.

Proof. Take a sequence $\{G_n\} \subset \mathcal{P}_h$ so that $\sum_n \|G_n - F\|_{L^p(\mu)} < \infty$. Then we have $\sum_n \|G_{n+1} - G_n\|_{L^p(\mu)} < \infty$. Define \tilde{F} by $\tilde{F} := \text{Lim}_n G_n$ as in Theorem 3. Now it is clear that \tilde{F} satisfies the required conditions.

Theorem 5. Let $F \in \mathcal{H}^p$ and \tilde{F} be any p -regular version of F . Then we have

(i)

$$\tilde{F}(h) = \int_B F(z+h) \mu(dz), \quad h \in H,$$

(ii) for $A \in \mathcal{L}$,

$$\tilde{F}(Az) = \Gamma(A^*)F(z), \quad \mu\text{-a.e.},$$

in particular,

$$\tilde{F}(\sqrt{t}z) = T_{-\log t}F(z), \quad \mu\text{-a.e.}, \quad 0 < t \leq 1.$$

Proof. Take a sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\sum_n \|G_n - F\|_{L^p(\mu)} < \infty$, and define a p -regular version \tilde{F} as in Theorem 3. We have only to show the assertions for this version \tilde{F} .

(i) Take any $h \in H$. Since $\{h\}$ is not holomorphically exceptional set, we

have $G_n(h) \rightarrow \tilde{F}(h)$. On the other hand,

$$G_n(h) = \int_B G_n(z+h) \mu(dz) \rightarrow \int_B F(z+h) \mu(dz).$$

Thus we see $\tilde{F}(h) = \int_B F(z+h) \mu(dz)$.

(ii) Since $N^p(\{G_n\})$ is of μ_A -measure 0, we have $G_n(Az) \rightarrow \tilde{F}(Az)$, for μ -a.e. z . On the other hand,

$$G_n(Az) = \Gamma(A^*)G_n(z) \rightarrow \Gamma(A^*)F(z), \quad \text{in } L^p(\mu).$$

Now it is easy to see $\tilde{F}(Az) = \Gamma(A^*)F(z)$, μ -a.e. z .

4. Regular version and B -valued Brownian motion

Let $(Z_t)_{0 \leq t \leq 1}$ be a B -valued independent increment process defined on a probability space (Ω, \mathcal{F}, P) such that $Z_0 = 0$ and the distribution of $Z_t - Z_s$, $t > s$, is μ_{t-s} . Then the process $(Z_t)_{0 \leq t \leq 1}$ becomes a diffusion process on B and it is called a B -valued Brownian motion (see for example, [3]).

Theorem 6. (i) For any $h \in H$, the process $(Z_t + h)_{0 \leq t \leq 1}$ does not hit any $N \in \mathcal{N}_h^p$ with probability 1. Namely,

$$P(Z_t + h \notin N \text{ for } \forall t \in [0, 1]) = 1.$$

(ii) Let $F \in \mathcal{H}^p$ and \tilde{F} be any p -regular version of F . Then the process $(\tilde{F}(Z_t))_{0 \leq t \leq 1}$ is a continuous L^p -conformal martingale.

(iii) For any $h \in H$ and for any $1 < p' < p$, the process $(\tilde{F}(Z_t + h))_{0 \leq t \leq 1}$ is a continuous $L^{p'}$ -conformal martingale.

Proof. (i) Take any sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\sum_n \|G_n\|_{L^p(\mu)} < \infty$. It is sufficient to show that

$$\mathbf{E} \left[\sup_{0 \leq t \leq 1} \sum_n |G_n(Z_t + h)| \right] < \infty,$$

where \mathbf{E} stands for the expectation with respect to the probability P . Let $1 < p' < p$. Since each $G_n(\cdot + h)$ is essentially defined on a finite dimensional complex space and it is $L^{p'}$ -holomorphic, the process $(G_n(Z_t + h))_{0 \leq t \leq 1}$ is a continuous $L^{p'}$ -conformal martingale (see for example, [4], Chapter IV-6). It therefore follows from Doob's inequality that $c_{p'} > 0$ being some constant,

$$\begin{aligned} \mathbf{E} \left[\sup_{0 \leq t \leq 1} |G_n(Z_t + h) - G_n(h)| \right] &\leq \mathbf{E} \left[\sup_{0 \leq t \leq 1} |G_n(Z_t + h) - G_n(h)|^p \right]^{1/p'} \\ &\leq c_{p'} \mathbf{E} \left[|G_n(Z_1 + h) - G_n(h)|^p \right]^{1/p'} \\ &= c_{p'} \|G_n(\cdot + h) - G_n(h)\|_{L^{p'}(\mu)} \\ &\leq c_{p'} (\|G_n(\cdot + h)\|_{L^{p'}(\mu)} + |G_n(h)|). \end{aligned}$$

Note that $\sum_n |G_n(h)| < \infty$ and

$$\|G_n(\cdot + h)\|_{L^{p'}(\mu)} \leq \|G_n\|_{L^p(\mu)} \|M(h, \cdot)\|_{L^{p'}(\mu)}^{1/p'},$$

where $p'/p + 1/q = 1$ and $M(h, z)$ is the Cameron-Martin density. Then we have

$$\mathbf{E} \left[\sum_n \sup_{0 \leq t \leq 1} |G_n(Z_t + h) - G_n(h)| \right] \leq c_{p'} \sum_n (\|G_n\|_{L^p(\mu)} \|M(h, \cdot)\|_{L^{p'}(\mu)}^{1/p'} + |G_n(h)|) < \infty.$$

Again by $\sum_n |G_n(h)| < \infty$, we have $\mathbf{E} [\sum_n \sup_{0 \leq t \leq 1} |G_n(Z_t + h)|] < \infty$, which implies the required inequality.

(ii) Take a sequence $\{G_n\} \subset \mathcal{P}_h$ such that $\sum_n \|G_n - F\|_{L^p(\mu)} < \infty$. On account of (i), it is sufficient to prove the assertion for a particular p -regular version of F . Hence we may assume \tilde{F} to be the p -regular version defined in Theorem 3 using $\{G_n\}$.

As mentioned above, the process $(G_n(Z_t))_{0 \leq t \leq 1}$ is a continuous L^p -conformal martingale. Again by Doob's inequality, we have

$$\mathbf{E} \left[\sum_{n=1}^{\infty} \sup_{0 \leq t \leq 1} |G_{n+1}(Z_t) - G_n(Z_t)| \right] \leq c_p \sum_{n=1}^{\infty} \|G_{n+1} - G_n\|_{L^p(\mu)} < \infty.$$

This implies that the sequence $\{(G_n(Z_t))_{0 \leq t \leq 1}\}_n$ converges to a continuous L^p -conformal martingale, say $(Y_t)_{0 \leq t \leq 1}$, on some $\Omega_0 \subset \Omega$ with $P(\Omega_0) = 1$. Let $\Omega_1 \subset \Omega$ be

$$\Omega_1 := \{\omega \in \Omega_0 \mid Z_t(\omega) \in N^p(\{G_{n+1} - G_n\}) \text{ for } \forall t \in [0, 1]\}.$$

Then we see $P(\Omega_1) = 1$ and that $Y_t(\omega) = \tilde{F}(Z_t(\omega))$ for each $t \in [0, 1]$ and each $\omega \in \Omega_1$, which completes the proof of (ii). The assertion (iii) easily follows from (ii).

Remark 2. In the above theorem, the process $(Z_t)_{0 \leq t \leq 1}$ need not be a Brownian motion, if it satisfies the following condition: For each $G \in \mathcal{P}_h$, $G(Z_t)$ is a conformal martingale and $\mathbf{E}[|G(Z_1)|^p]^{1/p} \leq c_p \|G\|_{L^p(\mu)}$ for some constant $c_p > 0$.

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