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On the law of entropy increasing of a one-dimensional infinite system II

By

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1. Introduction

In the paper [1] we considered a one dimensional hard-points infinite system on $\boldsymbol{Z}$ whose particles have several colors and velocities with unit magnitude. We showed there that Boltzmann type entropies increase for the initial states which do not have any spatial correlation. However we had to assume a condition which is of very technical character. The condition was that they have initially constant density 1 on $\boldsymbol{Z}$.

In this paper we consider the same system and show that the same result holds without the above condition, namely, the total entropy increases for the initial states which have initially no spatial correlation. It will be also shown that in general Boltzmann type entropy of a single color can decrease.

In section 2, for the sake of reader's convenience, we describe the system and the definitions of entropies (see [1] for details). In section 3, we state our main results and prove them. In section 4 we derive master equations from our particle model by taking various scaling limits.

2. Description of the dynamical system and the definition of entropy

Let $\boldsymbol{Z}$ be the set of all integers, and S be a "color" space with different $k(k \geq 2)$ colors:

$$S=\{\phi, c_1, c_2, \cdots, c_k\}$$

Let $X=\{\omega ; \omega: \boldsymbol{Z} \rightarrow S \times S\}$. We write

$$\omega(n)=(\omega(n,-), \omega(n,+)) \in S \times S \ (n \in \boldsymbol{Z}).$$

X can be identified with the product space $X=\prod_{n \in \boldsymbol{Z}} X_n$, where $X_n=S_n^- \times S_n^+$, $S_n^- = S_n^+ = S$. The time evolution mapping $T: X \rightarrow X$ in the phase space X is defined by $T=CT_0$, where

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$$(T_0\omega)(n, \pm)=\omega(n\mp 1, \pm),$$

and

$$((C\omega)(n, -), (C\omega)(n, +))=\begin{cases}((\omega(n, +), \omega(n, -)), \text{ if } \omega(n, -)\neq\phi \text{ and } \\ \qquad\qquad\qquad\qquad\qquad \omega(n, +)\neq\phi \\ ((\omega(n, -), \omega(n, +)), \text{ otherwise}\end{cases}$$

The set of state space $\mathcal{M}$ of the dynamical system (X, T) consists of all probability measures on X. Let

$$\bar{\mathcal{M}}=\{\mu\in\mathcal{M}\,;\ \mu=\bigotimes_{n\in \boldsymbol{Z}}(\mu_n^-\times\mu_n^+)\},$$

where $\mu_n^\pm$ is a probability measure on $S_n^\pm$, respectively. The elements of $\bar{\mathcal{M}}$ are called locally equilibrium states on X.

Hereafter we use following notations: for $\mu\in\mathcal{M}$; $c\in S$; $\epsilon=\pm$; $n, m\in \boldsymbol{Z}$

$$P_\epsilon^c(n, m)=P_\epsilon^c(n, m\,;\ \mu)=\mu\{\omega\,;\ (T^m\omega)(n, \epsilon)=c\}$$

$$d_\epsilon(n, m)=d_\epsilon(n, m\,;\ \mu)=\sum_{c\neq\phi}P_\epsilon^c(n, m\,;\ \mu)=1-P_\epsilon^\phi(n, m\,;\ \mu)$$

(Briefly

$$P_\epsilon^c(n, 0)=P_\epsilon^c(n)=P_\epsilon^c(n\,;\ \mu)$$

$$d_\epsilon(n, 0)=d_\epsilon(n)=d_\epsilon(n\,;\ \mu).)$$

The Boltzmann type entropy $H(\mu)$ of μ is defined by

$$H(\mu)=\overline{\lim_{N\to\infty}}\frac{-1}{2N+1}\sum_{\substack{|n|\leq N\\ \epsilon=\pm}}\sum_{c\in S}P_\epsilon^c(n\,;\ \mu)\log P_\epsilon^c(n\,;\ \mu).$$

We also define $H_c(\mu)$ of μ by

$$H_c(\mu)=\overline{\lim_{N\to\infty}}\frac{-1}{2N+1}\sum_{\substack{|n|\leq N\\ \epsilon=\pm}}P_\epsilon^c(n\,;\ \mu)\log P_\epsilon^c(n\,;\ \mu).$$

Remark. In the paper [1] we defined $H(\mu)$, $H_c(\mu)$ and $h_c(\mu)$ in slightly different manner in which we had to assume the existence of the limit.

3. Main results

Theorem ("Entropy increasing law"). *Assume that $\mu\in\bar{\mathcal{M}}$. Then we have*

$$H(T^{m+1}\mu)\geq H(T^m\mu)\qquad \textit{for } m\geq 0.$$

For the proof of the theorem we need some concepts which were introduced in [1]. Namely,

let π be a projection from $\mathcal{M}$ to $\overline{\mathcal{M}}$, $\pi: \mathcal{M}\to\overline{\mathcal{M}}$ defined by

$$P_\epsilon^c(n\,;\,\pi\mu)=P_\epsilon^c(n\,;\,\mu) \qquad \text{for } \forall c\in S,\ \epsilon=\pm,\ n\in\boldsymbol{Z}.$$

We also defined the K-S type entropy $h(\mu)$ of $\mu\in\mathcal{M}$ by

$$h(\mu)=\overline{\lim_{N\to\infty}}\frac{-1}{2N+1}\sum_{A\in\mathcal{U}_N}\mu(A)\log\mu(A),$$

where $\mathcal{U}_N$ is the partition of X on $[-N, N]$, i.e., $\mathcal{U}_N$ is the partition into the sets A of a following form,

$$A=\{\omega\,;\,\omega(n,\epsilon)=c(n,\epsilon\,;\,A) \qquad \text{for } \epsilon=\pm \text{ and } -N\leq n\leq N\},$$

where $c(n,\epsilon\,;\,A)\in S$.

Now in general let P_k be a probability measure on the finite set Ω_k, $k=1,\cdots,K$. Let $\mathcal{M}(P_1,\cdots,P_K)$ be the set of such probability measures $\overline{P}$ on the product set $\Omega_1\times\cdots\times\Omega_K$ that $\overline{P}\pi_k^{-1}=P_k$, where π_k is the natural projection from the product set $\Omega_1\times\cdots\times\Omega_K$ onto $\Omega_k(k=1,\cdots,K)$.

In general we define as usually the entropy $\epsilon(P)$ of the probability measure P on a finite set $\Omega=\{\omega_1,\cdots,\omega_I\}$ by

$$\epsilon(P)=-\sum_{i=1}^{I}P\{\omega_i\}\log P\{\omega_i\}.$$

Following lemma ("maximal entropy principle") is well known and can be easily proven by using Jensen's inequality.

Lemma. *For* $\forall\,\overline{P}\in\mathcal{M}(P_1,\cdots,P_K)$, *we have*

$$\epsilon(\overline{P})\leq\epsilon(P_1\times\cdots\times P_K).$$

The equality holds iff $\overline{P}=P_1\times\cdots\times P_K$.

From this lemma we obtain easily the following inequality:

$$h(\pi\mu)\geq h(\mu) \qquad \text{for } \mu\in\mathcal{M},$$

equality holds if (and essentially only if) $\mu\in\overline{\mathcal{M}}$.

Now from the definition we have ([1])

$$H(\mu)=h(\pi\mu) \qquad \text{for } \forall\,\mu\in\mathcal{M}.$$

We proved in [1] that $h(\mu)$ is T-invariant, i.e.,

$$h(\mu)=h(T\mu) \qquad \text{for } \forall\,\mu\in\mathcal{M}.$$

We also proved in [1] following

Theorem. *For $\mu\in\bar{\mathcal{M}}$, we have*

$$\pi(T^{m}(\pi(T^{m'}\mu)))=\pi(T^{m+m'}\mu) \qquad \textit{for } \forall m,\ m'\geq 0.$$

Now we can prove our theorem by using these results. Let $\mu\in\bar{\mathcal{M}}$. Then we have

$$\begin{aligned} H(T^{m+1}\mu)&=h(\pi T^{m+1}\mu)=h(\pi T\pi T^{m}\mu)\\ &\geq h(T\pi T^{m}\mu)=h(\pi T^{m}\mu)\\ &=H(T^{m}\mu) \text{ for } m\geq 0. \end{aligned}$$

We can also prove the theorem by direct computation. For that sake we need the following fundamental lemma which was proven in [1].

Fundamental lemma. *Let $\mu\in\bar{\mathcal{M}}$. Then we have following recursive formulas :*

for $m\geq 1$, $c\neq\phi$

$$\begin{aligned} P_{+}^{c}(n,\ m)&=(1-d_{-}(n+m))P_{+}^{c}(n-1,\ m-1)\\ &\quad+d_{+}(n-m)P_{-}^{c}(n+1,\ m-1) \end{aligned}$$

and

$$\begin{aligned} P_{-}^{c}(n,\ m)&=d_{-}(n+m)P_{+}^{c}(n-1,\ m-1)\\ &\quad+(1-d_{+}(n-m))P_{-}^{c}(n+1,\ m-1). \end{aligned}$$

Proof of the Theorem. For the simplicity we set

$$d_{+}=d_{+}(n-m),\ d_{-}=d_{-}(n+m)$$

Firstly we consider a case of $d_{+}\neq 0$ and $d_{-}\neq 0$. Let

$$Q_{\pm}(n,\ m)=P_{\pm}^{c}(n,\ m)/d_{\pm},$$

then from the fundamental lemma we have

$$Q_{+}(n,\ m)=(1-d_{-})Q_{+}(n-1,\ m-1)+d_{-}Q_{-}(n+1,\ m-1)$$

$$Q_{-}(n,\ m)=d_{+}Q_{+}(n-1,\ m-1)+(1-d_{+})Q_{-}(n+1,\ m-1)$$

From these relations we have

$$\begin{aligned} &\mathrm{P}_{+}^{c}(n,\ m)\log P_{+}^{c}(n,\ m)+P_{-}^{c}(n,\ m)\log P_{-}^{c}(n,\ m)\\ &\quad=d_{+}Q_{+}(n,\ m)\log d_{+}Q_{+}(n,\ m)+d_{-}Q_{-}(n,\ m)\log d_{-}Q_{-}(n,\ m)\\ &\quad=P_{+}^{c}(n,\ m)\log d_{+}+P_{-}^{c}(n,\ m)\log d_{-}+d_{+}Q_{+}(n,\ m)\log Q_{+}(n,\ m) \end{aligned}$$

$$+d_-Q_-(n, m) \log Q_-(n, m)$$

By the convexity of the function $x \log x \equiv f(x)$ we have

$$\begin{aligned}
Q_+(n, m) \log Q_+(n, m) &= f(Q_+(n, m)) \\
&= f((1-d_-)Q_+(n-1, m-1)+d_-Q_-(n+1, m-1)) \\
&\leq (1-d_-)f(Q_+(n-1, m-1))+d_-f(Q_-(n+1, m-1)),
\end{aligned}$$

$$\begin{aligned}
Q_-(n, m) \log Q_-(n, m) \leq d_+f(Q_+(n-1, m-1)) \\
+(1-d_+)f(Q_-(n+1, m-1)).
\end{aligned}$$

Therefore we obtain

$$\begin{aligned}
&P_+^c(n, m) \log P_+^c(n, m)+P_-^c(n, m) \log P_-^c(n, m) \\
&\leq P_+^c(n, m) \log d_+ + P_-^c(n, m) \log d_- \\
&\quad +d_+(1-d_-)Q_+(n-1, m-1) \log Q_+(n-1, m-1) \\
&\quad +d_+d_-Q_-(n+1, m-1) \log Q_-(n+1, m-1) \\
&\quad +d_-d_+Q_+(n-1, m-1) \log Q_+(n-1, m-1) \\
&\quad +d_-(1-d_+)Q_-(n+1, m-1) \log Q_-(n+1, m-1) \\
&= P_+^c(n, m) \log d_+ + P_-^c(n, m) \log d_- \\
&\quad +P_+^c(n-1, m-1) \log P_+^c(n-1, m-1)/d_+ \\
&\quad +P_-^c(n+1, m-1) \log P_-^c(n+1, m-1)/d_- \\
&= P_+^c(n-1, m-1) \log P_+^c(n-1, m-1) \\
&\quad +P_-^c(n+1, m-1) \log P_-^c(n+1, m-1) \\
&\quad +\log d_+\{P_+^c(n, m)-P_+^c(n-1, m-1)\} \\
&\quad +\log d_-\{P_-^c(n, m)-P_-^c(n+1, m-1)\} \\
&= P_+^c(n-1, m-1) \log P_+^c(n-1, m-1) \\
&\quad +P_-^c(n+1, m-1) \log P_-^c(n+1, m-1) \\
&\quad +\log d_+\{-d_-P_+^c(n-1, m-1)+d_+P_-^c(n+1, m-1)\} \\
&\quad +\log d_-\{d_-P_+^c(n-1, m-1)-d_+P_-^c(n+1, m-1)\} \\
&= P_+^c(n-1, m-1) \log P_+^c(n-1, m-1)
\end{aligned}$$

$$+P_-^c(n+1,\ m-1)\log P_-^c(n+1,\ m-1)$$

$$+(d_+P_-^c(n+1,\ m-1)-d_-P_+^c(n-1,\ m-1))\log d_+/d_-.$$

Furthermore, for $c\neq\phi$ we obtain

$$-\sum_{c\neq\phi}\{P_+^c(n,\ m)\log P_+^c(n,\ m)+P_-^c(n,\ m)\log P_-^c(n,\ m)\}$$

$$\geq-\sum_{c\neq\phi}\{P_+^c(n-1,\ m-1)\log P_+^c(n-1.\ m-1)$$

$$+P_-^c(n+1,\ m-1)\log P_-^c(n+1,\ m-1)\}-(d_+d_--d_-d_+)\log\frac{d_+}{d_-}$$

$$=-\sum_{c\neq\phi}\{P_+^c(n-1,\ m-1)\log P_+^c(n-1.\ m-1)$$

$$+P_-^c(n+1,\ m-1)\log P_-^c(n+1,\ m-1)\}$$

Note that $P_+^\phi(n,\ m)=P_+^\phi(n-1,\ m-1)$, $P_-^\phi(n,\ m)=P_-^\phi(n+1,\ m-1)$. Hence we have

$$-\sum_{c\in S}\{P_+^c(n,\ m)\log P_+^c(n,\ m)+P_-^c(n,\ m)\log P_-^c(n,\ m)\}$$

$$\geq-\sum_{c\in S}\{P_+^c(n-1,\ m-1)\log P_+^c(n-1.\ m-1)$$

$$+P_-^c(n+1,\ m-1)\log P_-^c(n+1,\ m-1)\}$$

For the case of $d_+=d_+(n-m)=0$ or $d_-=d_-(n+m)=0$ (say $d_+=0$) we have

$$P_+^c(n,\ m)=P_+^c(n-1,\ m-1)=0\ (c\neq\phi)$$

Hence from the fundamental lemma we have for any $c\in S$

$$P_-^c(n,\ m)=P_-^c(n+1,\ m-1)$$

So we have in this case,

$$-\sum_{c\in S}\{P_+^c(n,\ m)\log P_+^c(n,\ m)+P_-^c(n,\ m)\log P_-^c(n,\ m)\}$$

$$=-\sum_{c\in S}\{P_+^c(n-1,\ m-1)\log P_+^c(n-1,\ m-1)$$

$$+P_-^c(n+1,\ m-1)\log P_-^c(n+1,\ m-1)\}$$

Now it is easy to obtain the inequality

$$H(T^m\mu)\geq H(T^{m-1}\mu),\ m\geq1$$

(q.e.d.)

Remark. In the case $\mu\in\bar{\mathcal{M}}$, $d_+(n)=d_-(n)=d$ for all $n\in\boldsymbol{Z}$, we have

$$P_+^c(n,\ m)\log P_+^c(n,\ m)+P_-^c(n,\ m)\log P_-^c(n,\ m)$$
$$\leq P_+^c(n-1,\ m-1)\log P_+^c(n-1,\ m-1)$$
$$+P_-^c(n+1,\ m-1)\log P_-^c(n+1,\ m-1)$$

Hence we have, for all $c\in S$,

$$H_c(T^m\mu)\geq H_c(T^{m-1}\mu).$$

In general $H_c(T^m\mu)$ does not need to increase. In this respect we have the following.

Proposition. *Let $\mu\in\bar{\mathcal{M}}$. Moreover we assume*
1. $d_+(n\ ;\ \mu)=d_+$, $d_-(n\ ;\ \mu)=d_-$ *for all* $n\in\boldsymbol{Z}$, *and*
2. $P_+^c(n\ ;\ \mu)=p_+$, $P_-^c(n\ ;\ \mu)=p_-$ *for all* $n\in\boldsymbol{Z}$,

then we have

$$\begin{pmatrix}p_m\\ q_m\end{pmatrix}=\begin{pmatrix}1-d_- & d_+\\ d_- & 1-d_+\end{pmatrix}\begin{pmatrix}p_{m-1}\\ q_{m-1}\end{pmatrix},$$

where $p_m=P_+^c(n,\ m\ ;\ \mu)$, $q_m=P_-^c(n,\ m\ ;\ \mu)$, $m\geq 0$.

Proof. This is a direct consequence of the fundamental lemma.

Corollary. *Under the same assumptions as in the proposition, if* $d_+\neq d_-$, $d_++d_-\neq 1$ *and* $p_+\neq p_-$, *then* $H_c(T^m\mu)$ *can decrease in time m.*

Froof. From the proposition, we can easily obtain

$$p_m=\frac{d_+}{\varDelta}(p_++p_-)+\frac{d_-p_+-d_+p_-}{\varDelta}(1-\varDelta)^m,$$

$$q_m=\frac{d_-}{\varDelta}(p_++p_-)-\frac{d_-p_+-d_+p_-}{\varDelta}(1-\varDelta)^m,$$

where $\varDelta=d_++d_-$, and

$$H_c(T^m\mu)=-p_m\log p_m-q_m\log q_m.$$

Note that $0<\varDelta<2$, $\varDelta\neq 1$, hence the trajectory $(p_m,\ q_m)$, $m\geq 0$ moves on the line $p+q=p_++p_-$ and converges to the point $\left((p_++p_-)\frac{d_+}{\varDelta},\ (p_++p_-)\frac{d_-}{\varDelta}\right)$. Note that the function $h(p,\ q)=-p\log p-q\log q$ takes its maximum at $(p,\ q)=\left(\frac{p_++p_-}{2},\ \frac{p_++p_-}{2}\right)$ on the line $p+q=p_++p_-$.

Hence, for instance, if $p_+>p_-$, $d_->d_+$, $\varDelta<1$ then $H(T^m\mu)$ increases monotonically up to $m\leq M\equiv\dfrac{\log\dfrac{(d_--d_+)(p_++p_-)}{2(d_-p_+-d_+p_-)}}{\log(1-\varDelta)}$ and decreases monotonically for $m>M$. (q.e.d)

4. Derivation of Master equations

We want to remark that from our model it can be driven some interesting master equations by taking various hydrodynamic limits. Namely, we could consider our model as a diffusion process of the components in a fluid, and can derive heuristically master equations from our particle model by taking hydrodynamic limits. Firstly we assume the fluid has the constant density 1 and a velocity $2k$, so we set

$$d_+(n)=d_+=\frac{1}{2}+k\Delta x,$$

and

$$d_-(n)=d_-=\frac{1}{2}-k\Delta x.$$

Let

$$\rho_c(n\Delta x,\ m\Delta t)=P_+^c(n,\ m)+P_-^c(n,\ m)$$

and

$$u_c(n\Delta x,\ m\Delta t)\Delta x=P_+^c(n,\ m)-P_-^c(n,\ m),$$

where $\Delta t=(\Delta x)^2$. $\rho_c(x,\ t)$ may be interpreted as the density of particles with the color c in the fluid and $u_c(x,\ t)$ as its momentum of the fluid.

Let

$$(n\Delta x,\ m\Delta t)=(x,\ t),$$

then we obtain

$$\begin{aligned}P_+^c(n\pm 1,\ m)&=\frac{1}{2}\{\rho_c((n\pm 1)\Delta x,\ m\Delta t)+\Delta x u_c((n\pm 1)\Delta x,\ m\Delta t)\}\\&=\frac{1}{2}\Big\{\rho_c(x,\ t)+\Delta x u_c(x,\ t)\pm\frac{\partial\rho_c}{\partial x}(x,\ t)\Delta x+\frac{1}{2}\frac{\partial^2\rho_c}{\partial x^2}(\Delta x)^2\\&\qquad\pm\frac{\partial u_c}{\partial x}(\Delta x)^2+o((\Delta x)^2)\Big\}.\end{aligned}$$

Similarly we have

$$P_-^c(n\pm 1,\ m)=\frac{1}{2}\Big\{\rho_c(x,\ t)-\Delta x u_c(x,\ t)\pm\frac{\partial\rho_c}{\partial x}\Delta x+\frac{1}{2}\frac{\partial^2\rho_c}{\partial x^2}(\Delta x)^2$$

$$\mp\frac{\partial u_c}{\partial x}(\Delta x)^2+o((\Delta x)^2)\Big\},$$

and

$$P_+^c(n,\ m+1)=\frac{1}{2}\Big\{\rho_c(x,\ t)+\Delta x u_c(x,\ t)+\frac{\partial\rho_c}{\partial t}\Delta t+\Delta x\frac{\partial u_c}{\partial t}\Delta t+o(\Delta t)\Big\},$$

$$P_-^c(n,\ m+1)=\frac{1}{2}\Big\{\rho_c(x,\ t)-\Delta x u_c(x,\ t)+\frac{\partial\rho_c}{\partial t}\Delta t-\Delta x\frac{\partial u_c}{\partial t}\Delta t+o(\Delta t)\Big\}.$$

Hence from the fundamental lemma we obtain

$$(1)\begin{cases} u_c=2k\rho_c \\ \dfrac{\partial\rho_c}{\partial t}=-2k\dfrac{\partial\rho_c}{\partial x}+\dfrac{\partial^2\rho_c}{\partial x^2}\end{cases}$$

This is the well known diffusion equation with drift term.

We could also consider our models as a dilute gas with constant density D and velocity V. In this case we may set

$$d_+=R\Delta x=\frac{1}{2}(V+D)\Delta x,\quad d_-=L\Delta x=\frac{1}{2}(D-V)\Delta x.$$

Let

$$\rho_c(n\Delta x,\ m\Delta t)=P_+^c(n,\ m)+P_-^c(n,\ m),$$

$$u_c(n\Delta x,\ m\Delta t)=P_+^c(n,\ m)-P_-^c(n,\ m),$$

and $\Delta x=\Delta t$. Similarly as above, we can get

$$(2)\begin{cases} \dfrac{\partial\rho_c}{\partial t}=-\dfrac{\partial u}{\partial x} \\ \dfrac{\partial u_c}{\partial t}=-\dfrac{\partial\rho_c}{\partial x}+V\rho_c-Du_c\end{cases}$$

This can be seen as a wave equation with friction. We can easily solve this equation under a periodic boundary condition.

Let

$$\rho_c(x,\ t)=\sum_{n\in\mathbf{Z}}\rho_n(t)e^{inx}$$

and

$$u_c(x,\ t)=\sum_{n\in\mathbf{Z}}u_n(t)e^{inx},$$

then (2) can be writen as follows:

$$\begin{cases} \dfrac{d\rho_n}{dt} = -inu_n \\ \dfrac{du_n}{dt} = -in\rho_n + V\rho_n - Du_n \end{cases} \qquad n \in \boldsymbol{Z},$$

i.e.,

$$\frac{d}{dt}\begin{pmatrix} \rho_n \\ u_n \end{pmatrix} = \begin{pmatrix} 0 & -in \\ V-in & -D \end{pmatrix}\begin{pmatrix} \rho_n \\ u_n \end{pmatrix}.$$

This equation can be easily solved, so that we can see the properties of its solutions.

As we have verified for our simple particle model, we note that there can exist several macroscopic descriptions for a microscopic molecular system, depending on the scaling laws, namely the hierarchy of observations.

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