Classification of the irreducible representations 
of the affine Hecke algebra of type $B_2$ with 
unequal parameters

By
Naoya ENOMOTO

1. Introduction

The representation theory of the affine Hecke algebras has two different 
approaches. One is a geometric approach and the other is a combinatorial one.

In the equal parameter case, affine Hecke algebras are constructed using 
equivariant K-groups, and their irreducible representations are constructed on 
Borel-Moore homologies. By this method, their irreducible representations are 
parameterized by the index triples ([CG], [KL]). On the other hand, G. Lusztig 
classified the irreducible representations in the unequal parameter case. His 
ideas are to use equivariant cohomologies and graded Hecke algebras ([Lus89], 
[LusI], [LusII], [LusIII]).

Although the geometric approach will give us a powerful method for the 
classification, but it does not tell us the detailed structure of irreducible repre-
sentations. Thus it is important to construct them explicitly in combinatorial 
approach.

Using semi-normal representations and the generalized Young tableaux, 
A. Ram constructed calibrated irreducible representations with equal param-
eters ([Ram1]). Furthermore C. Kriloff and A. Ram constructed irreducible 
calibrated representations of graded Hecke algebras ([KR]). However, in gen-
eral, we don’t know the combinatorial construction of non-calibrated irreducible 
representations.

A. Ram classified irreducible representations of affine Hecke algebras of 
type $A_1$, $A_2$, $B_2$, $G_2$ in equal parameter case ([Ram2]). But there are some 
mistakes in his list of irreducible representations and his construction of induced 
representation of type $B_2$. For example, he missed the case $\chi_d^{(5)}$ (see Example 
3.1).

In this paper, we will correct his list about type $B_2$ and also classify the 
irreducible representations in the unequal parameter case. There are three 
one-parameter families of calibrated irreducible representations and some other
irreducible representations. We will use the Kato’s criterion for irreducibility (see Theorem 2.1).

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2. Preliminaries

2.1. Affine Hecke algebra

We will use following notations.

\[(R, R^+, \Pi) \text{ a root system of finite type, its positive roots and simple roots,} \]
\[Q, P \text{ the root lattice and the weight lattice of } R,\]
\[Q^\vee, P^\vee \text{ the coroot lattice and the coweight lattice of } R,\]
\[W \text{ the Weyl group of } R,\]
\[\ell(w) \text{ the length of } w \in W.\]

We put \(\Pi = \{\alpha_i\}_{i \in I}\), and denote by \(s_i\) the simple reflection associated with \(\alpha_i\).

First we define the Iwahori-Hecke algebra of \(W\).

**Definition 2.1.** Let \(\{q_i\}_{i \in I}\) be indeterminates. Then the *Iwahori-Hecke algebra* \(H\) of \(W\) is the associative algebra over \(\mathbb{C}(q_i)\) defined by following generators and relations;

- generators \(T_i \quad (i \in I)\)
- relations \((T_i - q_i)(T_i + q_i^{-1}) = 0 \quad (i \in I),\)
- \(m_{ij} T_i T_j \cdots T_i \cdots = T_j T_i \cdots T_j \cdots,\)

where \(m_{ij} = 2, 3, 4, 6\) according to \(\langle \alpha_i, \alpha_j^\vee \rangle \langle \alpha_j, \alpha_i^\vee \rangle = 0, 1, 2, 3.\)

**Remark 1.** The indeterminates \(q_i, q_j\) must be equal if and only if \(\alpha_i, \alpha_j\) are in the same \(W\)-orbit in \(R\). If all \(q_i\) are equal, we call the *equal parameter case*, and otherwise, *the unequal parameter case*.

For a reduced expression \(s_{i_1}s_{i_2} \cdots s_{i_r} \text{ of } w \in W\), we define \(T_w = T_{i_1} T_{i_2} \cdots T_{i_r}.\) This does not depend on the choice of reduced expressions.

Let us define the affine Hecke algebras.

**Definition 2.2.** The *affine Hecke algebra* \(\hat{H}\) is the associative algebra
Irreducible representations of affine Hecke algebra of type $B_2$ over $\mathbb{C}(q_i; i \in I)$ defined by following generators and relations;

- Generators $T_w X^\lambda$ ($w \in W, \lambda \in P^\vee$),
- Relations $(T_i - q_i)(T_i + q_i^{-1}) = 0$ ($i \in I$),
  $T_w T_{w'} = T_{ww'}$ if $\ell(w) + \ell(w') = \ell ww'$ ($w, w' \in W$),
  $X^\lambda X^\mu = X^{\lambda + \mu}$ ($\lambda, \mu \in P^\vee$),
  $X^\lambda T_i = T_i X^{s_i \lambda} + (q_i - q_i^{-1}) X^\lambda - X^{s_i \lambda} \frac{X^\lambda - X^{\alpha_i}}{1 - X^{-\alpha_i}}$ ($i \in I$).

2.2. Principal series representations and their irreducibility

Let us put $X^P = \{X^\lambda | \lambda \in P^\vee\}$ and let $\chi : X^{P^\vee} \to \mathbb{C}^*$ be a character of $X^{P^\vee}$.

**Definition 2.3.** Let $C v_\chi$ be the one-dimensional representation of $C[X]$ defined by

$$X^\lambda \cdot v_\chi = \chi(X^\lambda) v_\chi.$$ 

We call $\text{Ind}_{\text{C}[X]}^{\hat{H}} C v_\chi = \hat{H} \otimes_{\text{C}[X]} C v_\chi$ the principal representation of $\hat{H}$ associated with $\chi$.

Note that $\text{Res}_{\text{H}}^{\hat{H}} M(\chi)$ is isomorphic to the regular representation of $H$, so that $\dim M(\chi) = |W|$. We put $q_\alpha = q_i$ for $\alpha^\vee \in W \alpha_i^\vee$ ($i \in I$).

**Theorem 2.1 (Kato's Criterion of Irreducibility).** Let us put $P(\chi) = \{\alpha^\vee > 0 | \chi(X^{\alpha^\vee}) = q_\alpha^{\pm 2}\}$. Then $M(\chi)$ is irreducible if and only if $P(\chi) = \phi$.

For any finite-dimensional representation of $\hat{H}$ we put

$$M = \bigoplus_{\chi} M^\text{gen}_\chi$$

Then $M$ is the generalized weight decomposition of $M$.

**Proposition 2.1.** If $M$ is a simple $\hat{H}$-module with $M_{\chi} \neq 0$, then $M$ is a quotient of $M(\chi)$.

**Definition 2.4.** A finite-dimensional representation $M$ of $\hat{H}$ is calibrated (or $X$-semisimple) if $M^\text{gen}_\chi = M_\chi$ (for all $\chi$).
2.3. W-action Lemma
Let us define the action of Weyl group $W$ as the following:

$$(w \cdot \chi)(X^\lambda) = \chi(X^{w^{-1} \lambda}) \ (w \in W, \lambda \in P^\vee).$$

The following proposition is well known.

**Proposition 2.2** (W-action Lemma [Ram1], [Rog]).
1. If $M(\chi) \cong M(\chi')$, then there exists $w \in W$ such that $\chi' = w \chi$.
2. The representations $M(\chi)$ and $M(w \chi)$ have the same composition factors.

2.4. Specialization lemma
Let $\mathbb{K}$ be a field and $\mathcal{S}$ a discrete valuation ring such that $\mathbb{K}$ is the fraction field of $\mathcal{S}$. Let us denote the $\mathfrak{m} = (\pi)$ the maximal ideal of $\mathcal{S}$ and let $F = \mathcal{S}/\mathfrak{m}$ be the residue field of $\mathcal{S}$. Let $K(\mathcal{H}_2\text{-mod})$ be the Grothendieck group of the category of finite-dimensional representations of $\mathcal{H}_2$.

the following lemma is well-known (e.g. see [Ari, Lemma 13.16].)

**Lemma 2.1** (Specialization Lemma). Let $V$ be an $\mathcal{H}_2\text{-module}$ and $L$ an $\mathcal{H}_2\text{-submodule}$ of $V$ which is an $\mathcal{S}$-lattice of full rank. Then $[L \otimes F] \in K(\mathcal{H}_2\text{-mod})$ is determined by $V$ and does not depend on the choice of $L$.

2.5. Key results for type $B_2$
Let us consider the type $B_2$:

$$P^\vee = \mathbb{Z} \varepsilon_1 \oplus \mathbb{Z} \varepsilon_2, \ R^\vee = \{\alpha_1^\vee = \varepsilon_1 - \varepsilon_2, \alpha_2^\vee = 2\varepsilon_2\}, \ X_i = X^{\varepsilon_i}.$$

$$s_1 \varepsilon_1 = \varepsilon_2, \ s_1 \varepsilon_2 = \varepsilon_1, \ s_2 \varepsilon_1 = \varepsilon_1, \ s_2 \varepsilon_2 = -\varepsilon_2$$

Let us recall the definition of affine Hecke algebra of type $B_2$ with unequal parameters.

**Definition 2.5.** The affine Hecke algebra $\mathcal{H}$ of type $B_2$ is the associative algebra over $\mathbb{C}(p, q)$ defined by the following generators and relations:

- **generators**: $T_1, T_2, X_1, X_2$
- **relations**:
  - $(T_1 - q)(T_1 + q^{-1}) = 0$, \quad $\ (T_2 - p)(T_2 + p^{-1}) = 0$,
  - $T_1T_2T_1T_2 = T_2T_1T_1T_2$,
  - $T_1X_2T_1 = X_1$, \quad $T_2X_2^{-1}T_2 = X_2$,
  - $T_2X_1 = X_1T_2$, \quad $X_1X_2 = X_2X_1$.

We will use the following four subalgebras of $\mathcal{H}(B_2)$:

- $\mathcal{H}_1 = \langle T_1, X_1, X_2 \rangle$,
- $\mathcal{H}_2 = \langle T_2, X_1, X_2 \rangle$,
- $\mathcal{H} = \langle T_1, T_2 \rangle$,
- $\mathbb{C}[X_1, X_2] \subset \mathcal{H}$.

**Lemma 2.2** (Decomposition Lemma). Suppose $\chi(X^{\alpha_i}) = q_i^2$, and let $\rho_1, \rho_2$ be the following 1-dimensional representations of $\mathcal{H}_i = \langle T_i, X_j(1 \leq j \leq 2) \rangle \subset \mathcal{H}$:

$$\rho_1(X_j) = \chi(X_j), \ \rho_1(T_i) = q_i, \ \rho_2(X_j) = (s_i \chi)(X_j), \ \rho_2(T_i) = -q_i^{-1}.$$
Then there exists the following short exact sequence:

\[ 0 \to \text{Ind}_{\tilde{H}_i}^\tilde{H} \rho_2 \to M(\chi) \to \text{Ind}_{\tilde{H}_i}^\tilde{H} \rho_1 \to 0 \]

3. Classification

3.1. Method

Let \( M \) be an irreducible representation which is not principal. Then \( M \) appears in some \( M(\chi) \). By Kato’s criterion (Theorem 2.1), \( P(\chi) \neq \phi \). Using W-action Lemma (Lemma 2.2), we may assume \( P(\chi) \ni \alpha_i \) or \( \alpha_2 \), thus we obtain the following Lemma. We will use the notation \( -\chi \) defined by \( (\chi)(X_i) = -\chi(X_i) \) \((i = 1, 2)\).

**Lemma 3.1.** Except irreducible principal series representations, any finite-dimensional irreducible representation appears in the principal representations associated with the following characters as their composition factors:

\[
\begin{array}{cccccccccc}
\chi & \chi_a & \chi_b & \chi_c & \chi_d^{(1)} & \chi_d^{(2)} & \chi_d^{(3)} & \chi_d^{(4)} & \chi_d^{(5)} & \chi_f(v) & \chi_g(u) \\
\chi(X_1) & q^\pm & q^\pm p^{-1} & -p^{-1} & q^2 & q & p & 1 & -1 & pv & q^\pm u \\
\chi(X_2) & p & p^{-1} & p & q^{-1} & p & p & p & p & q^{-1} & u \\
\end{array}
\]

and \( -\chi_a, -\chi_b, -\chi_c, -\chi_d^{(1)}, -\chi_d^{(2)}, -\chi_d^{(3)}, -\chi_d^{(4)}, -\chi_d^{(5)}, -\chi_f(v) \), where

\[
v \neq \pm p^{-2}, \pm p^{-1}, \pm 1, q^{\pm 2}, q^{\pm 2} p^{-2}, \]

\[
u \neq \pm p^{\pm 1}, \pm 1, \pm q^{-2}, \pm q^{-2} p^{\pm 1}.
\]

**Note 1.** Two principal series representations \( M(-\chi_a) \) and \( M(\chi_a) \) have same composition factors, because of W-action lemma (Lemma 2.2). By replacing \( u \) with \( -u \), we don’t need to consider \( -\chi_g(u) \).

Finally, we must determine the composition factors of \( M(\chi) \) for above characters, and we must prove their irreducibility. But using the decomposition lemma, we consider the representations induced from \( \tilde{H}_i \). We will show the examples and some proofs in the following section.

3.2. Some examples and proofs

**Example 3.1.** We consider the principal series representation \( M(\chi_d^{(5)}) \).

Let \( \rho_1^{(5)} \) and \( \rho_2^{(5)} \) be the following 1-dimensional representations of \( \tilde{H}_2 \):

<table>
<thead>
<tr>
<th>\</th>
<th>\chi_1 \</th>
<th>\chi_2</th>
<th>\chi_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>\rho_1^{(5)}</td>
<td>-1</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>\rho_2^{(5)}</td>
<td>-1</td>
<td>-p^{-1}</td>
<td>-p^{-1}</td>
</tr>
</tbody>
</table>

Since \( \chi_d^{(5)}(\alpha_2^\prime) = p^2 \), we can apply the decompose lemma (Lemma 2.2) to \( M(\chi_d^{(5)}) \).
Lemma 3.2. Suppose $p \neq -q^{\pm 2}$. Then $\text{Ind}_{\bar{H}_2}^{R_2} \rho_1^{d(5)}$ and $\text{Ind}_{\bar{H}_2}^{R_2} \rho_2^{d(5)}$ are 4-dimensional non-calibrated irreducible representations.

Proof. We consider the case of $\text{Ind}_{\bar{H}_2}^{R_2} \rho_1^{d(5)}$. These simultaneous eigenvalues of $X_1$ and $X_2$ are $(p, -1), (-1, p)$, and the multiplicity of each eigenvalue is two. We can find the following representation matrices:

\[
T_1 = \begin{pmatrix}
\frac{p(q^2 - 1)}{(1 + p)q} & -\frac{(p - 1)(q^2 - 1)}{(1 + p)q} & 1 & -\frac{p(q^2 - 1)^2}{(1 + p)q^2} \\
0 & \frac{p^2(q^2 - 1)}{(1 + p)q^2} & 0 & \frac{pq}{(1 + p)q^2} \\
\frac{(p + q^2)(1 + pq^2)}{(1 + p)q^2} & (1 - p - pq^2)(q^2 - 1)^2 & \frac{(q^2 - 1)^2}{(1 + p)q} & (p - 1)(q^2 - 1)(pq + 1)(1 + p)q^2 \\
0 & 1 & 0 & \frac{(q^2 - 1)^2}{(1 + p)q}
\end{pmatrix},
\]

\[
X_1 = \begin{pmatrix} p \\ -1 - \frac{(p - 1)(q^2 - 1)(1 + pq^2)}{p(1 + pq^2)} \end{pmatrix}, \quad X_2 = \begin{pmatrix} -1 \frac{p^2 - 1}{p(q^2 - 1)} \end{pmatrix}.
\]

Since $p \neq -q^{\pm 2}$ and $p, q$ are not a root of unity, the non-diagonal component with respect to $(p, -1), (-1, p)$ in $X_1$ and $X_2$ don’t vanish. Thus the dimension of each simultaneous eigenspaces is just one. Let $v_1, v_2$ be the simultaneous eigenvectors with respect to $(p, -1), (-1, p)$. We have

\[
T_1v_1 = \frac{p(q^2 - 1)}{(1 + p)q} v_1 + \frac{(p + q^2)(1 + pq^2)}{(1 + p)q^2} v_2, \quad T_1v_2 = \frac{q^2 - 1}{(1 + p)q} v_2 + v_1,
\]

and $p \neq -q^{\pm 2}$. If there exists a submodule $0 \neq U$ of $\text{Ind}_{\bar{H}_2}^{R_2} \rho_1^{d(5)}$, then $U$ contains $v_1$ or $v_2$. If $v_2$ is contained in $U$, then $v_1$ is contained in $U$, and vice versa. Therefore $(v_1, v_2, T_1v_1, T_1v_2) \subset U$. This implies that $U = \text{Ind}_{\bar{H}_2}^{R_2} \rho_1^{d(5)}$, and $\text{Ind}_{\bar{H}_2}^{R_2} \rho_1^{d(5)}$ is irreducible. Similarly, we can show that $\text{Ind}_{\bar{H}_2}^{R_2} \rho_2^{d(5)}$ is irreducible. \hfill \Box

Example 3.2. We consider $M(\chi_a)$. Let $\rho_1^a$ and $\rho_2^a$ be the following 1-dimensional representations of $\bar{H}_2$:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1^a$</td>
<td>$q^2p$</td>
<td>$p$</td>
</tr>
<tr>
<td>$\rho_2^a$</td>
<td>$q^2p$</td>
<td>$-p^{-1}$</td>
</tr>
</tbody>
</table>

Since $\chi_a(\alpha_2^a) = p^2$, we can apply the decompose lemma (Lemma 2.2) to $M(\chi_a)$.

Lemma 3.3. Suppose $p \neq \pm q^{-1}, \pm q^{-2}, p^2 \neq -q^{-2}$. Then $\text{Ind}_{\bar{H}_2}^{R_2} \rho_1^a$ and $\text{Ind}_{\bar{H}_2}^{R_2} \rho_2^a$ have 1- and 3-dimensional calibrated irreducible composition factors. More precisely,

1. $\text{Ind}_{\bar{H}_2}^{R_2} \rho_1^a$ have two composition factors which are presented by the following representation matrices;
1-dimensional representations of affine Hecke algebra of type $B_2$

- $X_1 = pq^2$, $X_2 = p$, $T_1 = q$, $T_2 = p$.
- $U_1$: 
  \[ X_1 = \begin{pmatrix} p & p^2 \\ p & p^2 \end{pmatrix}, \quad X_2 = \begin{pmatrix} q^2 & q^2 \\ p & p \end{pmatrix}, \quad T_1 = \begin{pmatrix} \frac{q^2(p^2+1)}{p^2(p^2-1)} & \frac{q^2(p^2+1)}{p^2(p^2-1)} \\ \frac{q^2(p^2+1)}{p^2(p^2-1)} & \frac{q^2(p^2+1)}{p^2(p^2-1)} \end{pmatrix}, \quad T_2 = \begin{pmatrix} \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \\ \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \end{pmatrix}. \]

(2) $\text{Ind}_{\tilde{H}^2}^H \rho_2^q$ have two composition factors which are presented by the following representation matrices:
- $X_1 = p^{-1}q$, $X_2 = p^{-1}$, $T_1 = -q^{-1}$, $T_2 = -p^{-1}$.
- $U_2$: 
  \[ X_1 = \begin{pmatrix} p^{-1} & p^2 \\ p & p^2 \end{pmatrix}, \quad X_2 = \begin{pmatrix} p^{-1} & p^2 \\ p & p^2 \end{pmatrix}, \quad T_1 = \begin{pmatrix} \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \\ \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \end{pmatrix}, \quad T_2 = \begin{pmatrix} \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \\ \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \end{pmatrix}. \]

**Example 3.3.** We consider $M(\chi_b)$. Let $\rho_1^b$ and $\rho_2^b$ be the following 1-dimensional representations of $\tilde{H}_1$:

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$T_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1^b$</td>
<td>$q^p p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$q$</td>
</tr>
<tr>
<td>$\rho_2^b$</td>
<td>$p^{-1}$</td>
<td>$q^p p^{-1}$</td>
<td>$-q^{-1}$</td>
</tr>
</tbody>
</table>

Since $\chi_a(\alpha'_1) = q^2$, we can apply the decompose lemma (Lemma 2.2) to $M(\chi_b)$.

**Lemma 3.4.** (1) Suppose $p \neq \pm q, \pm q^2, p^2 \neq -q^2$. Then $\text{Ind}_{\tilde{H}_1}^H \rho_1^b$ and $\text{Ind}_{\tilde{H}_1}^H \rho_2^b$ have 1- and 3-dimensional calibrated irreducible composition factors which are calibrated and presented by the following representation matrices:

(i) case $\text{Ind}_{\tilde{H}_1}^H \rho_1^b$:
- $X_1 = q^2 p^{-1}$, $X_2 = p^{-1}$, $T_1 = q$, $T_2 = -p^{-1}$.
- $U_1$: 
  \[ X_1 = \begin{pmatrix} q^2 & p^{-1} \\ p & p \end{pmatrix}, \quad X_2 = \begin{pmatrix} p & q^2 \\ p^{-2} & p^{-1} \end{pmatrix}, \quad T_1 = \begin{pmatrix} \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \\ \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \end{pmatrix}, \quad T_2 = \begin{pmatrix} \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \\ \frac{q^2(p^2+1)}{p^2(q^2-1)} & \frac{q^2(p^2+1)}{p^2(q^2-1)} \end{pmatrix}. \]

(ii) case $\text{Ind}_{\tilde{H}_1}^H \rho_2^b$:
- $X_1 = pq^{-2}$, $X_2 = p$, $T_1 = -q^{-1}$, $T_2 = -p$. 
\( \bullet U_0^2: \)

\[
X_1 = \begin{pmatrix} pq^{-1} \\ p^{-1} \\ p^{-1} \\ p^{-1} \end{pmatrix}, \quad X_2 = \begin{pmatrix} p^{-2} \\ pq \\ q \end{pmatrix},
\]

\[
T_1 = \begin{pmatrix} -2(q^2-1) \\ (q^2-p^2)^2 \\ q \\ q^2-1 \end{pmatrix}, \quad T_2 = \begin{pmatrix} -q^{-1} \\ -q^{-1} \\ q^{-1} \\ -q^{-1} \end{pmatrix}.
\]

(2) Suppose \( p = q \). Then they have 1-dimensional composition factor and 3-dimensional non-calibrated composition factor which are presented by the following representation matrices;

(i) case \( \text{Ind}^G_{H_1, \rho_1^1}: \)

- \( X_1 = q, \quad X_2 = q^{-1}, \quad T_1 = q, \quad T_2 = -q^{-1}. \)
- \( U_0^1: \)

\[
X_1 = \begin{pmatrix} q \\ q^2 \\ q \end{pmatrix}, \quad X_2 = \begin{pmatrix} q^{-1} \\ \frac{1+2a^2}{q} \\ \frac{1}{q} \end{pmatrix},
\]

\[
T_1 = \begin{pmatrix} q \\ q^{-1} \\ \frac{1+2a^2}{q^2} \\ q^{-1} \\ q^{-1} \end{pmatrix}, \quad T_2 = \begin{pmatrix} -q^{-1} \\ 1 \\ \frac{1+2a^2}{q} \end{pmatrix}.
\]

(ii) case \( \text{Ind}^G_{H_1, \rho_2^1}: \)

- \( X_1 = q^{-1}, \quad X_2 = q, \quad T_1 = -q^{-1}, \quad T_2 = q. \)
- \( U_0^2: \)

\[
X_1 = \begin{pmatrix} q^{-1} \\ q^2-1 \\ q \end{pmatrix}, \quad X_2 = \begin{pmatrix} q^{-1} \\ q \end{pmatrix},
\]

\[
T_1 = \begin{pmatrix} q(q^2+q^2) \\ -q^{-1} \\ -q^{-1} \end{pmatrix}, \quad T_2 = \begin{pmatrix} -q^{-1} \\ q^{-1} \\ q(q^2+1) \\ -q^{-1} \end{pmatrix}.
\]

(3) Suppose \( p = q^2 \). Then they have 1-dimensional composition factor and 3-dimensional non-calibrated composition factor which are presented by the following representation matrices;

(i) case \( \text{Ind}^G_{H_1, \rho_1^1}: \)

- \( X_1 = 1, \quad X_2 = q^{-2}, \quad T_1 = q, \quad T_2 = -q^{-2}. \)
1-dimensional representations of case (ii) \( \chi \).

Since the following representation matrices

\[
\begin{align*}
X_1 &= \begin{pmatrix} 1 & q^2 \\ q & 1 \end{pmatrix}, & X_2 &= \begin{pmatrix} q^2 & \frac{q^2 - 1}{q} \\ 1 & 1 \end{pmatrix}, \\
T_1 &= \begin{pmatrix} -q^{-1} & \frac{(q^2 + 1)^2}{q} \\ q & \frac{q^2 + 1}{q} \end{pmatrix}, & T_2 &= \begin{pmatrix} q^2 & \frac{1}{q^2} \\ 1 & 1 \end{pmatrix}.
\end{align*}
\]

(ii) case Ind_{\mathcal{H}_1} \rho_2^k:

- \( X_1 = 1, \ X_2 = q^2, \ T_1 = -q^{-1}, \ T_2 = q^2. \)

- \( U_0^k: \)

\[
\begin{align*}
X_1 &= \begin{pmatrix} q^{-2} & q^{-2} \\ 1 & 1 \end{pmatrix}, & X_2 &= \begin{pmatrix} 1 & \frac{q^2 - 1}{q} \\ \frac{q^2 + 1}{q} & 1 \end{pmatrix}, \\
T_1 &= \begin{pmatrix} -q^{-1} & \frac{q^2 + 1}{q} \\ -q^{-1} & \frac{q^2 + 1}{q} \end{pmatrix}, & T_2 &= \begin{pmatrix} 1 & \frac{q^2 - 1}{q} \\ -q^{-2} & -q^{-2} \end{pmatrix}.
\end{align*}
\]

**Example 3.4.** We consider \( M(\chi_c) \). Let \( \rho_1^k \) and \( \rho_2^k \) be the following 1-dimensional representations of \( \mathcal{H}_2 \):

<table>
<thead>
<tr>
<th>( \rho_1^k )</th>
<th>( \rho_2^k )</th>
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<tbody>
<tr>
<td>( X_1 )</td>
<td>( X_2 )</td>
</tr>
<tr>
<td>( X_1 )</td>
<td>( X_2 )</td>
</tr>
<tr>
<td>( \rho_1^k )</td>
<td>( \rho_2^k )</td>
</tr>
<tr>
<td>( p^{-1} )</td>
<td>( -p^{-1} )</td>
</tr>
<tr>
<td>( p )</td>
<td>( -p^{-1} )</td>
</tr>
</tbody>
</table>

Since \( \chi_c(\alpha^*_2) = p^2 \), we can apply the decompose lemma (Lemma 2.2) to \( M(\chi_c) \).

**Lemma 3.5.** (1) Suppose \( p^2 \neq -q^{\pm 2} \). Ind_{\mathcal{H}_2} \rho_1^k \) and Ind_{\mathcal{H}_2} \rho_2^k \) have two 2-dimensional irreducible calibrated composition factors which are presented by the following representation matrices:

**composition factors of Ind_{\mathcal{H}_2} \rho_1^k:**

<table>
<thead>
<tr>
<th>( \rho_1^k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
</tr>
<tr>
<td>( U_0^1 )</td>
</tr>
</tbody>
</table>

**composition factors of Ind_{\mathcal{H}_2} \rho_2^k:**

<table>
<thead>
<tr>
<th>( \rho_2^k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
</tr>
<tr>
<td>( U_0^2 )</td>
</tr>
<tr>
<td>( U_0^3 )</td>
</tr>
</tbody>
</table>
(2) Suppose $p^2 = -q^2$. They have one 2-dimensional irreducible calibrated composition factor and two 1-dimensional composition factors. And their representation matrices are obtained by putting $p^2 = -q^2$ in above matrices, since specialization lemma (Lemma 2.1). More precisely, $U_{\epsilon}^1, U_{\gamma}^2$ are irreducible, but $U_{\epsilon}^2, U_{\gamma}^4$ have two 1-dimensional composition factors.

### 3.3. Classification Theorem

By the preceding Examples and Lemmas, we obtain the following classification theorem.

First, let us define the 1-dimensional representations of $\mathcal{H}_4$ in addition to the notation in the preceding Examples and Lemmas:

<table>
<thead>
<tr>
<th>$\mathcal{H}_1$</th>
<th>$\rho_1^{(1)}(\epsilon)$</th>
<th>$\rho_1^{(2)}(\gamma)$</th>
<th>$\rho_1^{(3)}(\gamma)$</th>
<th>$\rho_1^{(4)}(\gamma)$</th>
<th>$\rho_1'(u)$</th>
<th>$\rho_1''(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$q^4$</td>
<td>$q^4$</td>
<td>$q^4$</td>
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<td>$q^4$</td>
<td>$q^4$</td>
</tr>
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<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$q$</td>
<td>$q^{-1}$</td>
<td>$q$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\mathcal{H}_2$</th>
<th>$\rho_1^{(1)}(\epsilon)$</th>
<th>$\rho_1^{(2)}(\gamma)$</th>
<th>$\rho_1^{(3)}(\gamma)$</th>
<th>$\rho_1^{(4)}(\gamma)$</th>
<th>$\rho_1'(v)$</th>
<th>$\rho_1''(v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$p$</td>
<td>$p^{-1}$</td>
<td>$1$</td>
<td>$1$</td>
<td>$pv$</td>
<td>$pv$</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$p$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$p$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
</tr>
</tbody>
</table>

**Theorem 3.1.** Suppose that $p$ and $q$ are not a root of unity. The finite-dimensional irreducible representations of type $B_2$ with unequal parameters are given by the following lists depending on the relation of parameters.

(0) The principal series representations $M(\chi)$, where $\chi \neq \pm \chi_a, \pm \chi_b, \pm \chi_c, \pm \chi_d^{(j)} (1 \leq j \leq 5), \pm \chi_f(v), \chi_g(u)$ and their $W$-orbits, are irreducible.

(1) For any $p, q$, there are eight 1-dimensional (irreducible) representations defined by

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$q^p$</th>
<th>$q^{-1}p^{-1}$</th>
<th>$q^{-1}p^{-1}$</th>
<th>$q^p$</th>
<th>$q^{-1}p^{-1}$</th>
<th>$q^{-1}p^{-1}$</th>
<th>$q^{-1}p^{-1}$</th>
<th>$q^{-1}p^{-1}$</th>
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<tbody>
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<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
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<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>$q$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
<td>$q^{-1}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$p$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
<td>$p^{-1}$</td>
</tr>
</tbody>
</table>

(2) For any $p, q$,

\[
\text{Ind}_{\mathcal{H}_2}^{\mathcal{H}_1} \rho_1'(v), \text{Ind}_{\mathcal{H}_2}^{\mathcal{H}_1} \rho_1''(v), \text{Ind}_{\mathcal{H}_2}^{\mathcal{H}_1} (-\rho_1'(v)), \text{Ind}_{\mathcal{H}_2}^{\mathcal{H}_1} (-\rho_1''(v))
\]

with $v \neq \pm p^{-2}, \pm p^{-1}, \pm 1, \pm q, \pm q^{-2}, \pm q^{-1}, \pm q^{-2} p^{-2}$

\[
\text{Ind}_{\mathcal{H}_1}^{\mathcal{H}_1} \rho_0'(u), \text{Ind}_{\mathcal{H}_1}^{\mathcal{H}_1} \rho_0''(u) \text{ and } u \neq \pm p, \pm 1, \pm q, \pm q^{-1}, \pm q^{-2}, \pm q^{-2} p^{-1}
\]

are 4-dimensional one parameter families of irreducible representations and calibrated. They are not isomorphic to each other.

(3) When $p, q$ are generic i.e. $p \neq \pm q, \pm q^{-1}$ and $p^2 \neq -q^2$, the remaining finite-dimensional irreducible representations are the following:
(I) $U_{c}^{i} (1 \leq i \leq 4)$ which are 2-dimensional and calibrated.

(II) $U_{a}^{i}, U_{b}^{i}, U_{-a}^{i}, U_{-b}^{i} (i = 1, 2)$ which are 3-dimensional and calibrated.

(III)

$$\text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 1, 2),$$

$$\text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 3, 4, 5)$$

which are 4-dimensional and non-calibrated.

(4) When $p = q^2$, the remaining finite-dimensional irreducible representations are the following:

(I) $U_{c}^{i} (1 \leq i \leq 4)$ which are 2-dimensional and calibrated.

(II) $U_{a}^{i}, U_{b}^{i}, (i = 1, 2)$ which are 3-dimensional and calibrated.

(III) $U_{b}^{i}, U_{-b}^{i}, (i = 1, 2)$ which are 3-dimensional and non-calibrated.

(IV)

$$\text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 2),$$

$$\text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 3, 5)$$

which are 4-dimensional and non-calibrated.

(5) When $p = q$, the remaining finite-dimensional irreducible representations are the following:

(I) $U_{c}^{i} (1 \leq i \leq 4)$ which are 2-dimensional and calibrated.

(II) $U_{a}^{i}, U_{b}^{i}, (i = 1, 2)$ which are 3-dimensional and calibrated.

(III) $U_{b}^{i}, U_{-b}^{i}, (i = 1, 2)$ which are 3-dimensional and non-calibrated.

(IV)

$$\text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 1),$$

$$\text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 4, 5)$$

which are 4-dimensional and non-calibrated.

(6) When $p^2 = -q^2$, the remaining finite-dimensional irreducible representations are the following:

(I) $U_{c}^{i} (i = 1, 2)$ which are 2-dimensional and calibrated.

(II) $U_{a}^{i}, U_{b}^{i}, (1 \leq i \leq 2)$ which are 3-dimensional and calibrated.

(III)

$$\text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{1}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 1, 2),$$

$$\text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} \rho_{j}^{d(i)}, \text{Ind}_{\mathcal{H}_{2}}^{\tilde{\mathcal{H}}} (-\rho_{j}^{d(i)}) (j = 1, 2, i = 3, 4, 5)$$

which are 4-dimensional and non-calibrated.

(7) Using the following automorphisms of $\tilde{\mathcal{H}}$

$$X_1 \mapsto X_1, X_2 \mapsto X_2, T_1 \mapsto T_1, T_2 \mapsto -T_2, q \mapsto q, p \mapsto \pm p^{\pm 1}$$
the cases of \( p = \pm q^{-2}, -q^2 \) reduces the case (4). Similarly, the cases of \( p = \pm q^{-1}, -q \) reduces the case (5). The case of \( p^2 = -q^{-2} \) also reduces the case (6).

Note 2. In [Ram2], Ram dealt equal parameter case. However he missed the case \( \chi_d^{(5)} \) and did not explicitly list the isomorphism classes of irreducible representations \(-\chi_a, -\chi_b, -\chi_d^{(j)} \) and \(-\chi_f \).

4. Tables of irreducible representations

We will summarize about the dimension of composition factors and their calibratability. Note that we will omit the principal series representation \( M(-\chi) \) and their composition factors in the following tables.

4.1. \( p, q \) generic case (i.e. \( p \neq \pm q^{\pm 1}, \pm q^{\pm 2} \) and \( p^2 \neq -q^{\pm 2} \))
4.2. $p = q$ case; equal parameter case

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$\chi(X_1)$</th>
<th>$\chi(X_2)$</th>
<th>$P(\chi)$</th>
<th>dim</th>
<th>calibrated?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_a$</td>
<td>$q^2$</td>
<td>$q$</td>
<td>${a_1, a_2}$</td>
<td>1</td>
<td>$\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>$\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>$\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\chi_b$</td>
<td>$q$</td>
<td>$q^{-1}$</td>
<td>${a_1, a_2, 2a_1 + a_2}$</td>
<td>1</td>
<td>$\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>$\times$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>$\times$</td>
</tr>
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<td></td>
<td></td>
<td>1</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\chi_c$</td>
<td>$-q^{-1}$</td>
<td>$q$</td>
<td>${a_2, 2a_1 + a_2}$</td>
<td>2</td>
<td>$\circ$</td>
</tr>
<tr>
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<td>2</td>
<td>$\circ$</td>
</tr>
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<td></td>
<td></td>
<td></td>
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<td>$\circ$</td>
</tr>
<tr>
<td>$\chi_d^{(1)}$</td>
<td>$q^2$</td>
<td>$1$</td>
<td>${a_1, a_1 + a_2}$</td>
<td>4</td>
<td>$\times$</td>
</tr>
<tr>
<td>$\chi_d^{(1)}$</td>
<td>$1$</td>
<td>$q$</td>
<td>${a_2}$</td>
<td>4</td>
<td>$\times$</td>
</tr>
<tr>
<td>$\chi_d^{(2)}$</td>
<td>$-1$</td>
<td>$p$</td>
<td>${a_2}$</td>
<td>4</td>
<td>$\times$</td>
</tr>
<tr>
<td>$\chi_d^{(3)}$</td>
<td>$q^2v$</td>
<td>$q$</td>
<td>${a_2}$</td>
<td>4</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\chi_g(u)$</td>
<td>$q^2u$</td>
<td>$u$</td>
<td>${a_1}$</td>
<td>4</td>
<td>$\circ$</td>
</tr>
</tbody>
</table>

4.3. $p = q^2$ case

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$\chi(X_1)$</th>
<th>$\chi(X_2)$</th>
<th>$P(\chi)$</th>
<th>dim</th>
<th>calibrated?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_a$</td>
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<td>$q^2$</td>
<td>${a_1, a_2}$</td>
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<td>$\circ$</td>
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<tr>
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<td>1</td>
<td>$\circ$</td>
</tr>
<tr>
<td>$\chi_b$</td>
<td>$1$</td>
<td>$q^{-2}$</td>
<td>${a_1, a_2, a_1 + a_2}$</td>
<td>1</td>
<td>$\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>$\times$</td>
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<td>$\circ$</td>
</tr>
<tr>
<td>$\chi_d^{(2)}$</td>
<td>$q$</td>
<td>$q^{-1}$</td>
<td>${a_1}$</td>
<td>4</td>
<td>$\times$</td>
</tr>
<tr>
<td>$\chi_d^{(3)}$</td>
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<td>$q^2$</td>
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<td>$\chi_f(v)$</td>
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<tr>
<td>$\chi_g(u)$</td>
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</table>
### 4.4. \( p^2 = -q^2 \) case

<table>
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<tr>
<th>( \chi(\chi_1) )</th>
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<th>( \dim )</th>
<th>Calibrated?</th>
</tr>
</thead>
<tbody>
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<td>( \chi^* )</td>
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<td>{( \alpha_1, \alpha_2 )}</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( -p^{-1} )</td>
<td>{( \alpha_1, \alpha_2, 2\alpha_1 + \alpha_2 )}</td>
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</tr>
<tr>
<td>( \chi_1^{(1)} )</td>
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<td>{( \alpha_1, \alpha_1 + \alpha_2 )}</td>
<td>4</td>
</tr>
<tr>
<td>( \chi_2^{(2)} )</td>
<td>( \pm p \sqrt{-1} )</td>
<td>( \pm p \sqrt{-1} )</td>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>( -p^du )</td>
<td>( u )</td>
<td>{( \alpha_1 )}</td>
<td>4</td>
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Research Institute for Mathematical Sciences  
Kyoto University  
Kitashirakawa-Oiwakecho  
Sakyoku, Kyoto 606-8502  
JAPAN  
e-mail: henon@kurims.kyoto-u.ac.jp

### References


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