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THE $\overline{\partial}$-NEUMANN OPERATOR AND THE KOBAYASHI METRIC

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ABSTRACT. We introduce a condition, called Property -K, which encodes information about the holomorphic structure of fat subdomains. We obtain an equivalence between this condition and the compactness of the $\overline{\partial}$-Neumann operator in any convex domain. We also exhibit a local property of the Kobayashi metric under which the domain is locally a product space.

1. Introduction

In this paper we study a condition on the Kobayashi metric near a boundary point p of a pseudoconvex domain in $\mathbb{C}^n$ that is related to the compactness of the $\overline{\partial}$-Neumann operator. We call this condition Property -K. The precise definition of Property -K is given in the next section.

If Ω is a bounded pseudoconvex domain in $\mathbb{C}^n$ that is Kobayashi complete near p, and if Ω has a compact $\overline{\partial}$-Neumann operator, then Property -K is necessarily satisfied.

Product domains are examples of domains that have a non-compact $\overline{\partial}$-Neumann operator. If a domain Ω does not have Property -K near a boundary point p, then Ω can be well approximated near p by a product domain, in a sense made precise in Lemma 3 below.

In particular, we have the following result about the Kobayashi metric and product domains. Let D denote the open unit disk in $\mathbb{C}^1$. When Ω is a domain in $\mathbb{C}^n$ and v is a vector, $d_v(z)$ denotes the radius of the largest affine disk in Ω with center z and direction v, that is, $d_v(z) = \sup\{r \mid z + rDv \subset \Omega\}$.

THEOREM 1. *If Ω is a bounded domain in $\mathbb{C}^n$ that is Kobayashi complete near a boundary point p and there exist $\epsilon < 1$, $v \in \mathbb{C}^n$, and a constant $C > 0$*

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such that for all z close enough to p the Kobayashi metric K satisfies

$$K(z,v) < C\frac{1}{d_v(z)^\epsilon}$$

(when $\epsilon = 1$ and $C = 1$, this is always true), then near p, Ω is locally a product space.

We also give an example in which Property -K is not satisfied (and hence the $\overline{\partial}$-Neumann operator is not compact).

In Section 2, we define terminology. Section 3 contains the main results. Some applications are given in Section 4.

2. Terminology and definition of Property -K

Definition (Fat subdomain with mass at p). Let G be a domain in $\mathbb{C}^n$, and let A be a subdomain of G. If there is a sequence $\{f_j\}$ of holomorphic functions in the unit ball of $L^2(G)$ such that no subsequence of $\{f_j\}$ converges in $L^2(A)$, then A is said to be a *fat subdomain* of G. In other words, A is a fat subdomain of G if the restriction operator $L^2(G) \cap \mathcal{O}(G) \to L^2(A)$ is not a compact operator. If, for any open neighborhood U of p in $\mathbb{C}^n$, $A \cap U$ is also a fat subdomain of Ω, then we say that A *has mass at p*.

For example, if there is a point p in the boundary of G and a neighborhood U of p in $\mathbb{C}^n$ such that $A \cap U = G \cap U$, then A is a fat subdomain of G. This condition is not necessary, however. For instance, let D be the unit disk $\{z : |z| < 1\}$ in $\mathbb{C}$ and let A be $\{(x, y) \in D : 0 < x < 1$ and $0 < y < (1-x)^p\}$, where $p > 0$. Then A is a fat subdomain of D if (and only if) $p \leq 1$. One can easily check this by taking the sequence of holomorphic functions $\{f_j\}$ to be the sequence of normalized Bergman kernel functions $\{B_D(z, p_j)/\sqrt{B_D(p_j, p_j)}\}$, where $B_D(z, w)$ is the Bergman kernel on D and the sequence $\{p_j\}$ approaches the point $(1, 0)$.

We recall the definition of the Bergman kernel function. Let $H(\Omega)$ denote the space of square-integrable holomorphic functions on a domain Ω in $\mathbb{C}^n$. By the Riesz representation theorem, for each fixed point w in Ω there is a unique element of $H(\Omega)$, denoted by $B_\Omega(\cdot, w)$, such that

$$f(w) = (f, B_\Omega(\cdot, w)) = \int_\Omega f(z)\overline{B_\Omega(z, w)}\, dV_z$$

for all $f \in H(\Omega)$. This function $B_\Omega(z, w)$ is called the Bergman kernel function for Ω.

The following lemma contained in [5] gives explicit and small fat subdomains of a convex domain.

Lemma 1. *If Ω is a bounded convex domain in $\mathbb{C}^n$ and $0 < R \leq 1$, then:*

(1) *For any points $p_0 \in \partial\Omega$ and $p_1 \in \Omega$ there exist positive constants C and δ_0 such that the Bergman kernel function B_Ω satisfies the inequality*
$$B_\Omega(p_\delta, p_\delta) > CB_\Omega(p_{\delta/R}, p_{\delta/R})$$
for any $\delta \in (0, \delta_0)$, where $p_\delta := p_0 + \delta(p_1 - p_0)/\|p_1 - p_0\|$.

(2) *If $0 \in \partial\Omega$, then the scaled domain $R\Omega$ is a fat subdomain of Ω.*

We now introduce briefly the $\overline{\partial}$-Neumann operator. When a domain Ω is bounded and pseudoconvex, the (unbounded) self-adjoint, surjective operator $\overline{\partial}\overline{\partial}^* + \overline{\partial}^*\overline{\partial}$ has a bounded inverse operator acting on $(0, q)$-forms. This operator $N = N_q$ is called the $\overline{\partial}$-Neumann operator. We refer the reader to [4] and [7], the recent survey [1], and the book [2] for background on the $\overline{\partial}$-Neumann problem. In this paper we consider only N_1. The compactness condition can be reformulated in the following way.

LEMMA 2. *Let Ω be a bounded pseudoconvex domain. Then the following are equivalent.*

(1) *The $\overline{\partial}$-Neumann operator N_1 is compact from $L^2_{(0,1)}(\Omega)$ to itself.*

(2) *The canonical solution operators $\bar{\partial}^* N_1 : L^2_{(0,1)}(\Omega) \to L^2_{(0,0)}(\Omega)$ and $\bar{\partial}^* N_2 : L^2_{(0,2)}(\Omega) \to L^2_{(0,1)}(\Omega)$ are compact.*

Let Ω be a domain in $\mathbb{C}^n$ and $(p, v) \in \Omega \times \mathbb{C}^n$. The Kobayashi metric $K(p, v)$ is defined by

$$K(p, v) = \inf\left\{\frac{1}{|c|} : \text{ there exists } f : D \to \Omega \text{ holomorphic}, \quad f(0) = p,\ f'(0) = cv\right\}.$$

For $z', z'' \in \Omega$, we put

$$k_\Omega(z', z'') = \inf\left\{\int_0^1 K(r(t), r'(t))dt : \ r \text{ is a piecewise } C^1\text{-curve in } \Omega \text{ from } z' \text{ to } z''\right\}.$$

We call k_Ω the Kobayashi pseudodistance on Ω.

A domain Ω is called Kobayashi complete if any Cauchy sequence $\{z_j\}_{j \in N}$ with respect to the Kobayashi pseudodistance converges to a point $z_0 \in \Omega$, i.e., if $\{k_\Omega(z_j, z_0)\}$ converges to 0.

Examples of Kobayashi complete domains are strongly pseudoconvex domains, convex domains and bounded pseudoconvex Reinhart domains containing 0. It is an open problem whether every bounded balanced domain with C^∞-Minkowski function is Kobayashi complete. It is also still open whether every bounded pseudoconvex domain with C^∞-smooth boundary is Kobayashi complete [8].

Definition (Property -K). We say that Ω has Property -K near a point $p \in \partial\Omega$ if the following condition is satisfied:

For any fat subdomain A having mass at p and for all $v \in \mathbb{C}^n$, $C > 0$, and $\epsilon < 1$, there is a sequence $\{q_n\}$ approaching p in $\Omega \cap (A + vD)$ such that

$$K(q_n, v) \geq C \frac{1}{d_v(q_n)^\epsilon}.$$

The following theorem gives examples of domains that have Property -K and examples of domains that do not have this property. We will prove this theorem in Section 4.

Theorem 2. *If Ω is a bounded convex domain in $\mathbb{C}^n$, then the following are equivalent:*

(1) *Ω has Property -K at any point p in $\partial\Omega$.*
(2) *The $\overline{\partial}$-Neumann operator N_1 is compact.*
(3) *There is no affine complex disc in the boundary of Ω.*

Although in convex domains Property -K and the compactness of N_1 are equivalent, this equivalence does not hold in general. An example of a pseudoconvex Reinhart domain in $\mathbb{C}^2$ with compact $\overline{\partial}$-Neumann operator N_1 is

$$\Omega = \{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1|^2 + |z_2|^2 < 1, 0 < |z_1| < 1\}.$$

This domain is not Kobayashi complete and also does not have Property -K near $(0, 0)$ in $\partial\Omega$ [6, p. 150].

3. The main result

Theorem 3. *If Ω is a bounded pseudoconvex domain in $\mathbb{C}^n$ that is Kobayashi complete near p and has smooth boundary near p in $\partial\Omega$ and Ω does not satisfy Property -K, then Ω has a non-compact $\overline{\partial}$-Neumann operator.*

In order to prove this result, we need two lemmas.

Lemma 3. *Suppose that Ω is a bounded domain in $\mathbb{C}^n$ that is Kobayashi complete near p, and Ω does not have Property -K near p in $\partial\Omega$. Then there is a fat subdomain A having mass at p such that after linearly changing the coordinate system there is a coordinate system $(z_1, \ldots, z_n)$ on some open neighborhood U_0 of p in $\mathbb{C}^n$ such that there exists a constant C_0 for which $(1/4)C_0 D \times \pi(\{A \cap U_0 \mid |z_1| < (1/2)C_0\}) \subset \Omega$, where $\pi : \Omega \to \mathbb{C}^{n-1}$ is the natural projection $\pi(z) = (z_2, \ldots, z_n)$.*

Proof. By the hypotheses there is a fat subdomain A having mass at p, a non-zero vector $v \in \mathbb{C}^n$, a number $\epsilon < 1$, and a constant C_ϵ, such that

$$K(z, v) \leq \frac{C_\epsilon}{d_v(z)^\epsilon} \tag{1}$$

when $z \in (A + Dv) \cap (\Omega \cap U)$, where U is a small enough open neighborhood of p in $\mathbb{C}^n$. (When $\epsilon = 1$, the above inequality is always true.) To establish the lemma, we need to prove the following two steps.

STEP 1. *There is an open set $U_0 \subset\subset U$ containing p such that for all $z \in U_0 \cap A$ we have $C_0 Dv + z \subset \Omega$.*

STEP 2. *After linearly changing the coordinate system, v is a unit vector in z_1-direction and $(1/4)C_0 D \times \pi(\{A \cap U_0 \mid |z_1| < (1/2)C_0\}) \subset \Omega$.*

Proof of Step 1. Fix an open set $U_0 \subset\subset U$ in $\mathbb{C}^n$. We may assume that $\Omega \cap U$ is Kobayashi complete. Choose C_0 such that $C_0 = \frac{1}{\|v\|} \min(d(U_0, \Omega \cap U^c), 1)$. Suppose that there exists $q \in A \cap U_0$ such that $C_0 Dv + q \not\subset \Omega$. Let $|z_0|$ be the minimum value in $\{|z| \mid zv + q \in \partial\Omega\}$; obviously $|z_0| < C_0$. We define a curve r from q to $z_0 v + q$ in $\mathbb{C}^n$ by $r(s) = sz_0 v + q$. We have $r(0) = q$, $r(1) = z_0 v + q$, and $r'(s) = z_0 v$. Then $\|sz_0 v\| < d(U_0, \Omega \cap U^c)$ implies $r(s) \in (A + C_0 Dv) \cap (\Omega \cap U)$ if $s < 1$. We choose a sequence s_j approaching 1. By the property of the Kobayashi pseudodistance, inequality (1) and the inequality $d_v(r(s)) \geq (1 - s)(|z_0|)$ we have

$$\begin{aligned} k_\Omega(q, r(s_j)) &\leq \int_0^{s_j} K(r(s), r'(s))ds \leq |z_0| \int_0^{s_j} K(r(s), v)ds \\ &\leq |z_0| \int_0^{s_j} \frac{C_\epsilon}{d_v(r(s))^\epsilon} ds \leq \frac{|z_0|}{|z_0|^\epsilon} \int_0^{s_j} \frac{C_\epsilon}{(1-s)^\epsilon} ds \leq M. \end{aligned}$$

Now $r(s_j)$ is approaching $z_0 v + q \in \partial\Omega$ as j goes to infinity, contradicting the Kobayashi completeness of Ω near p. So we have $C_0 Dv + z \subset \Omega$ for all $z \in U_0 \cap A$. □

Proof of Step 2. We choose coordinates $(z_1, \ldots, z_n)$, such that v is a unit vector in z_1 -direction. By Step 1 we have, for fixed $z = (z_1, \ldots, z_n) \in U_0 \cap A$ and $|w| < C_0$, $wv + z \in \Omega$, that is, $wv + z = (w + z_1, z_2, .., z_n) \in \Omega$. If we choose any $(z_1, z') \in (1/4)C_0 D \times \pi(\{A \cap U_0 \mid |z_1| < (1/2)C_0\})$, then there is a point $(z_1^0, z') \in A \cap U_0$ with $|z_1^0| < (1/2)C_0$. This implies that $|z_1 - z_1^0| \leq C_0$. By the above argument, $(z_1, z') = (z_1 - z_1^0)v + (z_1^0, z') \in \Omega$, so $(1/4)C_0 D \times \pi(\{A \cap U_0 \mid |z_1| < (1/2)C_0\}) \subset \Omega$. □

With Steps 1 and 2 the proof of Lemma 3 is complete. □

LEMMA 4. *If Ω has a fat subdomain A having mass at p in $\partial\Omega$ which is a product space and has smooth boundary near p, then Ω has non-compact $\overline{\partial}$-Neumann operator.*

Proof. We give a sketch of the proof, which is based on a standard argument [6]. We may assume that there is a coordinate system $(z_1, \ldots, z_n)$ on a neighborhood U of p such that $p = 0$ and $A = C_0 D \times W$, where W is an open set in $\mathbb{C}^{n-1}$. The set $(C_0/2)D \times W$ is still a fat subdomain having mass at p. We may also assume that there is a holomorphic sequence $\{f_j\}_{j=1}^{\infty}$ which lies in the unit ball of $L^2(\Omega)$ and has no subsequence that converges in $L^2((C_0/2)D \times W)$. Denote by $\chi(t)$ a smooth cut-off function that is identically 1 for $0 \le t \le (C_0/2)$ and identically 0 for $t \ge 2C_0/3$. Let $z' = (z_2, \ldots, z_n)$. Let α_j be $\overline{\partial}(\chi(|z_1|) f_j((z_1, z')))$, which is $\overline{\partial}$ closed on Ω, and let $g_j = \overline{\partial}^* N \alpha_j$. Suppose that N is a compact operator. By Lemma 2 we may assume that $\overline{\partial}^* N$ is also a compact operator. After passing to a subsequence, we may assume that $\{g_j\}_{j=1}^{\infty}$ converges in $L^2(\Omega)$. Let $h_j((z_1, z')) = \chi(|z_1|) f_j((z_1, z')) - g_j((z_1, z'))$. Then h_j is holomorphic. If $|z_1| \ge 2C_0/3$ on A, then $h_j = g_j$. Using the mean value property of holomorphic functions, we see that h_j converges in $L^2(A)$, and hence that χf_j converges on $L^2((C_0/2)D \times W)$. This is a contradiction. □

Proof of Theorem 3. Lemma 3 implies that Ω has a fat subdomain with mass at p that is a product domain. By Lemma 4, Ω has a non-compact $\overline{\partial}$-Neumann operator. □

4. Application

Theorem 1 and Theorem 4 below are both applications of Lemma 3. We first prove Theorem 1.

Proof of Theorem 1. By the hypotheses there is a sufficiently small open neighborhood U of p in $\mathbb{C}^n$ and a constant C_ϵ such that for $z \in U \cap \Omega$

$$K(z, v) \le \frac{C_\epsilon}{d_v(z)^\epsilon}.$$

We may assume that $U \cap \Omega$ is Kobayashi complete. The hypothesis of Theorem 1 gives inequality (1) in the proof of Lemma 3 with the set A replaced by the set $U \cap \Omega$. We may assume that v is a unit vector in z_1 -direction.

Now we can follow directly the argument of Lemma 3. There is a constant C_0 and an open neighborhood U_0 of p in $\mathbb{C}^n$ such that $(1/4)C_0 D \times \pi(\{\Omega \cap U_0 \mid |z_1| < (1/2)C_0\}) \subset \Omega$. Hence, if we suppose that for all $z = (z_1, \ldots, z_n) \in U_0$, we have $|z_1| \le (1/4)C_0$, then we get $\Omega \cap U_0 = U_0 \cap ((1/4)C_0 D \times \pi(\{\Omega \cap U_0 \mid |z_1| < (1/2)C_0\}))$. Near $p = 0$, Ω is locally a product space. □

THEOREM 4. *Suppose that Ω is K-complete near p in $\partial\Omega$ and a bounded domain. The following are equivalent.*

(1) *For z close enough to p there exist $\epsilon < 1$ and a constant C_ϵ such that*

$$K(z, v) \le \frac{C_\epsilon}{d_v(z)^\epsilon}.$$

(2) *For z close enough to p and for all $\epsilon < 1$ there exists a constant C_ϵ such that*

$$K(z, v) \leq \frac{C_\epsilon}{d_v(z)^\epsilon}.$$

(3) *There is a small open neighborhood U of p in $\mathbb{C}^n$ such that there exists M so that for all $z \in U \cap \Omega$*

$$K(z, v) \leq M.$$

(4) *Ω is locally a product space as follows. Let $p = 0$. After linearly changing the coordinate system, there is a constant C_0, an open neighborhood U of 0 in $\mathbb{C}^n$, and an open set W in $\mathbb{C}^{n-1}$ such that $U \cap (C_0 D \times W) = \Omega \cap U$.*

Proof. By Theorem 1, (1) implies (4). We next show that (4) implies (3). Suppose that (4) is true. Then there is a constant C and an open neighborhood U_0 of 0 such that if $z = (z_1, z') \in U_0 \cap ((C/2)D \times W)$, then $d_v(z) \geq C/2$. So we get $K(z, v) \leq 1/d_v(z) \leq 2/C$. Hence (3) is satisfied.

Now we show that (3) implies (2). The domain Ω is bounded, so there is a M_0 such that $d_v((z_1, z')) \leq M_0$, $1 \leq M_0^\epsilon (1/d_v(z)^\epsilon)$ for all ϵ. This implies that $K(z, v) \leq M M_0^\epsilon (1/d_v(z)^\epsilon)$. Let $C_\epsilon = M M_0^\epsilon$. Then $K(z, v) \leq C_\epsilon / d_v(z)^\epsilon$.

Obviously (2) implies (1), so the proof of Theorem 4 is complete. □

One can ask whether property (3) in Theorem 4 is invariant under biholomorphism. Theorem 4 gives a negative answer because the product structure is not preserved under biholomorphism, as shown by the following example. The domain $\Omega = \{(z, w) : z \in D, |w| < R(z)\}$ can be mapped onto the unit dicylinder by some biholomorphism if $-\ln R(z)$ is harmonic in D [11]. This example is enough to show that the product structure is not invariant under biholomorphism.

We now introduce the localization principle of the Bergman Kernel in the case of a bounded pseudoconvex domain with smooth boundary in $\mathbb{C}^n$ (see [3], [10]). Let Ω be a bounded pseudoconvex domain with smooth boundary in $\mathbb{C}^n$, $z^0 \in \partial\Omega$. Then for any sufficiently small neighborhood U of z^0 for $z \in U' \cap \Omega$, where U' is a smaller neighborhood $U' \subset\subset U$, we have:

$$\frac{1}{c} B_{U \cap \Omega}(z, z) \leq B_\Omega(z, z) \leq B_{U \cap \Omega}(z, z).$$

Corollary. *If Ω is a smooth bounded pseudoconvex domain in $\mathbb{C}^n$ that is Kobayashi complete near p and there is a neighborhood U, a nonzero vector v, and a constant $M > 0$ such that for $q \in \Omega \cap U$,*

$$K(q, v) \leq M,$$

then, after linearly changing the coordinate system such that $p = 0$ and v is a unit vector in the z_1-direction, there are two constants C_1 and C_2 such that *for* $z = (z_1, z') \in \Omega \cap U_1, z' \in \mathbb{C}^{n-1}$,

$$B_{C_0D}(z_1, z_1) \times B_{\pi(U_0)}(z', z') < C_1 B_\Omega(z, z) < C_2 B_{C_0D}(z_1, z_1) \times B_{\pi(U_0)}(z', z'),$$

where U_0 *and* $U_1 \subset U_0$ *are subneighborhoods of* U *containing* p *and* C_0 *is the constant from Theorem 4.*

Geometrically, $\pi(U_0)$ is the intersection of the hypersurface supported by v at p and $\Omega \cap U_0$. The point of the Corollary is that the rate of blow-up of the Bergman kernel $B_\Omega(z, z)$ as $z \to p$ is comparable to the rate of blow-up of the Bergman kernel $B_{\pi(U_0)}$ of a lower-dimensional domain. Theorem 4 and the argument above imply this Corollary.

Proof of Theorem 2. A proof of the implication (2) $\Longleftrightarrow$ (3) can be found in [5, Therem 1.1.]. We now show that (2) implies (1). Suppose that Ω does not have Property -K near some point p in $\partial\Omega$. The convexity of Ω implies Kobayashi completeness. By Theorem 3 (when the domain is convex, smoothness is not necessary), Ω has a non-compact $\overline{\partial}$-Neumann operator.

To prove that (1) implies (3), we assume that there is an affine complex disk on the boundary of Ω. After linearly changing the coordinate system, we may assume that $0 \in \partial\Omega$. Let $A = \{z_1 \in \mathbb{C}^1 \mid |z_1| < 1\} \times \Omega_2 \subset \Omega$, where $\Omega_2 = (1/2)\{z^{'} \in \mathbb{C}^{n-1} \mid (0, z') \in \Omega\}$. By part (2) of Lemma 1, the convexity of Ω implies that A is a fat subdomain having mass at 0 of Ω. A complete proof of the existence of a fat subdomain, which is a product space after linearly changing the coordinate system, is contained in [5]. Set $v = (1, 0 \ldots, 0)$. For all $z \in A \cap \{(z_1, z') \mid |z_1| < (1/4)\}$, $d_v(z)$ is uniformly bounded from above and below. There exist M_1, M_2 such that $M_1 < 1/d_v(z) < M_2$. This implies that $K(z, v) \leq 1/d_v(z) \leq M_2(M_1^{-\epsilon})1/d_v(z)^\epsilon$. Thus Ω does not have Property -K. □

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