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SOME PROPERTIES OF MEAN CURVATURE VECTORS FOR CODIMENSION-ONE FOLIATIONS

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ABSTRACT. Given a codimension-one foliation $\mathcal{F}$ of a closed manifold M and a vector field X on M, we show that if X is transverse to $\mathcal{F}$, then there are many functions f on M so that fX is the mean curvature vector of $\mathcal{F}$ with respect to some Riemannian metric on M. Further we give a necessary and sufficient condition for X to become the mean curvature vector of $\mathcal{F}$ with respect to some Riemannian metric on M.

1. Introduction

Let $\mathcal{F}$ be a foliation of any codimension of a compact manifold M and X be a vector field on M. Recently, P. Schweitzer and P. Walczak [9] provided some necessary and sufficient conditions for X to become the mean curvature vector of $\mathcal{F}$ with respect to some Riemannian metric on M. In this paper, we focus on codimension-one foliations and study related topics. Given a codimension-one foliation $\mathcal{F}$ of a closed manifold M and a vector field X on M, we first show that if X is transverse to $\mathcal{F}$, then there are many functions f on M so that fX is the mean curvature vector of $\mathcal{F}$ with respect to some Riemannian metric on M. Here we can take f such that $\mathrm{supp}(f) = M$, where $\mathrm{supp}(f)$ is the closure in M of the set $\{x \in M | f(x) \neq 0\}$. Further we give a necessary and sufficient condition for X to become the mean curvature vector of $\mathcal{F}$ with respect to some Riemannian metric on M. This condition is similar to the conditions given in the author's papers [4], [5], [6].

In Section 2 we shall give some definitions and preliminaries and state our results. We shall prove the results in Section 3. An example and some remarks are given in Section 4.

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2. Preliminaries and results

In this paper, we work in the C^∞-category. In what follows, we always assume that foliations are of codimension-one and transversely oriented, and that the ambient manifolds are closed, connected, oriented and of dimension $n+1 \geq 2$, unless otherwise stated (see [1], [11] for generalities on foliations).

Let g be a Riemannian metric of M. Then there is a unique unit vector field orthogonal to $\mathcal{F}$ whose direction coincides with the given transverse orientation. We denote this vector field by N. Orientations of M and $\mathcal{F}$ are related as follows: Let $\{X_1, X_2, \dots, X_n\}$ be an oriented local frame of $T\mathcal{F}$. Then the orientation of M coincides with the one given by $\{N, X_1, X_2, \dots, X_n\}$.

We denote the mean curvature of a leaf L at x with respect to g and N by $h_g(x)$, that is,

$$h_g = \sum_{i=1}^{n} \langle \nabla_{E_i} E_i, N \rangle,$$

where $\langle , \rangle$ means $g(,)$, ∇ is the Riemannian connection of (M, g) and $\{E_1, E_2, \dots, E_n\}$ is an oriented local orthonormal frame of $T\mathcal{F}$. The vector field $H_g = h_g N$ is called the *mean curvature vector* of $\mathcal{F}$ with respect to g. A smooth function f on M is called *admissible* if $f = -h_g$ for some Riemannian metric g (cf. [4], [12]). We also call a vector field X on M *admissible* if $X = H_g$ for some Riemannian metric g. First we shall show that there are many admissible vector fields for any codimension-one foliations of closed manifolds.

THEOREM 1. *For any vector field Z transverse to a codimension-one foliation $\mathcal{F}$ of a closed oriented manifold M, there is a smooth function f on M with* $\operatorname{supp}(f) = M$ *so that fZ is admissible.*

A characterization of admissible functions is given in [6] (see also [4], [5], [12]). We shall give a similar but rather complicated characterization of admissible vector fields.

Define an n-form $\chi_{\mathcal{F}}$ on M by

$$\chi_{\mathcal{F}}(V_1, \dots, V_n) = \det(\langle E_i, V_j \rangle)_{i,j=1,\dots,n} \text{ for } V_j \in TM.$$

The restriction $\chi_{\mathcal{F}}|L$ is the volume element of $(L, L|g)$ for $L \in \mathcal{F}$.

PROPOSITION (Rummler [7]). $d\chi_{\mathcal{F}} = -h_g dV(M, g) = \operatorname{div}_g(N) dV(M, g)$, *where $dV(M, g)$ is the volume element of (M, g) and* $\operatorname{div}_g(N)$ *is the divergence of N with respect to g, that is,* $\operatorname{div}_g(N) = \sum_{i=1}^{n} \langle \nabla_{E_i} N, E_i \rangle$.

Now recall the set-up introduced by Sullivan [10]. Let D_p be the space of p-currents, and D^p be the space of differential p-forms on M with the C^∞ topology. It is well known that D^p is the dual space of D_p (cf. Schwartz [8]). Let $x \in M$ and $\{e_1, \dots, e_n\}$ be an oriented basis of $T_x\mathcal{F}$. We define the Dirac current $\delta_{e_1\wedge\cdots\wedge e_n}$ by

$$\delta_{e_1\wedge\cdots\wedge e_n}(\phi) = \phi_x(e_1 \wedge \cdots \wedge e_n) \text{ for } \phi \in D^n,$$

and the set $C_{\mathcal{F}}$ to be the closed convex cone in D_n spanned by the Dirac currents $\delta_{e_1\wedge\cdots\wedge e_n}$ for all oriented basis $\{e_1, \ldots, e_n\}$ of $T_x\mathcal{F}$ and $x \in M$. We denote a base of $C_{\mathcal{F}}$ by $\mathbf{C}$, which is an inverse image $L^{-1}(1)$ of a suitable continuous linear functional $L : D_n \to \mathbf{R}$. It is known that the base $\mathbf{C}$ is compact if L is suitably chosen (see Sullivan [10]). In the following, we assume that $\mathbf{C}$ is compact.

Let X be a vector field on M. Define the closed linear subspace $P(X)$ of D_n generated by all the Dirac currents $\delta_{X(x)\wedge v_1\wedge\cdots\wedge v_{n-1}}$ with $v_1, \ldots, v_{n-1} \in T_x\mathcal{F}$ and $x \in M$ (see [9] for more details), where $\delta_{X(x)\wedge v_1\wedge\cdots\wedge v_{n-1}}$ is defined by

$$\delta_{X(x)\wedge v_1\wedge\cdots\wedge v_{n-1}}(\phi) = \phi_x(X(x) \wedge v_1 \wedge \cdots \wedge v_{n-1}) \text{ for } \phi \in D^n.$$

Let $\partial : D_{n+1} \to D_n$ be the boundary operator and set $B = \partial(D_{n+1})$. Within this setting, we characterize admissible vector fields on M.

Theorem 2. *For a vector field X on M, the following two conditions are equivalent.*

(1) *X is admissible.*
(2) *There are a volume element dV, a non-vanishing vector field Z transverse to $\mathcal{F}$ whose direction coincides with the given transverse orientation of $\mathcal{F}$, a smooth function f on M, and a neighborhood U of $0 \in D_n$ such that*
  (i) $X = -fZ$,
  (ii) $\int_M f dV = 0$,
  (iii) $\int_c f dV = 0$ *for all* $c \in \partial^{-1}(P(X) \cap B)$,
  (iv) $\inf\{\int_c f dV \mid c \in \partial^{-1}((\mathbf{C} + P(X) + U) \cap B)\} > 0$.

Note that conditions (ii) and (iv) in this theorem mean that the function f is admissible. In Section 4, by giving a simple example, we shall show that the condition $X = -fZ$ with f being admissible is not sufficient for X to be admissible.

3. Proof of the theorems

To prove Theorem 1, we need some lemmas. As the first two lemmas are easy to prove, we omit the proofs.

Lemma 1. *Let M be a closed manifold and N be a non-vanishing vector field on M. There is a smooth function φ on M such that* $\operatorname{supp} N(\varphi) = M$.

Lemma 2. *Let M, N and φ be as in Lemma 1. For any smooth function h on M there is a positive constant $\alpha > 0$ so that* $\operatorname{supp}(h - \alpha N(\varphi)) = M$.

The following lemma is proved in [3, Lemma 3], where the equality $H' = e^{-2\psi}H$ in (ii) should be replaced by $H' = e^{-\psi}H$.

LEMMA 3. *Let $\mathcal{F}$ be a codimension-one foliation of a Riemannian manifold (M, g), N be the unit vector field orthogonal to $\mathcal{F}$ defined as in Section 2, and h be the mean curvature function of $\mathcal{F}$ with respect to g.*

(i) *If $\bar{g} = e^{2\psi}g$, then $\bar{h} = e^{-\psi}(h - N(\psi))$, where $\bar{h}$ is the mean curvature function of $\mathcal{F}$ with respect to $\bar{g}$ and the unit vector field $\bar{N}$ orthogonal to $\mathcal{F}$ with respect to $\bar{g}$ defined as in Section 2.*
(ii) *If $\bar{g}|T\mathcal{F} \otimes TM = g|T\mathcal{F} \otimes TM$ and $\bar{g}(U, V) = e^{2\psi}g(U, V)$ for U and V orthogonal to $\mathcal{F}$, then $\bar{h} = e^{-\psi}h$.*
(iii) *Let $Z = \varphi N + F$ be a vector field on M with $\varphi > 0$ and $F \in \Gamma(T\mathcal{F})$. Define a Riemannian metric $\bar{g}$ on M as follows: $\bar{g} = g$ on $T\mathcal{F}$, and Z is the unit vector field orthogonal to $\mathcal{F}$ with respect to $\bar{g}$. Then we have $\bar{h} = \varphi h + F(\log \varphi) - \mathrm{div}_g(F)$.*

Proof of Theorem 1. We may assume that the direction of Z coincides with the transverse orientation of $\mathcal{F}$. First choose an arbitrary Riemannian metric g of M. Let N be the unit vector field orthogonal to $\mathcal{F}$ defined as in Section 2. Then $Z = \rho N + F$ for some positive smooth function $\rho > 0$ and $F \in \Gamma(\mathcal{F})$. Define a new Riemannian metric $\bar{g}$ as in Lemma 3 (iii). Then it follows that Z is the unit vector field orthogonal to $\mathcal{F}$ with respect to $\bar{g}$ and $\bar{h} = \rho h + F(\log \rho) - \mathrm{div}_g(F)$. By Lemma 1 and Lemma 2, there is a smooth function φ and a positive constant $\alpha > 0$ so that $\mathrm{supp}(\bar{h} - Z(\alpha\varphi)) = M$. Define a Riemannian metric g' as in Lemma 3 (i), that is, $g' = e^{2\alpha\varphi}g$. Then it follows that $h' = e^{-\alpha\varphi}(\bar{h} - Z(\alpha\varphi))$. As $\mathrm{supp}(h') = \mathrm{supp}(\bar{h} - Z(\alpha\varphi)) = M$ and Z is orthogonal to $\mathcal{F}$, $e^{-\alpha\varphi}h'Z$ is the mean curvature vector of $\mathcal{F}$ with respect to g'. This completes the proof. □

To prove Theorem 2, we follow the proof given in [4] with some modifications motivated by [9] (see also Sullivan [10]). To do this we need a Hahn-Banach theorem of the following form (cf. [2]):

THEOREM OF HAHN-BANACH. *Let V be a Fréchet space, W be a closed subspace of V, and C be a compact convex cone at the origin $0 \in V$. Let $\rho : W \to \mathbf{R}$ be a continuous linear functional of W with $\rho(v) > 0$ for $v \in C \cap W \setminus \{0\}$. Then there is a continuous linear extension $\eta : V \to \mathbf{R}$ of ρ so that $\eta(v) > 0$ for $v \in C \setminus \{0\}$.*

Proof of Theorem 2. (1)⇒(2): Assume that there is a Riemannian metric g of M so that X is the mean curvature vector of $\mathcal{F}$. Let N be the unit vector field orthogonal to $\mathcal{F}$, and $\chi_{\mathcal{F}}$ be the n-form defined in Section 2. If $\mathbf{C}$ is chosen to be $L^{-1}(1)$ of a continuous linear functional $L : D_n \to \mathbf{R}$ with $\mathbf{C}$ being compact, as $\chi_{\mathcal{F}} : D_n \to \mathbf{R}$ is also continuous, there is a positive constant $\varepsilon > 0$ such that $\chi_{\mathcal{F}} \geq \varepsilon > 0$ on $\mathbf{C}$. We choose $U = \chi_{\mathcal{F}}^{-1}(-\varepsilon/2, \varepsilon/2)$ as a neighborhood of $0 \in D_n$. Set $dV = dV(M, g)$, $Z = N$, and $f = \mathrm{div}_g(N)$. We show that dV, Z, f, and U satisfy conditions (i)–(iv) in (2). As $X = h_g N = -\mathrm{div}_g(N)N = -fZ$, this shows that condition (i) is satisfied. As M is closed and oriented, it follows that

$$\int_M f dV = \int_M \mathrm{div}_g(N) dV(M, g) = 0,$$

which implies that condition (ii) is satisfied. For $c \in \partial^{-1}(P(X) \cap B)$, as $d\chi_{\mathcal{F}} = f dV(M, g)$ by the Proposition, we have

$$\int_c f dV = \int_c f dV(M, g) = \int_c d\chi_{\mathcal{F}} = \int_{\partial c} \chi_{\mathcal{F}}.$$

Since $\chi_{\mathcal{F}}(X, V_1, \ldots, V_{n-1}) = \chi_{\mathcal{F}}(-fN, V_1, \ldots, V_{n-1}) = 0$ for any $V_1, \ldots, V_{n-1} \in T\mathcal{F}$, it follows that $\int_c f dV = 0$, which shows that condition (iii) is satisfied. For $c \in \partial^{-1}((\mathbf{C}+P(X)+U)\cap B)$ with $\partial c = v+z+u$ ($v \in \mathbf{C}$, $z \in P(X)$, $u \in U$), by the same argument as above, we have

$$\int_c f dV = \int_{\partial c} \chi_{\mathcal{F}} = \int_v \chi_{\mathcal{F}} + \int_z \chi_{\mathcal{F}} + \int_u \chi_{\mathcal{F}} = \int_v \chi_{\mathcal{F}} + \int_u \chi_{\mathcal{F}} > \varepsilon/2 > 0,$$

because $\chi_{\mathcal{F}} = 0$ on $P(X)$, $\chi_{\mathcal{F}} \geq \varepsilon$ on the compact set $\mathbf{C}$, and $|\int_u \chi_{\mathcal{F}}| < \varepsilon/2$. This shows that condition (iv) is satisfied.

(2)⇒(1): Let dV, Z, f, U satisfy the conditions of (2). Condition (ii) implies that $f dV = d\phi$ for some $\phi \in D^n$. By the duality of D_p and D^p due to Schwartz, we can regard ϕ as a continuous linear functional $k : D_n \to \mathbf{R}$. Note that the restriction of k on $B = \partial(D_{n+1})$ is independent of the choice of ϕ. By condition (iii), we may assume that $k|(P(X) \cap B) = 0$. Extend $k : B \to \mathbf{R}$ to a function $\tilde{k}$ defined on the subspace $P(X) + B$ by defining $\tilde{k}(z+b) = k(b)$ for $z \in P(X)$ and $b \in B$. As $k|(P(X)\cap B) = 0$, this extension is well-defined and is continuous on $P(X) + B$. Note that, by condition (iv), $\tilde{k} > 0$ on $C_{\mathcal{F}} \cap (P(X)+B) \setminus \{0\}$. Extend $\tilde{k}$ continuously to a function κ defined on the closed subspace $W = \overline{P(X) + B}$. We have to show that $\kappa(v) > 0$ for $v \in C_{\mathcal{F}} \cap W \setminus \{0\}$ in order to apply the Hahn-Banach Theorem quoted above to the case $V = D_n$, $W = \overline{P(X) + B}$, $C = C_{\mathcal{F}}$ and $\rho = \kappa$. For $v \in C_{\mathcal{F}} \cap W \setminus \{0\}$, as $\mathbf{C}$ is a base of $C_{\mathcal{F}}$, there is a positive number $a > 0$ so that $av \in \mathbf{C}$. As $\kappa(v) = \kappa(av)/a$ and $a > 0$, it is sufficient to show that $\kappa(v) > 0$ for $v \in \mathbf{C} \cap W$.

Take $v \in \mathbf{C} \cap W$ and a net $\{w_\lambda : \lambda \in \Lambda\}$ converging to v (cf. [2]). As $W = \overline{P(X) + B}$, we can take $w_\lambda = z_\lambda + b_\lambda$ with $z_\lambda \in P(X)$ and $b_\lambda \in B$. Set $u_\lambda = v - z_\lambda - b_\lambda$. Then, as $\{w_\lambda\}$ converges to v, u_λ converges to 0. Since U is a neighborhood of $0 \in D_n$, there is a $\lambda_0 \in \Lambda$ so that $v_\lambda \in U$ for all $\lambda \geq \lambda_0$. Thus $b_\lambda = v - z_\lambda - u_\lambda \in (\mathbf{C} + P(X) + U) \cap B$. By assumption, it follows that $\kappa(b_\lambda) \geq \varepsilon > 0$. Note that $u_\lambda \in W = \mathrm{Dom}(\kappa)$ because $v \in W$ and

$z_\lambda + b_\lambda \in P(X) + B \subset W$ for $\lambda \geq \lambda_0$. It follows that

$$\begin{aligned}\kappa(v) &= \kappa(z_\lambda + b_\lambda + u_\lambda) \\ &= \kappa(z_\lambda) + \kappa(b_\lambda) + \kappa(u_\lambda) \\ &= \kappa(b_\lambda) + \kappa(u_\lambda) \text{ for all } \lambda \geq \lambda_0.\end{aligned}$$

As $\{u_\lambda\}$ converges to 0, $\{\kappa(u_\lambda)\}$ converges to 0. Thus we have $\kappa(v) > 0$, since $\kappa(b_\lambda) \geq \varepsilon > 0$.

By applying the Hahn-Banach Theorem in this situation, we obtain a continuous linear map $\eta : D_n \to \mathbf{R}$ with $\eta|_B = k|_B$, $\eta(v) > 0$ for $v \in C_{\mathcal{F}} \setminus \{0\}$, and $\eta(z) = 0$ for $z \in P(X)$. By the duality due to Schwartz, we have an n-form χ on M so that $\chi > 0$ on $\mathcal{F}$, $d\chi = f dV$, and $\iota_X \chi = 0$, where ι_X is the interior product.

Now define a Riemannian metric g as follows: On each leaf $L \in \mathcal{F}$, $\chi|_L$ is the volume form of $(L, g|_L)$, $\ker \chi$ is orthogonal to $\mathcal{F}$, and on $\ker \chi$ the metric is determined by requiring $dV(M, g) = dV$, where dV is the n-form in condition (2). Choose the unit vector field N orthogonal to $\mathcal{F}$ as in Section 2. As $\iota_X \chi = 0$ and both $\ker \chi$ and N are orthogonal to $\mathcal{F}$, if $X(x) \neq 0$, then $X(x)$ and $N(x)$ are linearly dependent. Thus the directions of $Z(x)$ and $N(x)$ coincide on the set $\{x \in M \mid X(x) \neq 0\}$, and, consequently, on the set $\operatorname{supp}(X)$. Since Z and N are defined globally on M, there is a smooth function α on M so that $N = e^{-2\alpha} Z$ on $\operatorname{supp}(X)$. Thus, we have $fN = e^{-2\alpha} fZ$ on M. By the relation $f dV = d\chi = -h_g dV(M, g)$ and condition (i), it follows that $H_g = h_g N = -fN = -e^{-2\alpha} fZ = e^{-2\alpha} X$ on M. We deform this metric g into $\bar{g}$ as follows: $g|T\mathcal{F} \otimes TM = \bar{g}|T\mathcal{F} \otimes TM$ and $g(U, V) = e^{2\alpha} \bar{g}(U, V)$ for U and V orthogonal to $\mathcal{F}$. By Lemma 3 (ii), it follows that $H_g = e^{-2\alpha} H_{\bar{g}}$, where H_g (resp. $H_{\bar{g}}$) is the mean curvature vector of $\mathcal{F}$ with respect to the metric g (resp. $\bar{g}$). With respect to this metric $\bar{g}$ we have $X = e^{2\alpha} H_g = e^{2\alpha} e^{-2\alpha} H_{\bar{g}} = H_{\bar{g}}$, which completes the proof. □

Remark. Note that if the subspace $P(X) + B$ is already closed, it is easy to see that condition (iv) can be weakened to the following condition, which does not need any assumption on the existence of U:

$$\int_c f dV > 0 \text{ for all } c \in \partial^{-1}((\mathbf{C} + P(X)) \cap B).$$

However, the closedness of $P(X) + B$ seems to be not so easy to show.

4. Example and a concluding remark

In this section, we give a simple example which shows that the condition $X = -fZ$ with f being admissible is not sufficient for X to be admissible.

Let T^2 be the two dimensional torus with the canonical coordinate $\{x, y\}$. Define a foliation $\mathcal{F}$ by $\{S^1 \times \{y\} \mid y \in S^1\}$. As this foliation is taut, any smooth function f on T^2 with $f(x) \cdot f(y) < 0$ for some $x, y \in T^2$ is admissible.

Take the vector filed ∂_y as a transverse vector field Z to $\mathcal{F}$, choose a smooth function f which is positive except in a small neighborhood U of a fixed point $(x_0, y_0) \in T^2$, where $f(x_0, y_0) < 0$, and set $X = -fZ$. We show that X cannot be the mean curvature vector with respect to any Riemannian metric of T^2.

Assume that there is a Riemannian metric g of T^2 so that the mean curvature vector is $X = -fZ$. Take the unit normal vector field N to $\mathcal{F}$ such that $\langle N, Z\rangle > 0$. Then $\mathrm{div}_g(N) = -\langle X, N\rangle$. Take a compact domain $D = [a, b] \times S^1 \subset T^2$ with $D \cap U = \emptyset$. Then we have $\int_D \mathrm{div}_g(N) = \int_{\partial D} \langle \nu, N\rangle$, where ν is the unit vector field orthogonal to ∂D and is pointing outwards to D on ∂D. As ν is tangent to $\mathcal{F}$, $\langle \nu, N\rangle = 0$, which means that $\int_D \mathrm{div}_g(N) = 0$. On the other hand, since $\mathrm{div}_g(N) = -\langle X, N\rangle = f\langle Z, N\rangle > 0$ on D, we have $\int_D \mathrm{div}_g(N) > 0$. This is a contradiction.

As is explained in [9], if X has a closed orbit and the holonomy is expanding along the orbit, then X cannot be admissible, because the area of a piece of leaves decreases under the mean curvature flow. Note that this property of X is independent of the given codimension-one foliations. In this case, $(P(X)+B)\cap \mathbf{C}$ might not be empty even though $P(X)\cap \mathbf{C} = \emptyset$ and $B\cap \mathbf{C} = \emptyset$. Thus condition (iv) in Theorem 2 seems to be difficult to check. Further interesting and complicated examples are discussed in [9]. It seems to be of some interest to study the geometric conditions under which conditions (iii) and (iv) in Theorem 2 are satisfied.

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