## CORRECTION TO MY PAPER "FIXED-POINT THEOREMS FOR COMPACT CONVEX SETS"<sup>1</sup>

BY

MAHLON MARSH DAY<sup>2</sup>

Mr. Neil Rickert has pointed out to me that the remark on page 589 claiming validity for the converse of Theorem 3 stumbles on the fact that the adjoint mapping of the left regular representation of S over C(S) need not be  $(w^*$ -) continuous unless S is discrete or compact. (For example, over the real additive group G, a  $\mu$  in  $C(G)^*$  and an x in C(G) can easily be constructed such that  $\mu(x) = [l_0^*\mu](x) = 1$  but  $[l_g^*\mu](x) = 0$  for all  $g \neq 0$ ). Because the proof of Theorem 1 anchors on one element of K, this need not be catastrophic. The converse as claimed is false, but a related condition is equivalent to leftamenability of C(S). Recall that A(K) is the semigroup of affine, continuous mappings of the compact convex set K into itself.

DEFINITION. A homomorphism  $\tau$  of a topological semigroup S into A(K) is called *slightly continuous* if and only if there is at least one y in K such that  $[\tau s](y)$  is a continuous function from S into K.

The remark at the bottom of (c), p. 589, should then be replaced by

THEOREM 4. If S is a topological semigroup, there is a left-invariant mean on C(S) if and only if for each compact convex set K in each locally convex space X and for each slightly continuous homomorphism of X into A(K) there is in K a common fixed point p of all the transformations in the semigroup  $\Sigma = \tau(S)$ .

To sketch the proof: If  $\gamma$  is a left-invariant mean on C(S), let  $\delta$  be any extension of  $\gamma$  which is a mean on m(S). (For existence of  $\delta$  see reference [4, Theorem 1, p. 20].) Define V from  $m(\Sigma)$  to m(S) by: for all x,  $[Vx](s) = x(\tau s)$  for all s. If  $\mu = V^* \delta$ , then  $\mu$  is a mean on  $m(\Sigma)$ . If y is chosen in K to fit the condition imposed by slight continuity of  $\tau$ , the left-invariance of  $\gamma$  on C(S) implies that  $h(j\mu) = j\mu$  for every h in  $\Sigma$ ; that is, that  $j\mu = jV^*\delta$  is the desired fixed point in K.

The proof for the converse requires only the following lemma, which can be proved by observing that every evaluation functional  $\eta_{s_0}$ , defined by  $\eta_{s_0}(x) = x(s_0)$  for all x in C(S), is a mean on C(S) for which  $l_s^* \eta_{s_0}$  is a continuous function from S into  $C(S)^*$  with its  $w^*$ -topology.

Lemma. The adjoint of the left regular representation over C(S) is a slightly continuous representation of S in A(M), when M is the (w\*-compact, convex) set of means on C(S) with the w\*-topology.

Received May 7, 1964.

<sup>&</sup>lt;sup>1</sup> Illinois Journal of Mathematics, vol. 5 (1960), pp. 585-589.

 $<sup>^{2}</sup>$  This correction was written while the author was partially supported by a grant from the National Science Foundation.