THE TODD CHARACTER AND THE INTEGRALITY THEOREM FOR THE CHERN CHARACTER

BY
LARRY SMITH

In [5] the authors introduced a homomorphism called the Todd character, relating complex bordism homology theory to rational homology. Specifically, the Todd character is a family of homomorphisms

$$th : MU_n(X) \longrightarrow \oplus_{k=0}^{[n/2]} H_{n-2k}(X; Q),$$

the component morphisms being denoted by

$$th_{n-2k} : MU_n(X) \longrightarrow H_{n-2k}(X; Q).$$

By analogy with the Chern character we may seek an integrality theorem for the Todd character. This turns out to be a comparatively easy task given a few elementary facts concerning the classical Todd polynomials. The precise result that we will establish is:

INTEGRALITY THEOREM FOR THE TODD CHARACTER. *Let X be a cw-complex and $\alpha \in MU_{2n}(X)$. Then*

$$\mu_{n-k} th_{2k}(\alpha) \in H_{2k}(X; Z)/\text{torsion} \subset H_{2k}(X; Q)$$

where μ_t is the integer defined by

$$\mu_t = \prod\nolimits_{\text{primes}} p^{[t/p-1]}.$$

(A similar result holds for odd dimensional classes.)

In [5] and [6] several applications of the Todd character were made and it was often remarked that the Todd character and the Chern character are closely related. We will display one such relation in the present note by deriving the integrality theorem of J. F. Adams [1] for the Chern character from the corresponding result for the Todd character and elementary properties of the complex bordism homology and cohomology theories. Our treatment of the integrality theorem for the Chern character seems closely related to the special case treated by E. Dyer in [7].

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1. Definition and elementary properties of the Todd character

Let us begin by recalling the definition of the Todd character

$$th : MU_{**}(\quad) \longrightarrow H_{**}(\quad; Q)$$

as given in [5].

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Notations. We shall have occasion to deal with both Z and Z_2 graded modules. Let us therefore agree that for any Z graded module M_* that M_{**} denotes the Z_2 graded module given by weakening the grading, i.e.,

$$(M_*)_0 = \oplus_{i=-\infty}^{\infty} M_{2i} = (M_*)_{\text{even}}, \qquad (M_*)_1 = \oplus_{j=-\infty}^{\infty} M_{2j+1} = (M_*)_{\text{odd}}.$$

A similar notation applies to ground rings. Note that a Z graded Λ_* module M_* becomes a Z_2 graded Λ_{**} module M_{**} in the obvious way.

To define the Todd character as in [5] we suppose given a finite complex X. Recall that a homology class $x \in H_*(X; Q)$ is uniquely determined by the mapping

$$\langle -, x\rangle : H^*(X; Q) \to Q$$

given by the Kronecker product (duality pairing). Suppose that $\alpha = [M, f] \in MU_*(X)$. Then $th(\alpha) \in H_{**}(X; Q)$ is uniquely determined by the formula

$$\langle w, th(\alpha)\rangle = \langle f^*(w) Td(M), [M]\rangle$$

where $w \in H^{**}(X; Q)$ is arbitrary, $Td(M) \in H^{**}(M; Q)$ denotes the total Todd class of M [8] and $[M]$ the fundamental homology class of M. The definition is now extended to arbitrary complexes by taking limits and to $MU_{**}(X)$ by additivity. If X is a point $*$ then

$$th : MU_{**}(*) \to H_{**}(*; Q)$$

may be identified with the classical Todd genus

$$Td : MU_{**} \to Z \subset Q.$$

For a homogenous element $\alpha = [M, f] \in MU_n(X)$ the Todd character $th(\alpha) \in H_{**}(X; Q)$ has components $th_m(\alpha) \in H_m(X; Q)$ and one readily finds

$$th_m(\alpha) = 0 \quad \text{for } m > n, \qquad th_n(\alpha) = \mu(\alpha)$$

where $\mu : MU_*(X) \to H_*(X; Z)$ is the classical Thom homomorphism [4]. Therefore

$$th(\alpha) = \mu(\alpha) + \text{terms of lower degree.}$$

Thus in contrast to the Chern character, the *leading term* of the Todd character has a simple geometric interpretation. However, it is possible for the leading term of $th(\alpha)$ to vanish despite the fact that $th(\alpha) \neq 0$. Such examples may be constructed from [5, §8 Example 1] and are somewhat perplexing.

There is an alternate description of the Todd character in terms of spectra which clarifies some of its formal properties.

Notations. Let **MU** denote the Thom spectrum for the unitary group, **BU** the Bott spectrum for the unitary group, and $\mathbf{K}(A_*)$ the Eilenberg-MacLane spectrum for the graded abelian group A_*. There is the map of ring spectra [3].

$$\mu_C : \mathbf{MU} \to \mathbf{BU}$$

that expresses the K-theory orientability of complex vector bundles. The universal Chern character is induced by the canonical map of ring spectra

$$ch : \mathbf{BU} \to \mathbf{K}(\pi_*(BU) \otimes Q)$$

the composite of these is a map of ring spectra

$$th : MU \to \mathbf{K}(\pi_*(BU) \otimes Q)$$

that defines the transformations between generalized homology theories

$$MU_r(\quad) \to H_{r-2s}(\quad; Q)$$

that are the component morphisms of the Todd character.

Notations. Let us denote by $K^*(\quad)$ and $K_*(\quad)$ the Z_2 graded complex K cohomology and homology theories. There are the Z_2 graded derivatives of $H*(\quad; \mathbf{BU})$ and $H_*(\quad; \mathbf{BU})$ [11] respectively. We write

$$ch : K_*(\quad) \to H_{**}(\quad; Q), \qquad ch : K^*(\quad) \to H^{**}(\quad; Q)$$

for the homology and cohomology Chern characters.

The relation between the Todd character and the Chern character may be expressed as follows:

PROPOSITION 1.1. *Suppose that X and Y are Spanier-Whitehead dual in the n-sphere. Then the following diagram commutes*

$$\begin{tikzcd}
MU_k(X) \arrow[r, leftrightarrow, "D"'] \arrow[d] \arrow[dd, bend right=60, "th_m"'] & MU^{n-k-1}(Y) \arrow[d] \arrow[dd, bend left=60, "th_{n-m-1}"] \\
K_{[k]}(X) \arrow[r, leftrightarrow, "D"'] \arrow[d, "ch_m"] & K^{[n-k-1]}(Y) \arrow[d, "ch_{n-m-1}"] \\
H_m(X; Q) \arrow[r, leftrightarrow, "D"'] & H^{n-m-1}(Y; Q)
\end{tikzcd}$$

where D denotes the Spanier-Whitehead (Alexander) duality isomorphism and $[a]$ denotes the mod 2 residue of a.

LEMMA 1.2. *Let X be a cw-complex and $\alpha = [M, f] \in MU_*(X)$. Then for any $[N] \in MU_*$ we have $th([N]\alpha) = Td[N]th(\alpha) \in H_{**}(X; Q)$.*

Proof. This is immediate from the fact that th is induced by a map of ring spectra

$$th : \mathbf{MU} \to \mathbf{K}(\pi_*(BU) \otimes Q)$$

which on coefficients is the Todd genus. □

Notations. Let us denote by Z_{Td} the Z_2 graded MU_{**} module obtained via the morphism

$$th : MU_{**} \to Z, \qquad [M] \to Td[M].$$

PROPOSITION 1.3. *The Todd character induces a morphism*

$$\tilde{th} : Z_{Td} \otimes_{MU_{**}} MU_{**}(X) \to H_{**}(X; Q)$$

such that the diagram

$$\begin{tikzcd} Z_{Td} \otimes_{MU_{**}} MU_{**}(X) \arrow[d, "\tilde{\mu}_C"', "\cong"] \arrow[dr, "\tilde{th}"] & \\ K_*(X) \arrow[r, "ch"'] & H_{**}(X; Q) \end{tikzcd}$$

commutes.

Proof. The existence of $\tilde{th}$ follows from (1.2). The isomorphism $\tilde{\mu}_C$ was originally discovered by Conner and Floyd [3] (see [4; §9] for an alternate proof) while the commutativity of the diagram is immediate from the spectral definition of th. □

COROLLARY 1.4. *For any cw-complex X the induced map*

$$\tilde{th} : Q_{Td} \otimes_{MU_{**}} MU_{**}(X) \to H_{**}(X; Q)$$

is an isomorphism. □

2. Integrality theorems

For any space X note that $H_*(X; Z)$/torsion is a subgroup of maximal rank in $H_*(X; Q)$. We say that a class $x \in H_*(X; Q)$ is *integral* if it lies in the subgroup $H_*(X; Z)$/torsion of $H_*(X; Q)$. Since the subgroup $H_*(X; Q)$/torsion is of maximal rank in $H_*(X; Q)$ it follows that for any class $y \in H_*(X; Q)$ there is an integer a such that ay is integral. A natural question to ask is given $\alpha \in MU_r(X)$ what is the least integer λ such that $\lambda th_s(\alpha)$ is integral? The following result provides a universal answer to this question:

THEOREM 2.1. *Let X be a cw-complex and $\alpha \in MU_{2n}(X)$. Then*

$$\mu_{n-k} th_{2k}(\alpha) \in H_{2k}(X; Q)$$

is integral where μ_t is the integer defined by

$$\mu_t = \prod\nolimits_{\text{primes } p} p^{[t/p-1]}$$

where $[a/b]$ denotes the integral part of a/b. A similar result holds for odd dimensional classes.

As we shall see presently (2.1) implies the integrality theorem for the Chern character of J. F. Adams [1]. The proof of (2.1) requires the following result from algebraic geometry.

PROPOSITION 2.2. *The universal Todd polynomial*

$$Td_t(c_1, \cdots, c_t) \in H^{2t}(BU; Q)$$

can be written in a unique way as a polynomial with coprime integral coefficients divided by the positive integer μ_t *defined in* (2.1).

Proof. See for example [8; 1.7.3] or [2]. □

Proof of 2.1. It sufficies to consider the case where X is a finite complex. Clearly we need only show that any integral cohomology class w will take integral values on $\mu_{n-k} th_{2k}(\alpha)$. To this end note that

$$\mu_{n-k} Td_{n-k}(c_1, \cdots, c_{n-k}) \in H^{2n-2k}(BU; Z)$$

by (2.2). Thus writing $\alpha = [M, f]$ we have for any $w \in H^{2k}(X, Z)$ the formula

$$\langle w, \mu_{n-k} th_{2k}(\alpha) \rangle = \langle \mu_{n-k} Td_{n-k}(M) f^* w, [M] \rangle \in Z$$

since all classes on the right are integral. □

Let us recall now the theorem of Adams [1] on the integrality of the Chern character.

THEOREM (J. F. Adams). *Suppose that* X *is a* $2n - 1$ *connected cw-complex* and $\xi \in K(X)$. *Then*

$$\mu_t ch_{2n+2t}(\xi) \in H^{2n+2t}(X; Q)$$

is integral, where μ_t *is the integer defined in* (2.1).

In order to deduce this result from the integrality theorem for the Todd character we shall need the following technical result.

PROPOSITION 2.3. *Suppose that* X *is a* $2n - 1$ *connected finite cw-complex. Then*

$$\mu_{\mathbf{C}} : \tilde{M}U^{2n}(X) \to K(X)$$

is epic.

Proof. Suppose that $\xi \in K(X)$. According to Conner and Floyd [3; §9] there is an $\alpha \in \tilde{M}U^{**}(X)$ with $\mu_{\mathbf{C}}(\alpha) = \xi$. Now as X is $2n - 1$ connected it follows from work of Quillen [9] that $\tilde{M}U^*(X)$ is generated as an MU^*-module by classes of degree $\geq$ 2n. Thus we may write

$$\alpha = \sum [M_i]\alpha_i$$

where

$$\alpha_i \in \tilde{M}U^{2n_i}(X), \quad n_i \geq n, \qquad [M_i] \in MU^{-2m_i} = MU_{2m_i}.$$

We then have

$$\mu_{\mathbf{C}}(\alpha) = \sum \mu_{\mathbf{C}}([M_i]\alpha_i) = \sum Td[M_i]\mu_{\mathbf{C}}(\alpha_i).$$

Now define

$$\beta = \sum Td[M_i][\mathbf{C}P(1)]^{n-n_i}\alpha_i .$$

Note that

$$\deg\, [\mathbf{C}P(1)]^{n-n_i}\alpha_i = 2(n - n_i) + 2n_i = 2n.$$

Moreover

$$\mu_{\mathbf{C}}(\beta) = \sum Td[M_i]\mu_{\mathbf{C}}(\alpha_i) = \mu_{\mathbf{C}}(\alpha) = \xi,$$

and the result follows. □

Proof of the Integrality Theorem of Adams. Inspection of the universal example $BU(2n, \cdots, \infty)$ [1] shows that we may assume X has finite skeletons. Let us show that it suffices to consider the case where X has dimension $2n + 2t$. To this end denote by X^{2n+2t} the $2n + 2t$ dimensional skeleton of X. There is the cofibration

$$X^{2n+2t} \xrightarrow{i} X \xrightarrow{j} X_{2n+2t}$$

defining X_{2n+2t}. Observe that as X_{2n+2t} is $2n + 2t$ connected it follows from the universal coefficient theorem that

$$(*) \qquad H^i(X_{2n+2t}\,; Z) = 0, \quad i < 2n + 2t,$$

$$(**) \qquad H^{2n+2t+1}(X_{2n+2t}\,, Z) \text{ is free abelian.}$$

The cofibration preceding thus leads to the commutative diagram

$$\begin{array}{ccccccc}
H^{2n+2t+1}(X_{2n+2t}; Q) & \xleftarrow{\delta_Q} & H^{2n+2t}(X^{2n+2t}; Q) & \xleftarrow{i_Q^*} & H^{2n+2t}(X; Q) & \leftarrow & 0 \\
\uparrow \rho_{2n+2t} & & \uparrow \rho^{2n+2t} & & \uparrow \rho & & \\
H^{2n+2t+1}(X_{2n+2t}; Z) & \xleftarrow[\delta^*]{} & H^{2n+2t}(X^{2n+2k}; Z) & \xleftarrow[i^*]{} & H^{2n+2t}(X; Z) & \leftarrow & 0.
\end{array}$$

Assertion. Suppose $w \in H^{2n+2t}(X, Q)$ satisfies

$$i_Q^*(w) \in \operatorname{Im}\{\rho^{2n+2t} : H^{2n+2t}(X^{2n+2t}; Z) \to H^{2n+2t}(X^{2n+2t}; Q)\};$$

then

$$w \in \operatorname{Im}\{\rho : H^{2n+2t}(X; Z) \to H^{2n+2t}(X; Q)\}.$$

For suppose that

$$i_Q^* w = \rho^{2n+2t} y.$$

Then

$$0 = \delta_Q^* i_Q^* w = \delta_Q^* \rho^{2n+2t} y = \rho_{2n+2t}\delta^* y.$$

But $(**)$ then shows $\delta^* y = 0$ and therefore $y = i^*(x)$ which gives

$$i_Q^*\rho(x) = \rho^{2n+2t} i^*(x) = \rho^{2n+2t}(y) = i_Q^* w.$$

Since i_Q^* is monic this gives $\rho(x) = w$ as required.

Thus by replacing X by X^{2n+2t} and ξ by $\xi \mid_{X^{2n+2t}}$ we may suppose that X is a finite complex of dimension $2n + 2t$. As X is also $2n - 1$ connected we may apply a theorem of Stallings [10] to conclude that X embeds up to homotopy type in $S^{2n+4t+1}$. By replacing X by a homotopy equivalent space we may thus assume that $X \subset S^{2n+4t+1}$. Let Y be the complement of an open regular neighborhood of X in $S^{2n+4t+1}$. Then by (1.1) we have the following commutative diagram

$$\begin{tikzcd}[column sep=huge, row sep=large]
\widetilde{MU}^{2n}(X) \arrow[r, leftrightarrow, "\simeq", "D"'] \arrow[d, "\mu"] \arrow[dd, bend right=60, "th_{2n+2t}"'] & \widetilde{MU}_{4t}(Y) \arrow[d, "\mu_{\mathbf{C}}"] \arrow[dd, bend left=60, "th_{2t}"] \\
\tilde{K}^0(X) \arrow[r, leftrightarrow, "\simeq", "D"'] \arrow[d, "ch_{2n+2t}"] & \tilde{K}_0(Y) \arrow[d, "ch_{2t}"] \\
\tilde{H}^{2n+2t}(X; Q) \arrow[r, leftrightarrow, "\simeq", "D"'] & \tilde{H}_{2t}(Y; Q)
\end{tikzcd}$$

where D is the Alexander duality map. According to (2.3) $\xi = \mu_{\mathbf{C}}(\alpha)$ for some $\alpha \in \widetilde{MU}^{2n}(X)$. The above diagram thus shows that we need only demonstrate that

$$\mu_t th_{2t} D(\alpha) \in H_{2t}(Y; Q)$$

is integral. However this is immediate from (2.1). $\square$

3. More integrality theorems

Let us write **bu** for the connective Bott spectrum for the unitary group and $k_*(\ )$, $k^*(\ )$ for the associated homology and cohomology theories. The integrality theorem of Adams for the Chern character may be stated as follows: If $\xi \in k^{2r}(X)$ then

$$\mu_s ch_{2r+2s}(\xi) \in H^{2r+2s}(X; Q)$$

is actually integral. In view of our preceding work it is tempting to examine, and try to derive, a similar integrality theorem for the homology Chern character

$$ch_* : k_*(\ ) \to H_{**}(X; Q).$$

This is our objective in the present section.

The Chern character for **bu** may be defined in the following way: Let

$$ch : \mathbf{bu} \to \mathbf{K}(\pi_*(\mathbf{bu}) \otimes Q)$$

be the natural map of ring spectra. There is induced a family of natural transformations

$$ch_s : k_{**}(\ ) \to H_s(\ ; Q)$$

which are the components of the universal Chern character for **bu** homology.

One readily checks that the following diagram commutes:

$$\begin{tikzcd}
MU_{**}(X) \arrow[dr, "\mu"] \arrow[drr, "th"] \arrow[dd, "\zeta"'] & & \\
& K_*(X) \arrow[r, "ch"] & H_{**}(X; Q). \\
k_{**}(X) \arrow[ur, "\lambda"] \arrow[urr, "ch"'] & &
\end{tikzcd}$$

By analogy with (2.1) one expects the following result:

THEOREM 3.1. *Let X be a cw-complex and $\xi \in k_{2r}(X)$. Then*

$$\mu_s ch_{2r-2s}(\xi) \in H_{2r-2s}(X; Q)$$

is integral, with μ_s as in (2.1).

In view of the preceding commutative diagram it is tempting to try to deduce (3.1) from (2.1) and the diagram

$$\begin{tikzcd}
MU_{2r}(X) \arrow[dr, "th_{2r-2s}"] \arrow[dd, "\zeta"'] & \\
& H_{2r-2s}(X; Q) \\
k_{2r}(X) \arrow[ur, "ch_{2r-2s}"'] &
\end{tikzcd}$$

Unfortunately the mapping $\zeta : MU_{2r}(X) \to k_{2r}(X)$ need not be surjective [4; §10–11] and so a naive proof along these lines does not seem possible. We must therefore proceed in a slightly more devious manner. We shall require the following technical lemma:

LEMMA 3.2. *Let X be a cw-complex and $\xi \in k_n(X)$. Then there exists $\eta \in \tilde{k}_n(X^n)$ such that*

$$i_*(\eta) = \xi$$

where $i : X^n \to X$ is the canonical inclusion.

Proof. Consider the cofibration sequence

$$X^n \xrightarrow[i]{} X \xrightarrow[j]{} X_n$$

defining the coskeleton X_n. Since X_n is n-connected it follows quite easily that $\tilde{k}_m(X_n) = 0$, $m \leq n$. Therefore $j_*(\xi) = 0$ and exactness of the **bu** homology exact sequence of a cofibration yields the result. □

Proof of (3.1). It suffices (since we are working in homology) to consider the case where X is a finite complex. According to (3.2) there is an $\eta \in k_{2r}(X^{2r})$

such that $i_*(\eta) = \xi$. The commutative diagram

$$\begin{array}{ccc} k_{2r}(X^{2r}) & \xrightarrow{i_*} & k_{2r}(X) \\ \big\downarrow ch_{2r-2s} & & \big\downarrow ch_{2r-2s} \\ H_{2r-2s}(X^{2r}; Q) & \xrightarrow{i_*} & H_{2r-2s}(X; Q) \end{array}$$

shows that it will suffice to show that $\mu_s ch_{2r-2s}(\eta)$ is integral. Consider the cofibration

$$X^{2r-2s-2} \subset X^{2r} \xrightarrow[q]{} X^{2r}_{2r-2s-2}$$

defining $X^{2r}_{2r-2s-2}$. Then

$$q : H_{2r-2s}(X^{2r}; k) \to H_{2r-2s}(X^{2r}_{2r-2s-2}; k)$$

is an isomorphism for all coefficients k since $X^{2r-2s-2}$ is at most $2r - 2s - 2$ dimensional. Therefore it will suffice to show that $\mu_s ch_{2r-2s}(\alpha)$ is integral where $\alpha = q_*(\eta) \in k_{2r}(X^{2r}_{2r-2s-2})$. By the theorem of Stallings [10] we may embed $X^{2r}_{2r-2s-2}$ up to homotopy type in $S^{2r+2s+3}$. Since we seek a homotopy invariant conclusion we may as well assume that $X^{2r}_{2r-2s-2} \subset S^{2r+2s+3}$. Let Y be the complement of a regular neighborhood of $X^{2r}_{2r-2s-2}$ in $S^{2r+2s+3}$ and introduce the commutative diagram

$$\begin{array}{ccc} k_{2r}(X^{2r}_{2r-2s-2}) & \underset{\cong}{\overset{D}{\longleftrightarrow}} & k^{2s+2}(Y) \\ \big\downarrow ch_{2r-2s} & & \big\downarrow ch_{2s} \\ H_{2r-2s}(X^{2r}_{2r-2s-2}; Q) & \underset{D}{\overset{\cong}{\longleftrightarrow}} & H^{4s+2}(Y; Q) \end{array}$$

where D is the duality isomorphism. It is thus sufficient to show that $D(\mu_s ch_{2r-2s}(\alpha))$ is integral. But

$$D(\mu_s ch_{2r-2s}(\alpha)) = \mu_s ch_{2s} D(\alpha)$$

and the right handside is integral by the theorem of Adams. □

Since we have proven the integrality theorem of Adams from the corresponding result for the Todd character it follows that we have deduced (3.1) from (2.1). However this is certainly not apparent from the preceding proof. This suggests that one look for a more direct deduction of (3.1) from (2.1). The author has been singularly unsuccessful in such attempts.

Another point to note is that the commutative diagram

$$\begin{array}{ccc} MU_{2r}(X) & & \\ & \searrow^{th_{2r-2s}} & \\ \big\downarrow \zeta & & H_{2r-2s}(X; Q). \\ & \nearrow_{ch_{2r-2s}} & \\ k_{2r}(X) & & \end{array}$$

shows that (2.1) is implied by (3.1). This suggests the problem of providing an elementary proof of (3.1) independent of [1] and (2.1).

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UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA
AARHUS UNIVERSITY
AARHUS, DENMARK