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QUASICOMPACT COMPOSITION OPERATORS AND POWER-CONTRACTIVE SELFMAPS

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Abstract. Using the concept of power-contractive selfmaps of metric spaces, we investigate quasicompact composition operators on certain classes of Lipschitz algebras. As an application of our results, we obtain certain properties of power-contractive selfmaps of plane sets.

1. Introduction and preliminaries

Let A be a commutative unital Banach algebra and let Φ_A denote the character space of A, consisting of all nonzero homomorphisms h from A into the field of complex numbers $\mathbb{C}$. For $a \in A$, we denote by $\widehat{a}$ the Gelfand transform of a given by $\widehat{a}(h) = h(a)$ for all $h \in \Phi_A$. It is known that if A is semisimple and $T : A \to A$ is a unital endomorphism, then $T^*|_{\Phi_A}$ maps Φ_A into Φ_A and

$$\widehat{Ta} = \widehat{a} \circ \phi \quad \text{for all } a \in A,$$

where $\phi = T^*|_{\Phi_A} : \Phi_A \to \Phi_A$. Motivated by this, in general, for a semisimple Banach algebra A, an endomorphism $T : A \to A$ is said to be *induced* by a selfmap $\phi : \Phi_A \to \Phi_A$ if $\widehat{Ta} = \widehat{a} \circ \phi$ for all $a \in A$. When A is a natural Banach function algebra on a compact Hausdorff space X, the character space of A can be identified with the underlying set X through the homeomorphism $x \leftrightarrow \delta_x$ for all $x \in X$, where δ_x is the evaluation map on A defined by $\delta_x(f) = f(x)$ for all

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$f \in A$. In this case, by letting $\phi = T^*|_X : X \to X$ we have

$$Tf = f \circ \phi \quad \text{for all } f \in A.$$

This leads to the general definition of *composition operators*. For a Banach space A of functions on a compact Hausdorff space X, an operator $T : A \to A$ is said to be *induced* by the selfmap $\phi : X \to X$ if $Tf = f \circ \phi$ for all $f \in A$. In this case, we also say that $T : A \to A$ is a *composition operator* induced by the selfmap $\phi : X \to X$. The notation C_ϕ is widely used to denote the composition operator induced by the selfmap ϕ. Note that if A is a Banach function *algebra* on the compact Hausdorff space X, then the composition operator $C_\phi : A \to A$ is indeed an endomorphism (multiplicative).

Over the years, considerable interest has grown in the study of composition operators on different classes of Banach spaces (algebras). Boundedness and compactness of composition operators on certain Banach function spaces (algebras) have been studied in [2], [8], and [10]. For more results concerning composition operators and different generalizations of composition operators, see [7], [9], [11], [12].

In the present article we study *quasicompactness* of composition operators, defined as follows. For an infinite-dimensional Banach space E, we denote by $\mathcal{B}(E)$ and $\mathcal{K}(E)$ the Banach algebra of all bounded operators and compact operators on E, respectively. The essential spectrum $\sigma_e(T)$ of $T \in \mathcal{B}(E)$ is the spectrum of $T + \mathcal{K}(E)$ in the Calkin algebra $\frac{\mathcal{B}(E)}{\mathcal{K}(E)}$. The essential spectral radius $r_e(T)$ of $T \in \mathcal{B}(E)$ is the spectral radius of $T+\mathcal{K}(E)$ in $\frac{\mathcal{B}(E)}{\mathcal{K}(E)}$, which is given by the formula

$$r_e(T) = \lim_{n\to\infty} \big(\operatorname{dist}\big(T^n, \mathcal{K}(E)\big)\big)^{1/n}.$$

The operator $T \in \mathcal{B}(E)$ is called *quasicompact* if $r_e(T) < 1$. Equivalently, $T \in \mathcal{B}(E)$ is quasicompact if and only if

$$\|T^N - S\| < 1 \quad \text{for some } S \in \mathcal{K}(E) \text{ and } N \in \mathbb{N},$$

where $\|\cdot\|$ denotes the operator norm in $\mathcal{B}(E)$. If T^N is compact for some $N \in \mathbb{N}$, then T is called *power-compact*. Clearly, every power-compact operator is quasicompact. The quasicompactness of composition operators on certain Banach algebras has been studied in [1] and [3].

In the present article, composition operators are considered on certain algebras of Lipschitz functions, as per the following definition. Let (X, d) be a compact metric space with at least two points. The Lipschitz constant of a complex-valued function f on X is defined by

$$p(f) = \sup_{\substack{x,y\in X\\ x\neq y}} \frac{|f(x) - f(y)|}{d(x, y)}.$$

The Lipschitz algebra $\operatorname{Lip}(X, d)$ consists of those complex-valued functions f on (X, d) for which $p(f) < \infty$. When the underlying metric space X is a plane set, we write $\operatorname{Lip}(X)$ for $\operatorname{Lip}(X, |\cdot|)$. Note that the Lipschitz algebra $\operatorname{Lip}(X, d)$ is contained in $C(X)$, the algebra of all complex-valued continuous functions on

(X, d). The algebra $\text{Lip}(X, d)$ is a natural Banach function algebra on (X, d) if equipped with the norm

$$\|f\|_{\text{Lip}} = \|f\|_X + p(f),$$

where $\|f\|_X$ is the sup-norm given by

$$\|f\|_X = \sup_{x \in X} \big|f(x)\big|.$$

These Lipschitz algebras were first studied by D. R. Sherbert in 1963. Quasicompact composition operators on certain algebras of Lipschitz functions have been studied in [1].

In Section 2, we investigate quasicompactness of composition operators on certain algebras of Lipschitz functions. Our results in this section generalize some results of [1]. In Section 3, by applying some results of Section 2, we investigate selfmaps of certain plane sets.

2. Main results

We begin this section by recalling that a selfmap ϕ of a metric space (X, d) is said to be *contractive* if there exists a constant $0 \leq c < 1$ such that

$$d(\phi x, \phi y) \leq c d(x, y),$$

for all $x, y \in X$. These contractive selfmaps play an important role in the study of selfmaps of metric spaces. Next, we state a generalization of this concept which will be used variously throughout the rest of this paper. Before stating the next definition, we recall that for a selfmap $\phi : X \to X$ and a natural number n, ϕ^n denotes the nth iterate of ϕ.

Definition 2.1. A selfmap ϕ of the metric space (X, d) is said to be *power-contractive* if ϕ^N is contractive for some $N \in \mathbb{N}$, that is, if there exists a constant $0 \leq c_N < 1$ such that

$$d(\phi^N x, \phi^N y) \leq c_N d(x, y)$$

for all $x, y \in X$.

Clearly, every contractive selfmap is power-contractive. By constructing a power-contractive selfmap which is not contractive, in the next example we show that the converse is not true in general.

Example 2.2. Let $\overline{\mathbb{D}}$ be the closed unit disc of the complex plane and define the selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ by

$$\phi z = \frac{1 - z^2}{2} \quad \text{for all } z \in \overline{\mathbb{D}}.$$

Then, for each $z, w \in \overline{\mathbb{D}}$ with $z \neq w$, we have

$$\frac{|\phi z - \phi w|}{|z - w|} = \frac{|z + w|}{2}.$$

Hence, by choosing the sequences (z_n) and (w_n) in $\overline{\mathbb{D}}$ with $z_n \neq w_n$, $z_n \to 1$ and $w_n \to 1$, we have

$$\frac{|\phi z_n - \phi w_n|}{|z_n - w_n|} \to 1 \quad \text{as } n \to \infty,$$

meaning that ϕ is not contractive. Similarly, one can see that

$$\phi^2 z = \frac{4 - (1 - z^2)^2}{8} \quad \text{for all } z \in \overline{\mathbb{D}}$$

is not contractive. Finally, note that

$$\phi^3 z = \frac{64 - (4 - (1 - z^2)^2)^2}{128} \quad \text{for all } z \in \overline{\mathbb{D}},$$

and that, for each $z, w \in \overline{\mathbb{D}}$ with $z \neq w$, we have

$$\frac{|\phi^3 z - \phi^3 w|}{|z - w|} \leq \frac{3}{4}.$$

Consequently, ϕ is a power-contractive selfmap which is not contractive.

Using the concept of power-contractiveness, in the next theorem we characterize quasicompact composition operators on certain algebras of Lipschitz functions. Before stating the next theorem, we note that if C_ϕ is a composition operator induced by the selfmap ϕ, then C_ϕ^n is the composition operator induced by the selfmap ϕ^n—that is, $C_\phi^n = C_{\phi^n}$.

Theorem 2.3. *Let X be a compact connected plane set and let A be any natural closed subalgebra of* $\mathrm{Lip}(X)$ *containing the constant functions. Let $\phi \in A$ be a selfmap of X and let $C_\phi : A \to A$ be a composition operator induced by ϕ. Then, C_ϕ is quasicompact if and only if ϕ is power-contractive.*

Proof. Let $C_\phi : A \to A$ be quasicompact. Then, by [3, Theorem 1.2], ϕ has a unique fixed point $x_0 \in X$. Consider the operator (of rank 1) $S_0 : A \to A$ given by $S_0 f = f(x_0)1$ for all $f \in A$. By [3, Theorem 1.2], $\|C_{\phi^n} - S_0\| \to 0$ as $n \to \infty$. So, there exists $N \in \mathbb{N}$ such that

$$c = \|C_{\phi^N} - S_0\| \|\phi\|_{\mathrm{Lip}} < 1. \tag{2.1}$$

Let $x, y \in X$ and let $x \neq y$. Then, by applying (2.1), we have

$$\begin{aligned} \frac{|\phi^{N+1} x - \phi^{N+1} y|}{|x - y|} &\leq p(C_{\phi^N}\phi - S_0\phi) \\ &\leq \|C_{\phi^N}\phi - S_0\phi\|_{\mathrm{Lip}} \\ &\leq \|C_{\phi^N} - S_0\| \|\phi\|_{\mathrm{Lip}} \\ &= c < 1, \end{aligned}$$

meaning that ϕ is power-contractive.

Conversely, let $\phi : X \to X$ be power-contractive. Then, there exist $N_1 \in \mathbb{N}$ and $0 \leq c_1 < 1$ such that

$$|\phi^{N_1} x - \phi^{N_1} y| \leq c_1 |x - y| \tag{2.2}$$

for all $x, y \in X$. On the other hand, since ϕ is bounded, there exists $r > 0$ such that

$$|\phi x| \leq r \quad \text{for all } x \in X. \tag{2.3}$$

One can conclude from (2.2) that

$$p(\phi^{nN_1}) = \sup_{\substack{x,y \in X \\ x \neq y}} \frac{|\phi^{nN_1}x - \phi^{nN_1}y|}{|x-y|} \leq c_1^n \to 0 \quad \text{as } n \to \infty. \tag{2.4}$$

Also, by applying (2.3) and (2.4), we have

$$\begin{aligned}
\operatorname{diam}\big(\phi^{nN_1+1}(X)\big) &= \sup_{x,y \in X} |\phi^{nN_1+1}x - \phi^{nN_1+1}y| \\
&= \sup_{x,y \in X} |\phi^{nN_1}\phi x - \phi^{nN_1}\phi y| \\
&\leq c_1^n \sup_{x,y \in X} |\phi x - \phi y| \\
&\leq 2rc_1^n \to 0, \quad \text{as } n \to \infty.
\end{aligned} \tag{2.5}$$

Since $\phi^{m+1}(X) \subseteq \phi^m(X)$ for each $m \in \mathbb{N}$, (2.5) implies that $\operatorname{diam}(\phi^m(X)) \to 0$ as $m \to \infty$. Hence, there exists $x_0 \in X$ such that

$$\|\phi^m - x_0\|_X = \sup_{x,y \in X} |\phi^m x - x_0| \to 0 \quad \text{as } m \to \infty. \tag{2.6}$$

Consequently, by applying (2.4) and (2.6), one can find $N \in \mathbb{N}$ large enough, such that

$$\|\phi^N - x_0\|_X + p(\phi^N) < 1. \tag{2.7}$$

Now, consider the compact (of rank 1) operator $S_0 : A \to A$ given by $S_0 f = f(x_0)1$ for all $f \in A$. Next, we show that $\|C_{\phi^N} - S_0\| < 1$, which implies that $\operatorname{dist}(C_{\phi^N}, \mathcal{K}(A)) < 1$, meaning that ϕ is quasicompact. Let $f \in A$; then for each $x \in X$, we have

$$\begin{aligned}
\big|(C_{\phi^N} f)(x) - (S_0 f)(x)\big| &= \big|f(\phi^N x) - f(x_0)\big| \\
&\leq p(f)|\phi^N x - x_0| \\
&\leq \|f\|_{\text{Lip}} \|\phi^N - x_0\|_X,
\end{aligned}$$

which implies that

$$\|C_{\phi^N} f - S_0 f\|_X \leq \|f\|_{\text{Lip}} \|\phi^N - x_0\|_X. \tag{2.8}$$

On the other hand, for each $x, y \in X$ with $x \neq y$, we have

$$\begin{aligned}
&\frac{|((C_{\phi^N} f)(x) - (S_0 f)(x)) - ((C_{\phi^N} f)(y) - (S_0 f)(y))|}{|x-y|} \\
&= \frac{|f(\phi^N x) - f(\phi^N y)|}{|x-y|} \\
&\leq p(f) \frac{|\phi^N x - \phi^N y|}{|x-y|}
\end{aligned}$$

$$\leq p(f)p(\phi^N)$$
$$\leq \|f\|_{\mathrm{Lip}} p(\phi^N),$$

and hence

$$p(C_{\phi^N} f - S_0 f) \leq \|f\|_{\mathrm{Lip}} p(\phi^N). \tag{2.9}$$

Now, by applying (2.8) and (2.9), we get

$$\|C_{\phi^N} f - S_0 f\|_{\mathrm{Lip}} \leq \|f\|_{\mathrm{Lip}} \big(\|\phi^N - x_0\|_X + p(\phi^N)\big),$$

for all $f \in A$. This along with (2.7) implies that

$$\|C_{\phi^N} - S_0\| \leq \|\phi^N - x_0\|_X + p(\phi^N) < 1$$

which completes the proof. □

Note that if the algebra A in Theorem 2.3 contains the identity function Z on X, then $\phi = C_\phi Z \in A$. Hence, we get the following corollary.

Corollary 2.4. *Let X be a compact connected plane set and let A be any natural closed subalgebra of* $\mathrm{Lip}(X)$ *containing the identity function and constant functions. Let $C_\phi : A \to A$ be a composition operator induced by the selfmap $\phi : X \to X$. Then, C_ϕ is quasicompact if and only if ϕ is power-contractive.*

Remark 2.5. It is worth mentioning that the class of algebras A satisfying conditions of Corollary 2.4 includes the following subalgebras of $\mathrm{Lip}(X)$:

(1) The Lipschitz algebra $\mathrm{Lip}(X)$.
(2) The analytic Lipschitz algebra $\mathrm{Lip}_A(X) = \mathrm{Lip}(X) \cap A(X)$ (see [2, p. 304]) where $A(X)$ denotes the uniform algebra of all continuous complex-valued functions on X which are analytic on $\mathrm{int}(X)$.
(3) The rational Lipschitz algebra $\mathrm{Lip}_R(X)$ (see [5, p. 14]); or the polynomial Lipschitz algebra $\mathrm{Lip}_P(X)$ (see [5, p. 14]) when X is a polynomially convex plane set.

Remark 2.6. In Theorem 2.3 and Corollary 2.4, connectedness of X is used in the "only if" part while the "if part" holds without connectivity assumption on X.

Finally, we note that by applying a similar approach as in the proof of Theorem 2.3, one can get the next result for a general metric space (X, d) instead of a plane set X. But, note that by releasing this condition on X we get the next theorem only for the algebra $\mathrm{Lip}(X, d)$, not for its subalgebras A. (See also [4, Corollary 2.4], where the result of the next theorem is obtained using a different approach based on the essential spectral radius estimates.) It is worth mentioning that Section 3 is based on applying the result of Theorem 2.3, which is valid not only for the algebra $\mathrm{Lip}(X)$ but also for certain subalgebras A of $\mathrm{Lip}(X)$.

Theorem 2.7. *Suppose that (X, d) is a compact connected metric space and that $C_\phi : \mathrm{Lip}(X, d) \to \mathrm{Lip}(X, d)$ is a composition operator induced by the selfmap $\phi : X \to X$. Then C_ϕ is quasicompact if and only if ϕ is power-contractive.*

Remark 2.8. In Theorem 2.7, connectedness of (X, d) is used in the "only if" part while the "if part" holds without connectivity assumption on (X, d).

3. Applications

Contractive maps play an important role in the study of selfmaps of metric spaces (see, e.g., [6] and references therein). In this section, as an application of some results in Section 2, we obtain certain properties of contractive or power-contractive selfmaps of plane sets.

We start with selfmaps of the classic plane set $\overline{\mathbb{D}}$ and note that if a Lipschitz selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ is contractive, then the image of ϕ, $\phi(\overline{\mathbb{D}})$, satisfies the following inequality:

$$\operatorname{diam}\big(\phi(\overline{\mathbb{D}})\big) < \operatorname{diam}(\overline{\mathbb{D}}). \tag{3.1}$$

This simple observation leads to the following general question about contractive selfmaps of metric spaces:

"What is the relation between contractiveness or power-contractiveness of $\phi : X \to X$ and its (iterative) image's properties?"

Here, by "iterative image" of the selfmap $\phi : X \to X$, we mean $\phi^n(X)$ for some $n \in \mathbb{N}$. Having this question in mind, in the rest of this section we study the following related question:

"What sufficient conditions on the iterative image of $\phi : X \to X$ imply contractiveness or power-contractiveness of ϕ?"

Recalling inequality (3.1) as a necessary condition for the contractiveness of the selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$, the next theorem shows that

$$\phi(\overline{\mathbb{D}}) \subseteq \mathbb{D},$$

is a sufficient condition for the power-contractiveness of the Lipschitz selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ in $A(\overline{\mathbb{D}})$.

Theorem 3.1. *Suppose that the Lipschitz selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ belongs to $A(\overline{\mathbb{D}})$. If $\phi^N(\overline{\mathbb{D}}) \subseteq \mathbb{D}$ for some $N \in \mathbb{N}$, then ϕ is power-contractive.*

Instead of proving the above result for selfmaps of $\overline{\mathbb{D}}$, we next prove a more general case for selfmaps of compact connected plane sets X.

Theorem 3.2. *Suppose that X is a compact connected plane set and the Lipschitz selfmap $\phi : X \to X$ belongs to $A(X)$. If $\phi^N(X) \subseteq \operatorname{int}(X)$ for some $N \in \mathbb{N}$, then ϕ is power-contractive.*

Proof. The selfmap $\phi : X \to X$ belongs to $\operatorname{Lip}_A(X)$. Hence, ϕ induces the composition operator $C_\phi : \operatorname{Lip}_A(X) \to \operatorname{Lip}_A(X)$. Now, if $\phi^N(X) \subseteq \operatorname{int}(X)$ for some $N \in \mathbb{N}$, then by [2, Theorem 2.1], C_{ϕ^N} is compact. Consequently, C_ϕ is power-compact and hence quasicompact. Note that $\operatorname{Lip}_A(X)$ contains the coordinate function Z. So, by Corollary 2.4 and Remark 2.5, ϕ is power-contractive. □

Besides suitable constant functions, also the following non-constant example shows that the converse of Theorem 3.2 (or Theorem 3.1) is not true in general.

Example 3.3. Let $0 < r < 1$, and define

$$\phi z = rz + (1 - r) \quad \text{for all } z \in \overline{\mathbb{D}}.$$

Then, ϕ is a selfmap of the closed unit disc $\overline{\mathbb{D}}$. Indeed, ϕ maps the closed unit disc to the closed disc with radius r centered at $1-r$. Clearly, ϕ is contractive and hence power-contractive. But, $\phi^n 1 = 1$ for each $n \in \mathbb{N}$, and hence $\phi^n(\overline{\mathbb{D}}) \nsubseteq \mathbb{D}$.

One of the main assumptions in Theorem 3.2 (or Theorem 3.1) is that the selfmap $\phi : X \to X$ belongs to $A(X)$. By constructing the following example, we show that this assumption is necessary and cannot be removed in general.

Example 3.4. Let $0 < r < 1$, and consider the selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ defined as follows

$$\phi z = \begin{cases} \mathrm{Re}(z) & \text{if } -r \leq \mathrm{Re}(z) \leq r, \\ -r & \text{if } -1 \leq \mathrm{Re}(z) \leq -r, \\ r & \text{if } r \leq \mathrm{Re}(z) \leq 1. \end{cases}$$

First, note that the selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ does not belong to $A(\overline{\mathbb{D}})$, since it does not satisfy Cauchy–Riemann equations at any neighbourhood of zero. On the other hand, one can see that, for each $z, w \in \overline{\mathbb{D}}$, we have

$$|\phi z - \phi w| \leq |z - w|,$$

meaning that ϕ is a Lipschitz selfmap.

Note that for all $-r \leq x, y \leq r$, we have

$$\big|\phi(x,0) - \phi(y,0)\big| = |x - y| = \big|(x,0) - (y,0)\big|,$$

meaning that ϕ is not contractive. Also, for each $n \in \mathbb{N}$ we have $\phi^n = \phi$. Consequently, ϕ is not power-contractive.

Remark 3.5. By Theorem 3.2 (or Theorem 3.1)

$$\phi^N(X) \subseteq \mathrm{int}(X) \quad \text{for some } N \in \mathbb{N} \tag{3.2}$$

is a sufficient condition for the power-contractiveness of any Lipschitz selfmap $\phi : X \to X$ that belongs to $A(X)$. Hence, the simpler condition

$$\phi(X) \subseteq \mathrm{int}(X) \tag{3.3}$$

is also a sufficient condition for the power-contractiveness of such selfmap. Regarding these two conditions, it is worth mentioning the following points.

(i) Condition (3.3) implies condition (3.2), but in general these two conditions are not equivalent. For example, the selfmap $\phi : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ in Example 2.2 satisfies condition (3.2) for $N = 3$ but it does not satisfy condition (3.3), because $\phi i = 1$.

(ii) In Theorem 3.2 (or Theorem 3.1), if we replace condition (3.2) with condition (3.3), we can still conclude that the selfmap $\phi : X \to X$ is *power-contractive*, and not necessarily *contractive*. To see this, consider the selfmap $\phi^2 : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$, where ϕ is the selfmap given in Example 2.2. Then, one can see that $\phi^2(\overline{\mathbb{D}}) \subseteq \mathbb{D}$ but as mentioned in Example 2.2, $\phi^2 : \overline{\mathbb{D}} \to \overline{\mathbb{D}}$ is not contractive. Note that ϕ^2 is power-contractive, since $(\phi^2)^3 = (\phi^3)^2$ and ϕ^3 is contractive.

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