book leans more heavily toward CMA than does the latter, but neither one really exemplifies MMA.

SPECIFIC COMMENTS ON THE SCHERVISH REVIEW

1. Where Schervish discusses the development of power functions in multivariate analysis, it would have helped a bit if power were discussed in terms of how power functions are normally used in multivariate statistical practice, namely, from the viewpoint of someone trying to determine sample size for an experiment involving multiple (correlated) outcomes. How is this sample size selection problem best solved? There is not much discussion of this kind of question in the two books reviewed.

2. The author refers to Anderson's discussion of the Scheffé procedure (it was extended to the multivariate case by Bennett) for dealing with the multivariate Behrens-Fisher problem of testing equality of means in two normal populations with unequal variances, when he says, "Data is discarded with a vengeance." The issue here is that if we have M observations on one population, and N observations on another, and $M < N$, Scheffé suggests that we randomly match M observations from the two populations and discard the remaining $(N - M)$ in the matching process. Actually, all of the observations in each of the populations should be used to estimate each of the variances. If M and N are large there is of course no problem in ignoring $(N - M)$ observations in the testing. The only occasion when a problem arises is in the case of M and N small, and $M \ll N$. From the Bayesian point of view these types of issues never arise, at the tradeoff cost of having to develop prior information for the parameters.

3. Schervish suggests that "one other unfortunate feature of Section 5.5 is ... This test is simply not another example of the type of test proposed for the Behrens-Fisher problem." Here, Anderson suggests that we can test the hypothesis that two normal subvectors have equal means (with unequal covariance matrices, so that it is a Behrens-Fisher type problem) by forming the difference in the sample mean vectors, "and this statistic is most relevant to $(\mu^{(1)} - \mu^{(2)})$" (Anderson, page 178). This is a special case of the Scheffé/Bennett approach discussed in Item 2, above, for the case of $M = N$, where the two-sample problem is reduced to a one-sample problem by subtraction of the sample means.

4. Schervish's suggested alternative to a second principle of classical inference is a bit harsh. Although "unbiasedness" is not a particularly relevant property for situations in which we are going to have to estimate only once, or only a few times, and although unbiasedness is a property that violates the "likelihood principle," I believe that most any reasonable statistician who is in the position of having to recommend an estimator that will be close to the true value on the average, over many estimations of the same quantity, would find unbiasedness a compelling property when taken in conjunction with the requirement of small variance.

Summary

In summary, the review of these important books on multivariate analysis by Mark Schervish not only provides a helpful perspective from which to appreciate these contributions to our field, but also, is refreshing and entertaining.

Rejoinder

Mark J. Schervish

I wish to thank the discussants for taking the time to carefully read the review and offer their own views on the topics covered. They have each made it more informative and useful for the interested reader. Because some of the comments of the authors of the two books reviewed are in the way of rejoinder to my review, I will refrain from offering further commentary on those remarks. Much of the discussion provides the readers with brief overviews of areas that I failed to mention in my review. As my review already is a comment, at great length, on the work of many people, I will keep my comments on the discussion brief.

Because Professor Anderson's comments are almost entirely concerned with my review of his text, I will let him have the last word on the matter. I will thank him, however, for bringing to my attention part (b) of Problem 3 in Chapter 11 of his book, which indeed does suggest the predictive interpretation of principal components. A further suggestion of this interpretation appears in the paper of Kettenring (1971), whom I also thank for the reference.

Some of the discussants mention projection pursuit as a computationally intensive multivariate method that I did not discuss. Professor Goldstein remedies this omission by giving us a brief description of projection pursuit. Jones and Sibson (1987) give more detail and examples for those who wish to know more. The discussion after the Jones and Sibson paper is very enlightening. Several of the discussants to that paper stress the need to be careful not to overinterpret structure one finds by maximizing a projection index. On the other hand, one cannot underestimate the value of being able to look at a data set before trying to fit mathematical models. In their comments on the present review, Gnanadesikan and Kettenring also stress this point. In a sense, multivariate analysts have put the model-fitting cart before the exploratory analysis horse. This was due in part to lack of algorithms and computing power. The lack of computing power persists to a large extent, as the recent report of Eddy, Huber, McClure, Moore, Stuetzle and Thisted (1986) suggests. The lack of algorithms is partially being filled by methods like projection pursuit and the bootstrap, but there is still a great deal of uncertainty about how to interpret the results of these methods.

On the mathematical side, Professor Perlman gives us an introduction to group symmetry covariance models. Consonni and Dawid (1985) describe group symmetry mean structure models with particular emphasis on Bayesian inference. These models have potential applicability to hierarchical modeling of multivariate data. Group theory and invariance have always played an important role in the mathematical theory of multivariate analysis. Professor Sen reminds us of the books by Eaton (1983) and Muirhead (1982), which also stress applications of group theory to multivariate analysis. In addition, Eaton emphasizes vector-space methods and Muirhead uses differential topology extensively. I should also mention a new book by Stone (1987) that attempts a strictly coordinate-free presentation of multivariate analysis. These books, together with the work on group models described by Professor Perlman, illustrate just a few of the wide variety of mathematical approaches to multivariate analysis. Each of the approaches has its advantages for dealing with certain problems, and all of them deserve some attention from anyone interested in acquiring a thorough understanding of the mathematical theory.

Addressing both data analytical and mathematical issues together, Professor Press has offered a comparison of classical and modern multivariate analysis. This comparison helps to point out a serious shortcoming of my review that probably did not escape most readers, namely that the review is not particularly comprehensive. I devoted much of my effort to topics covered or mentioned in at least one of the two texts. Professor Sen is to be applauded for his Herculean effort in reviewing 16 books simultaneously. Some of these 16 books do more justice to modern multivariate analysis than either of the two that I reviewed. Some of them emphasize material quite different from anything mentioned in my review or the discussion. The field has grown so much that a single expression like "multivariate analysis" hardly narrows the focus of discussion at all. Had there been six more discussants, we would have been able to read about six other interesting topics that have received little or no mention in this discussion. Such topics, and others that Professor Press describes as modern multivariate analysis, are *slowly* making their way into introductory texts on the subject, and I believe that the book by Dillon and Goldstein is one example.

In conclusion, it seems clear that the field of multivariate analysis has grown phenomenally since 1958. Its focus has broadened to the extent that it is barely distinguishable from the subject of statistics as a whole. I do not believe that I am the most qualified person to lead a discussion of such an important area of research and application, but I thank the editor for giving me the opportunity to try and for getting six renowned multivariate analysts to round out the discussion. In expressing my appreciation to the discussants for their help in enhancing this review, I would also like to belatedly thank a man who enhanced the entire field of multivariate analysis until his untimely death earlier this year. P. R. Krishnaiah was a leader in this field, and his work will continue to inspire generations to come. I join the discussants in dedicating this discussion to his memory. We will miss him.

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