Harold Hotelling 1895–1973

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Harold Hotelling was a leading mathematical economist at a time when that subject was in its infancy; he also enjoyed a world-wide reputation as a mathematical statistician. In both fields he was renowned for his theoretical prowess and as a most effective and caring teacher. His wide influence upon the professions is felt not only through his publications (a large number of which are seminal contributions), but also through his students, among whom one can count many of the next generation's leading economists and statisticians.

Hotelling's career spans a most creative period in the disciplines of both mathematical economics and mathematical statistics, and few figures have displayed his originality and flair: fewer still have publication records that bear comparison. Most importantly, his published papers are today seen as the starting point of much contemporary research. Indeed, his name is familiar to a remarkably wide range of professionals, a range that runs from economists to statisticians but also includes educationalists and psychologists. In economics there is Hotelling's Lemma and Hotelling's Rule and in statistics he is particularly known through the Hotelling T^2 statistic.

In 1985 his widow donated the collection of Harold's correspondence and professional manuscripts to the Butler Library of Columbia University. It comprises a very large and impressive source of material for the historian of economic and statistical thought. This fascinating figure has never been the subject of any systematic study and the primary material at Columbia affords an exciting opportunity to examine the life and career of this most revered, distinguished and influential man.

Harold Hotelling was born on September 29, 1895 in Fulda, Minnesota, of ancestors long American but originally of English and Dutch extraction. When he was nine the family moved to Seattle. The Methodist Church became a focus of the lives of the newcomers, providing both a place of religious worship and a most hearty welcome. Hotelling records that he was stimulated as a very young man by "Methodist interest in social justice and race equality, and the family tradition of active concern with public problems." (This and other nonattributed quotations are from original

papers in the Hotelling Collection held at the Butler Library, Columbia University). Although not a church-goer, this stimulus stayed with him for his whole life, though perhaps with less intensity than when, in his youth, he attempted to organize a ban on dancing in public! Throughout his life he remained a staunch teetotaller, and looked unfavorably upon the consumption of tobacco. He was an avid reader, and his inquisitive nature knew few bounds. The Seattle Public Library was a most important part of his formative years: the building stood on wooden stilts in a swamp and, to the tune of croaking frogs, he read *Scientific American* and all the books on electricity. Economically, these were very hard times. The depression of 1907 took its toll on Seattle and on the Hotellings: Harold's father lost on his investments and on his hay business, which was being made obsolete by the coming of the automobile. Having attended high school in Seattle, Harold did electrical wiring work and was employed on small newspapers while studying at the University of Washington. He spent a year, from the fall of 1915, working on the *Puyallup Herald*, and returned in 1916 to major in journalism at Washington.

His university studies were interrupted by World War I and he noted that, on being called to war service "Having studied mathematics, science and classics at school and college, was considered by [the] Army authorities competent to care for mules. The result was [that] a temperamental mule temporarily broke his leg and thereby saved his life, as the rest of the division was sent to France and [was] wiped out." (The mule's name was Dynamite.) Hotelling was never engaged in active service and, on being discharged from the Army on February 4, 1919, resumed his course in journalism. In a letter of 1962 he wrote: "I have a first degree in journalism from Washington but at the time the School of Journalism was badly disorganized because of the First World War and they let me substitute so much economics for journalism that the degree might just have well been in it. Anyhow, the journalism degree got me a job on the *Washington Standard* in Olympia. Its economic component, plus later studies of mathematics at Seattle, Chicago and Princeton gave enough color to the idea that I was an economist to give me a job as Professor of Economics at Columbia University. I actually did teach economics there, but it was economics so mathematical that no member of the distinguished

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economics faculty there could understand it." In fact Hotelling studied just three economics courses, representing less than three-quarters of his final year studies.

He was awarded his BA in journalism in 1919, joined the staff of a local paper, but soon became disillusioned: "Journalism had seemed to offer both a means of livelihood and an opportunity to stimulate proper action on public matters. Later I concluded that it had been overrated in both respects." Much of his disillusionment stemmed from his inability to write sufficiently quickly. With the encouragement of one of the Washington faculty, Eric Temple Bell, the mathematician and historian of mathematics, Hotelling returned to the University in January 1920 to study mathematics at the Master's level, financing this by a badly paid job of teaching trigonometry and analytical geometry (on both of which he was, by his own admission, extremely hazy). He obtained his MA at the end of 1921 and applied for an Economics Fellowship at Columbia but was rejected. He did, however, successfully apply for a Mathematics Fellowship at Princeton. Hotelling had a long-standing strategy: he wrote "The combination of science and political economy led to the thought of applying the methods proven so useful in the exact sciences to discover new truth in economics and political science. Proficiency in these methods require" in the first place mathematics." Having achieved proficiency in the required mathematics at Washington, Hotelling had hoped to develop his understanding of economics and statistics at Princeton; however, he was frustrated in this: "actually I found there was no one there who knew anything about either subject. I therefore studied the topology, differential geometry, analysis, mathematical physics and astrophysics that Princeton then offered, and all of these have to some extent contributed to my later work." He pursued his doctorate studies, concentrating on analysis situs (known today as topology) under the direction of Oswald Veblen, the nephew of Thorstein Veblen, and was awarded his degree in June of 1924. Hotelling was then appointed as a junior associate at the Food Research Institute (FRI), Stanford; Hotelling's primary task was to help estimate crop yields and food requirements, particularly for wheat. He worked at Stanford for 7 years, although he gradually moved into the Mathematics Department. In 1927 he accepted a full time assistant professorship in Mathematics where he taught a unique combination of courses, including determinants and probability, the mathematics of statistical inference, analysis situs and differential geometry. In 1931 he moved to Columbia as Professor of Mathematical Economics, replacing Henry Ludwell Moore who had retired on grounds of ill-health a year earlier.

In 1920 he had married Floy Tracy. Their first child

was born in 1923 and was named Eric Bell, in honor of Eric Temple Bell "who had foreseen the direction my career should take and over a series of years had supplied the guidance and inspiration to offset many diverting cross-currents." Their second child was born at Stanford in 1925. Just after the move to New York, Floy fell ill and died in October of 1932 of causes whose ultimate origin was ascribed to a childhood attack of rheumatic fever. The severe personal blow of his wife's sudden death fell at the beginning of what was to be his most productive and creative period. The task of looking after his two young children was, however, eased by Floy's sister who, for the next few years, cared for them with her own children who were of similar ages.

Hotelling then had the good fortune to meet Susanna Porter Edmondson and in due course they were married in June 1934. She was born in Montgomery, Alabama, in 1909 and was educated at Randolph Macon Womans College in Virginia, having spent her junior year in France. She made two subsequent trips to France, financed by her winning essay competitions. Her MA was awarded by Wisconsin in 1930, and she then pursued work in romance languages at Bryn Mawr. However, having decided on a change in direction, statistics became her chosen subject. She was advised to go to Columbia where Hotelling, then recognized as one of the leading statisticians, was responsible for the teaching. She attended his statistics course (the same class, incidentally, as Milton Friedman, the 1976 Nobel Laureate in economics) and on marrying Harold she devoted herself to the role of homemaker. Six children were born of this marriage, the first being George Alfred in July of 1937; that summer they moved from their city apartment to a house at Mountain Lakes, New Jersey.

The demands made upon Sue's time as wife and mother did not diminish the time and energy she directed toward assisting with Harold's departmental role. She worked tirelessly as hostess to distinguished visitors (of whom there were innumerable) and as friend and hostess to his students. The second Sunday of every month was "Open House": Hotelling's students, members of faculty and visitors, along with all their families (including children) were invited out to tea. With pun intended, these became known as Hotelling Teas, after the Hotelling T^2 statistic. The great care he took of his students, and the gentle but persuasive encouragement he offered them, were legendary. He was as proud of his students' achievements as he was of his own (although of his own work he was always extremely modest).

Hotelling had the remarkable gift of identifying and nurturing talent in students and, in those dark days of Hitler's power, he was responsible for the admission of a large number of refugee scholars and students

from Central Europe to Columbia. He taught both mathematical statistics and mathematical economics, but with regard to the latter, the environment was most unfortunate. The Department of Economics had an active antitheoretical stance; most of its members were not concerned with theoretical research. There was, for example, no course offered in neoclassical price theory, which must have made Columbia unique. Nevertheless, many of the next generation's professional economists had the benefit of his teaching. Of his students who were to become academic economists, the most famous is Kenneth Arrow who was the 1972 Nobel Laureate.

He lectured widely throughout the States and abroad, and had a special relationship with India where he spent the winter of 1939–1940 as visiting lecturer and as consultant to the Government. His ties with India continued after the visit, and the Government of India maintained a program of sending a group of students to Hotelling for his supervision. He was also extensively involved with outside agencies. He once testified for over an hour and a quarter before the New York State Public Service Commission on the shortcomings of the New York Telephone Company's depreciation methods and he acted as consultant to a large number of United States Government agencies while at Columbia. These included the National Recovery Administration (NRA), the Division of Tax Research of the Treasury Department, the Wartime Office of Scientific Research and Development, the Bureau of the Budget and through it the Bureau of Labor Statistics. Of the NRA, he was, at times, most critical. In particular he argued the case for increasing production, in contrast to the Government's plan to reduce the production of certain items such as oil and agricultural products. In the *Columbia Alumni News* (1936a) he wrote: "The success of the government's recovery program ... must be judged, not in terms of price levels, but in terms of the quantities of physical goods and services which are put into the hands of consumers ... In this respect much that is being done at Washington is definitely in the wrong direction." In addition to this remarkable level of involvement with outside agencies, much of which stimulated his theoretical work, he also organized the Statistical Research Group (SRG) that did confidential War research at Columbia from 1942 to 1945. The SRG had been formed to act as a consultancy agency to address various statistical problems that arose as a consequence of the War effort (such as a statistical approach to quality control of bomb sights); it was composed of what surely must be the most extraordinary group of statisticians ever organized. Hotelling was closely associated with all its work, spending just over 3 years as the principal investigator. Of course, Hotelling was, at the same

time, most active in academic research and during this period he published his most important contributions.

Had the term been in common usage at the time, Hotelling would have been described as a "workaholic"; he worked for outside agencies, took roles of responsibility on numerous national academic associations (he was a founder member and sometime president of both the Econometric Society and the Institute of Mathematical Statistics) and maintained the very highest standards in his research. He was always active on several fronts at once and, perhaps because his mind was so full of the day's work, often had difficulties going to sleep. His own remedy centered on his favorite board game, Monopoly. His memory was phenomenal: he had a perfect recollection of the board, together with a set of random numbers that simulated the roll of the dice. He could play a "mind game" of Monopoloy, and he said he sometimes played a couple of games before sleep overtook him. When playing Monopoly with his family, he modified the traditional rules: rather than allow the players the right to buy properties at the stated price, he invented an auction, in which all players could take part. Thus the properties were sold to the highest bidder, not necessarily the one whose piece landed on the square. This simple device adds a remarkable dimension to the game, and removes an element of chance, replacing it with strategy and skill.

Hotelling's roles in establishing the place of mathematics in economics, in raising the standards of statistical research and in transforming mathematical statistics into an accepted academic discipline were paramount. His contributions to economic theory and to mathematical statistics are described, somewhat briefly, below. His role in establishing statistics as a subject area was equally important and he had a most profound effect upon the way in which statistics is taught. He vigorously sought to improve the standards of statistical teaching, and in 1940 delivered his famous paper on "The Teaching of Statistics" to the Institute of Mathematical Statistics (Hotelling, 1940). This was received with unanimous acclaim by the audience and it had a marked influence upon the profession, contributing to the establishment of some of the first departments of statistics in American universities. On his home ground of Columbia, he continually and consistently attempted to persuade the University to establish a statistics institute, and argued that statistics be seen as a discipline in its own right, as opposed to a service subject to other studies. In seeking to form an Institute of Statistics at Columbia he was frustrated; nevertheless, his grand design was put into practice very soon after his leaving Columbia, and the establishment of statistics as a subject with an independent existence owes significant debt to the seeds sown by Hotelling in the 1930s when he

established the secure foundation of a highly respected graduate school. Incidentally, he was also responsible for Columbia accepting Russian as an appropriate foreign language for doctoral work, a practice then copied by many other universities. However, this surely cannot explain the incident when, in the early 1950s, he applied for a passport renewal and, in order to get the renewal, was forced to sign an affidavit swearing that he had never been a member of the Communist Party.

After the War, Hotelling was attracted away from Columbia to take the role of co-director of the then newly founded Statistical Institute at North Carolina. Although he was not actively seeking a move from Columbia, he was attracted by the opportunity. The director, Gertrude Cox, had asked him what would induce him to move and he, taken with the idea in principle, stated his terms. These included a salary which would match that of the highest paid football coach! Cox consulted with the University president and met his terms in full. Thus, Hotelling accepted the new position. On resigning he was told that Columbia themselves could negotiate with him so that he might stay; however, Hotelling was a man of great principle, and he chose not to enter any such bargaining, saying that he had stated his terms, they had been met and that in consequence he was going. Hotelling's word was binding, and nothing could have shaken him from his commitment to North Carolina. He moved in 1946 and remained there for the rest of his life. He published very little thereafter, but rather devoted himself to the role of establishing the Institute as the premier center of mathematical statistical research. Hotelling drew together a group of statisticians at the Institute, a list of whom reads like a role call of honor. In creating a world center of excellence in mathematical statistical research Hotelling surely had no equal.

Hotelling's academic career covers the period from the early 1920s to the middle 1960s, but his publications cover a much shorter period, which largely coincides with his time at Columbia. Notwithstanding his enormous influence upon statistics and economics via his teaching, his lasting influence is, however, more keenly felt through his publications, many of which are considered as classics. He was never a prolific writer of economics, but what his work lacked in quantity it more than made up in quality. His output of mathematical statistics was, however, prodigious. Most of his contributions to economics are seen as seminal, and his mathematical statistics papers were truly pathbreaking.

In 1925, Hotelling published his first academic papers: one on manifolds (from his doctorate thesis), one on the derivation of the F statistic and one on the theory of depreciation. The latter introduced, for the first time, the definition of depreciation as the decrease in the discounted value of future returns. Of the papers he published while at Stanford, the most important include the paper of 1927 in which he demonstrated that trend projections of population were statistically inappropriate and introduced the estimation of differential equations subject to error. In a paper of 1929 (written jointly with Holbrook Working, then of the Institute), he developed these ideas within the context of an economic problem regarding the interpretation of trends. That same year, he published his famous paper on stability in competition (1929). This paper was essentially an exercise in game theory that introduced the notion of locational equilibrium in a duopolistic structure. Although subject to some corrections (see especially d'Aspremont, Gabszewicz and Thisse, 1979), this paper is the inspiration of much research today, not only in economics, but also in geography and politics.

Of his few economics papers, perhaps the most well known was published in 1931 (Hotelling, 1931a). (See Devarajan and Fisher (1981) for a review and an assessment of this paper.) Hotelling's paper concerns the optimal rate of depletion of an exhaustible resource, and uses the calculus of variations as the major analytical vehicle. The use of such a technique was wholly unfamiliar to economists at the time, and according to Hotelling's own later account, the paper was rejected by the *Economic Journal* because its mathematics were too difficult for that journal's readership. All the recent literature essentially takes Hotelling's model as the starting point. That year he also published his remarkable statistics paper (1931b) that generalizes Student's t statistic to the simultaneous testing of hypotheses concerning the means of variables having a joint normal distribution. This paper also introduced the "confidence interval," although not under that label.

In this period he became very involved with Henry Schultz (of Chicago) who was, then, writing his pioneering text *The Theory and Measurement of Demand*. Hotelling's influence upon the text is immeasurable: in correspondence he developed the theoretical constraints upon the demand equations and suggested that Schultz either use them as constraints in the estimation process (thus reducing the number of parameters and gaining efficiency) or use them as posterior checks on the validity of the underlying economic theory. The proposals of such a strategy were quite new to economists of the 1930s. Hotelling published much of this work in a series of papers (1932, 1933a, 1935): they are notable as a set for identifying the so-called "integrability conditions," highlighting the precise role to be played in economic theory by the second order conditions of optimization problems,

and form the basis for what is seen today as the most important technique of "duality." Indeed, Samuelson (the 1970 Nobel Laureate in economics) wrote in the second edition of his *Foundations of Economic Analysis* that Hotelling's 1932 paper was "the inspiration for this book's foray into duality theory" (1983, page 453).

In this same period, he published two important papers in mathematical statistics; in 1933 he developed the technique of principal components and in 1936 he published a paper that offered a most general approach to the analysis of relations between two sets of variates (1933b, 1936b). This paper, in particular, has been utilized by econometricians in analyzing simultaneous equation systems.

Hotelling's publications of the 1930s culminated in his famous 1938 paper that was his Presidential Address before the Econometric Society. He had very strong social interests that motivated much of his technical economics and this paper, especially, illustrated his genuine concern with general issues of welfare. His position was that of a nondogmatic market socialist and in this work he advocated "marginal cost pricing" as a necessary prerequisite for "Pareto efficiency." The mathematical analysis utilized line integrals as a generalization of consumers' and producers' surplus in the case of many commodities, and through such technical prowess he was able to demonstrate the soundness of his general argument. This argument generated great interest, and Hotelling entered into a debate with Frisch on the principles involved (see Hotelling, 1938, 1939a, 1939b; Frisch, 1939a, 1939b; see also Ruggles, 1949–1950, for an overview of this debate). Hotelling's proposal of marginal cost pricing implied that those industries that enjoyed decreasing average cost would make losses. Typically, such industries would be public utilities that yield great benefit to all sections of the population and, in order to cover such losses, he advocated the taxation of the rich (mainly via lump sum taxes). At the same time he was all too aware of the need for economic efficiency and the need to avoid waste.

His publications number nearly one hundred; only the most important have been identified above. Their lasting influence upon research in economics and statistics is ample testimony to their enormous worth and, in addition to this most impressive achievement, Hotelling taught many of the leading economists and statisticians of the generation that followed. Furthermore, he played the most important role in establishing mathematical statistics as a discipline in its own right.

He retired first in 1960 (when he was 65) but carried on teaching until he was 70. The flow of honors he received is simply too great to list. In 1965 he visited Buenos Aires to help establish a statistics program in the University. He fell ill and was obliged, after an unsuccessful operation, to return to Chapel Hill. He underwent subsequent operations but was never quite the same man again, although he continued to attend departmental seminars and social functions. In May of 1972, during family lunch, he suffered a stroke from which he never recovered. He was nursed at home by his wife with the utmost devotion for 19 months. On December 26, 1973 he died.

ACKNOWLEDGMENTS

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